ANALYSIS OF REINFORCED CONCRETE COLUMNS SUBJECTED TO BIAXIAL LOADS

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ABSTRACT. Numerical investigations for the analysis and the design of L-shaped short reinforced concrete columns subjected to combined axial load and bending were undertaken for the purpose of providing design aids for engineers. The use of a computer lends itself naturally to the solution of the problem which generally requires an iterative process. Therefore an attempt has been in this paper to computerize the analysis procedure for L-shaped column sections. This study based on the fibre method present the interaction diagrams with the limit state analysis. Many codes presented design aids only for square/rectangular and circular columns, but in the Algerian code C.B.A.98 no specifications are given. The presented computer analysis results have been compared with existing experimental and numerical data. It is indicated that the results from the proposed analysis correlate well.

Keywords: RC short columns, Biaxial bending, Axial load, Interaction curves.

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INTRODUCTION

The biaxial interaction diagrams of reinforced concrete rectangular columns have been investigated extensively by numerous researchers. Gouwens [1] developed simplified design aids for rectangular columns that are included in many textbooks. For L-shaped columns, Marin [2] presented design aids, Ramamurthy and Khan [3] suggested two methods of design: the *failure surface* and the *equivalent rectangular column* and Hsu [4] presented theoretical and experimental results. Channel and box shaped columns have been reported by Hsu [5] and Dundar [6]. L and T-shaped columns have been also reported by Mallikarjuna and Mahadivappa [7], they presented an interaction diagrams for the design and analysis. Most of the work developed by the aforementioned researchers was carried out using the Whitney's rectangular stress block for the concrete in compression and integral methods to derive the overall equilibrium at the sectional level as it is suggested by current practice Building.

Marin [8], using a second-degree parabola for the concrete's stress-strain relationship, presented closed form expressions for M_{nX}, M_{nY}, and P_n, but his formulation is limited to fully confined concrete, the descending curve of the concrete is not included, and to a perfectly elastoplastic stress-strain relationship for the steel reinforcement. With the availability of powerful and inexpensive micro-computers, the analysis of RC columns and composite sections under biaxial bending and axial load became amenable using the *fiber* approach based on the finite element method (FEM). Brondum Nielsen [9-1o] and Yen [11] presented methods for the analysis of arbitrary cross sections. Mallikarjuna and Mahadivappa [7] proposed an algorithm for analysis and design of L-shaped based on interaction curves for uniaxial and biaxial loading with an angle of 45°, this procedure will be developed in part of this work.

Recently, Barzegar and Erasito [12] developed a computer algorithm for an arbitrary cross section using the Whitney's rectangular stress block and integral methods. The most complete algorithm for the analysis of RC members under biaxial bending including the effects of both prestressed and regular reinforcements is that by Kawakami et al. [13]. More recently a general algorithm for the design and analysis of any shape sections is developed by Rodrigues and Aristizabal-Ochoa [14], they used nonlinear stress-strain relationships: a second-degree parabola and a trapezoidal shape for the concrete, and a multilinear elastoplastic relationship for the reinforcements.

The main objective of the present paper is to present a general approach by which the biaxial interaction diagrams of a square and L shape RC cross sections can be determined with minimum programming. Since the proposed method uses a nonlinear stress-strain relationship for the concrete and a multilinear elastoplastic one for the reinforcement, it can be utilized to study the effects of creep and confinement of the concrete by simply modifying the corresponding input parameters. The algorithm presented can be easily programmed . For this purpose, expressions for any point on the failure surface (M_{nX}-M_{nY}-P_n) are derived.

STRUCTURAL MODEL

Consider a reinforced concrete section with a uniform reinforcement arrangement along the contour as shown in Figure1. All boundaries of the cross section are straight lines, therefore, the contribution of the concrete under compression consists trapezoidal fibres define by the intersection between the section contour and the neutral axis x. The global X, Y axes of the cross section could have their origin chosen to suit the user's needs. The plastic centroid is generally taken as the origin, but Marin [8] claimed that the gross section's geometric centroid is a better choice. The global x, y system is utilized to define the strain variation of both materials, concrete and steel, and also to carry out the necessary integrations in the proposed analysis.

