Solution de la serie N°2

Exercice 1.

P(A) = 0.6, P(B) = 0.7 et P (A U B) = 0.9

$*P(A\ /\ B\ ) = \frac{P(A\cap B\ )}{P(B\ )}$.

$P(A\cap B\ ) =\ P(A) + P(B) - P(A\cup B\ ) = 0.6 + 0.7 - 0.9 = 0.4$ (parce que $P(A\cup B\ ) =\ P(A) + P(B) - P(A\cap B\ )$)

$\Longrightarrow P(A\ /\ B\ ) = \frac{P(A\cap B\ )}{P(B\ )} = \frac{0.4}{0.7} = 0.571\,43.$

$*P(B\ /\ A\ ) = \frac{P(B\cap A\ )}{P(A\ )} = \frac{P(A\cap B\ )}{P(A\ )} = \frac{0.4}{0.6} = 0.666\,67$ (parce que $(A\cap B\ ) = (B\cap A\ )$)

Exercice 2.

C : Être malade (Coroné)
V : Être vacciné

$P(V) = \frac{1}{3}$
$P(V\ /\ C\ ) = \frac{1}{10}$

$P(C) = 0.25$
$*P(C\ /\ V\ ) =?$
$P(C\ /\ V\ ) = \frac{P(C\ \cap\ V)}{P(V)}$
$P(C\ \cap\ V) = P(V\ \cap\ C)$ (parce que $(C\cap V\ ) = (V\cap C\ )$)
$P(V\cap C) =\ P(V\ /\ C\ ) \times P(C) = \frac{1}{10} \times 0.25 = 0.025$
(parce que $P(V\ /\ C\ ) = \frac{(V\ \cap C\ )}{P(C\ )} \Longrightarrow P(V\cap C) =\ P(V\ /\ C\ ) \times P(C)$)
$\Longrightarrow P(C\ /\ V\ ) = \frac{P(C\ \cap\ V)}{P(V)} = \frac{P(V\ \cap C)}{P(V)} = \frac{0.025}{\frac{1}{3}} = 0.075$

Exercice 3.

H_1 : Le vaccin est de type Johnson & Johnson.
H_2 : Le vaccin est de type Spoutnik.
H_3 : Le vaccin est de type Sinovac..

A : Être contaminé par le virus après avoir reçu le vaccin.

$P(H_1) = 0.27,\ P(H_2) = 0.33,\ P(H_3) = 0.40$

$P(A\ /\ H_1) = 0.03,\ P(A\ /\ H_2) = 0.04,\ P(A\ /\ H_3) = 0.05.$

Les conditions d'application (Théorème de Bayes):

$$\begin{cases} H_1 \cup H_2 \cup H_3 = \Omega \\ \vee\ i,j\ H_i \cap H_j = \oslash\ ((H_1 \cap H_2) = \oslash, (H_1 \cap H_3) = \oslash,\ (H_2 \cap H_3) = \oslash\ ) \\ \sum_{i=1}^{3} P(H_i) = 1\ .(P(H_1) + P(H_2) + P(H_3) = 0.27 + 0.33 + 0.40 = 1) \end{cases}$$

$A = (A \cap H_1) \cup (A \cap H_2) \cup (A \cap H_3)$

1- $P(A\ /\ H_2) = 0.04$ (D'après les données)

2- $P(A) = \sum_{i=1}^{3} P(A\ /\ H_i\ ) \times P(H_i) = P(A\ /\ H_1) \times P(H_1\ ) + P(A\ /\ H_2) \times P(H_2\ ) + P(A\ /\ H_3) \times P(H_3\ )$

$= (0.03 \times 0.27) + (0.04 \times 0.33) + (0.05 \times 0.40) = 0.041\,3$

3- $P(H_1\ /\ A\ ) = \frac{P(A\ /\ H_1) \times P(H_1\ )}{P(A)} = \frac{0.03 \times 0.27}{0.041\,3} = 0.196\,13$

Exercice 2.

H_1 : bons risques.
H_2 : les risques moyens.
H_3 : les mauvais risques.

A : Le client a eu un accident.

$P(H_1) = 0.20,\ P(H_2) = 0.50,\ P(H_3) = 0.30$
$P(A\ /\ H_1) = 0.05,\ P(A\ /\ H_2) = 0.15,\ P(A\ /\ H_3) = 0.30.$

Les conditions d'application (Théorème de Bayes):

$$\begin{cases} H_1 \cup H_2 \cup H_3 = \Omega \\ \vee\ i,j\ H_i \cap H_j = \oslash\ ((H_1 \cap H_2) = \oslash, (H_1 \cap H_3) = \oslash,\ (H_2 \cap H_3) = \oslash\ ) \\ \sum_{i=1}^{3} P(H_i) = 1\ .(P(H_1) + P(H_2) + P(H_3) = 0.20 + 0.50 + 0.30 = 1) \end{cases}$$

$A = (A \cap H_1) \cup (A \cap H_2) \cup (A \cap H_3)$

1- $P(A) = \sum_{i=1}^{3} P(A\ /\ H_i\ ) \times P(H_i) = P(A\ /\ H_1) \times P(H_1\ ) + P(A\ /\ H_2) \times P(H_2\ ) + P(A\ /\ H_3) \times P(H_3\ )$

$= (0.05 \times 0.20) + (0.15 \times 0.50) + (0.30 \times 0.30) = 0.175$

2- $P(H_1\ /\ \overline{A}\ ) = \frac{P(\overline{A}\ /\ H_1) \times P(H_1\ )}{P(\overline{A})}$

$P(A) + P(\overline{A}) = 1 \Longrightarrow P(\overline{A}) = 1 - P(A) = 1 - 0.175 = 0.825$

$P(\overline{A} \ / \ H_1) + P(A/ \ H_1) = 1 \Longrightarrow P(\overline{A} \ / \ H_1) = 1 - P(A \ / \ H_1) = 1 - 0.05 = 0.95$

$\Longrightarrow P(H_1 \ / \ \overline{A} \ ) = \frac{P(\overline{A} \ / \ H_1) \times P(H_1 \ )}{P(\overline{A})} = \frac{0.95 \times 0.20}{0.825} = 0.230\,3$