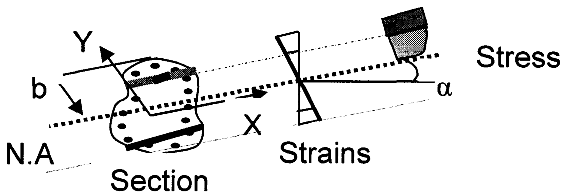

Figure 1 Structural model

Assumptions

The proposed method is based on the following three assumptions:

1. A linear strain distribution across the irregular cross section (Bernoulli's assumption that plane sections remain plane after loading), as shown in Figure 1 and indicated by:

$$\varepsilon_c = \varepsilon\ (y/c) \tag{1}$$

2. The stress-strain curve of the concrete under compression, Figure 2(a).

for $\varepsilon_c / \varepsilon_o \leq 1$

$$f_c = f_c'\ y/c\ \varepsilon/\varepsilon_o(2 - y/c\ \varepsilon/\varepsilon_o) \tag{2a}$$

for $\varepsilon_c / \varepsilon_o > 1$

$$f_c = f_c' \tag{2b}$$

This combination allows for the modeling of creep by simply varying the parameter ε_o. The concrete tensile strength is neglected.

3. A multilinear elastoplastic stress-strain relationship for the reinforcement, both in tension and in compression as shown in Figure 2(b).

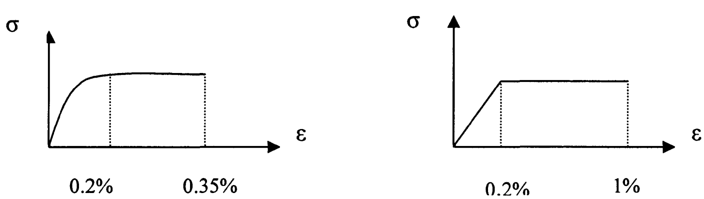

Figure 2 Uniaxial Constitutive Material Properties: (a) Stress-Strain Curve of Concrete under Compression; (b) Multilinear Elastic-Plastic Stress-Strain Relationship for Steel

Expressions for P_n, M_{nX}, and M_{nY}

$$P_n = \Sigma_n P_{ci} + \Sigma_n (f_{si} - f_{ci}) A_{si} \quad (3a)$$
$$M_{nX} = \text{Sin}\alpha\, \Sigma_n M_{ciY} + \text{Cos}\alpha\, \Sigma_n M_{ciX} + Y_a \Sigma_n P_{ci} + \Sigma_n (f_{si} - f_{ci}) A_{si} y_{si} \quad (3b)$$
$$M_{nY} = \text{Cos}\alpha\, \Sigma_n M_{ciY} + \text{Sin}\alpha\, \Sigma_n M_{ciX} + X_a \Sigma_n P_{ci} + \Sigma_n (f_{si} - f_{ci}) A_{si} y_{si} \quad (3c)$$

where P_n, M_{nX}, and M_{nY} are nominal total axial force strength and nominal bending moment strengths about the global X and Y axes, respectively. Notice that P_{ci}, M_{ciX}, and M_{ciY} represent the contribution of trapezoid i, Figure 1, to the axial force and bending moments, they can be calculated using (*4*a)–(*4*c), as follows:

$$dP_{ci} = \iint f\, dxdy \quad (4a)$$
$$dM_{cix} = \iint f_c\, y\, dx\, dy \quad (4b)$$
$$dM_{ciy} = \iint f_c\, x\, dx\, dy \quad (4c)$$

The nominal bending moment strength about the neutral axis (x,y) can be derived as:

$$M_n = \Sigma_n M_{ci} + \Sigma_n (f_{si} - f_{ci}) A_{bi} y_{si} \quad (5)$$

where P_{ci}, A_{bi}, and M_{ci} are the contribution of the fiber i about the neutral axis: $P_{ci} = A_{ci} f_{ci}$, $A_{bi} = \rho_{si} A_{si} /100$, and $M_{ci} = P_{ci}\ y_{ci}$

GENERATION OF FAILURE SURFACE

There are three different techniques available to determine the biaxial strength of an arbitrary RC cross section:

(1) interaction curves for a given bending moments ratio, $M_{nx} / M_{ny} = \tan\theta$;
(2) load contours for a given axial load P;
(3) isogonics or three-dimensional curves as proposed by Marin [2].

To determine the biaxial strength by the first two methods, the location of the neutral axis must be found. To accomplish this, (*3*a) and (*3*b) must be solved simultaneously for α and b (position of N.A). These are nonlinear equations with several parameters all coupled together, requiring an iterative approach such as the quasi-Newton method suggested by Yen [11]. The isogonics or 3D curve technique is direct, since the neutral axis (i.e., α and b) is assumed from the very start, requiring only the generation of points of the failure surface as given explicitly by (*3*a)–(*3*c). The technique of interaction curves permits one to find the nominal axial strength P_n of a particular short column if the eccentricities (e_X and e_Y), the boundaries of the cross section, and the reinforcement distribution are given. The nominal axial strength can be determined by making $M_{nX} = P_n e_Y$ and $M_{nY} = P_n e_X$ in (*3*b) and (*3*c), respectively, and then solving for P_n, α and b. From this solution, the designer is able to determine the interaction curves for each particular moments ratio or θ, $M_{nx} / M_{ny} = \tan\theta$. This technique generates plane interaction curves for columns under biaxial bending, which are much easier to plot and use in design. This technique consists of solving (*3*a) and (*3*b) for M_{nY}, α and *b* for different values of the axial load P_n that is varied from:

$$P_{max} = 0.85 f_c' (A_g - A_s) + f_y A_{st} \text{ to}$$
$$P_{min} = -f_y A_{st}$$

and for a given ratio of applied moments or a predetermined value of θ.

The technique adopted is an indirect method, with a given angle α, we vary the position of the neutral axis from y = D_c (the cover), the maximum axial tensile capacity, to y = 2.5B (B is the dimension of the section), the maximum axial compressive capacity, and for each position of the neutral axis, the compression zone is divided into fibers of a width equal unity, then P_n is derived (3a) and M_n (eq.5) from the gross section's geometric centroid. For each position the couple (P_n, M_n) gives a point of the curve. If we choose a range of reinforcement ratio from the minimum code requirement ρ_{smin} to the maximum code requirement ρ_{smax}, then we obtain a range of curves. This is a practical method because we have any problem of convergence and if the designer desires to check a given design, only a single interaction curve needs to be determined and plotted, the axial load versus the total moment; P_n - M_n or $M_{ny}(1 +\sqrt{(1+tn^2\,\theta)}]$. If it is an analysis problem, the point defined by the applied loading must fall inside the obtained interaction curve. On the other hand, if it is a design problem, such as to find the reinforcement required, the position of the point gives the curve and then the ratio.

EXPERIMENTAL RESULTS

The main objective of this validation is to compare the results obtained by the proposed algorithm to experimental data. For each specimen we plot the correspondent interaction curve, then we estimate the limit axial load (P) and we compare with the experimental data (P_{test}).

Table 1 Uniaxial loading for square sections, Hsu [15]

AUTHORS	SPECIMEN	B, mm	A_s, %	e_x, mm	P_{test}, kN	P, kN	FAILURE	P_{test} / P
Hognestad	A15a	254	4.80	317	391.44	386.06	T	1.01
Hsu and Hudson	HS1	101.6	2.75	127	286.68	270.32	T	1.06
	HS2	101.6	2.75	76.2	526.78	600.59	T	0.88
	N11	152.4	4.44	50.8	320.27	305.57	C	1.04
	N12	152.4	4.44	50.8	355.85	334.13	C	1.06
	N41	152.4	6.89	50.8	444.82	420.29	C	1.05
	N42	152.4	6.89	50.8	471.50	480.76	C	0.98
Heindahl and Bianchini	AR1	127	3.2	26.2	346.95	361.29	C	0.96
	AR2	127	3.2	26.9	362.52	361.81	C	1.00
	AR3	127	3.2	71.12	188.16	220.86	C	0.85
	AR4	127	3.2	69.2	204.62	220.86	C	0.93
	AR5	127	3.2	133.4	104.98	107.59	T	0.97
	AR6	127	3.2	134	105.42	107.59	T	0.98
	DR1	127	3.2	69.3	189.49	146.90	C	1.29

The results obtained in Table1, for the uniaxial loading give:

• The same failure modes by tension (T) or compression (C) for all the specimens,

• The axial strength with an error less than 6% :

+1% from Hognestad, +4.6% from Hsu, +2% from Hudson, and +6% from Bianchini and Al.

The results obtained in Table 2, for a biaxial loading show:

• The same failure modes by tension (T) or compression (C) for all the specimens,

• The axial strength with errors less than 6% thus:

+4% form Hsu, +1% from Ramamurthy, and +1.6% from Bianchini and Al.

Table 2 Biaxial loading for square sections, Hsu [15]

AUTHORS	SPECIMEN	B, mm	A_s, %	e_x, mm	e_y, mm	ANGLE	P_{test}, kN	P, kN	FAILURE	P_{test} / P
Anderson and Lee	SC4	101.6	5	71.6	71.6	45	60.05	53.03	T	1.13
Hsu	S1	101.6	2.75	25.4	38.1	33.7	93.41	103.3	C	0.9
	U1	101.6	2.81	63.5	90	35.2	42.70	56.98	T	0.75
	U2	101.6	2.81	76.2	90	40.2	38.70	50.22	T	0.8
	U3	101.6	2.81	90	90	45	35.58	41.65	T	0.85
	U4	101.6	2.81	51	51	45	63.61	62.51	C	1.02
	U6	101.6	2.81	12.7	178	29.2	27.76	35.39	T	0.78
	H1	108	4.87	76.2	50.8	56.3	61.83	53.90	T	1.14
	H2	108	4.87	82.6	57.2	55.3	52.49	61.64	T	0.85
	H3	108	4.87	63.5	76.2	40	60.49	60.48	T	1.00
Ramamurthy	B2	203.2	3.88	19.4	47	22.4	77.18	76.53	C	1.00
	B3	203.2	3.88	50.8	88	30	53.38	52.46	T	1.02
	B4	203.2	3.88	64	110	30	39.59	39.20	T	1.01
	B5	203.2	3.88	36	36	45	59.83	56.22	C	1.06
	B6	203.2	3.88	64.7	64.7	45	500.42	503.48	C	0.99
Heindahl and Bianchini	BR1	127	3.2	10.4	25.06	22.5	324.72	33.94	C	0.95
	BR2	127	3.2	10.3	25	22.5	342.51	339.37	C	1.01
	BR3	127	3.2	66.6	27.6	67.5	169.03	158.53	C	1.06
	BR4	127	3.2	66.6	27.6	67.5	159.24	158.53	C	1.00
	BR5	127	3.2	124	51.4	67.5	84.96	78.01	T	1.08
	BR6	127	3.2	128	53	67.5	78.29	78.01	T	1.00
	ER1	127	3.2	36.6	26.34	54.3	187.27	174.19	C	1.07
	FR1	127	3.2	48.7	48.71	45	169.92	175.82	C	0.96

THEORETICAL RESULTS

The results obtained by means of the proposed algorithm are compared with those obtained by the professional package PCACOL. For this purpose we choose a range of sections with B_l/B varies from 0.0 to 0.6 and percentages from 0.8% to 5%. The PCACOL gives some characteristics points: pure compression, pure flexure, balanced point, compression failure and tension failure. The comparison shows good agreement between interaction curves proposed (straight line) and the PCACOL points, Figures 6-7 for uniaxial (θ= 0) and biaxial loadings.

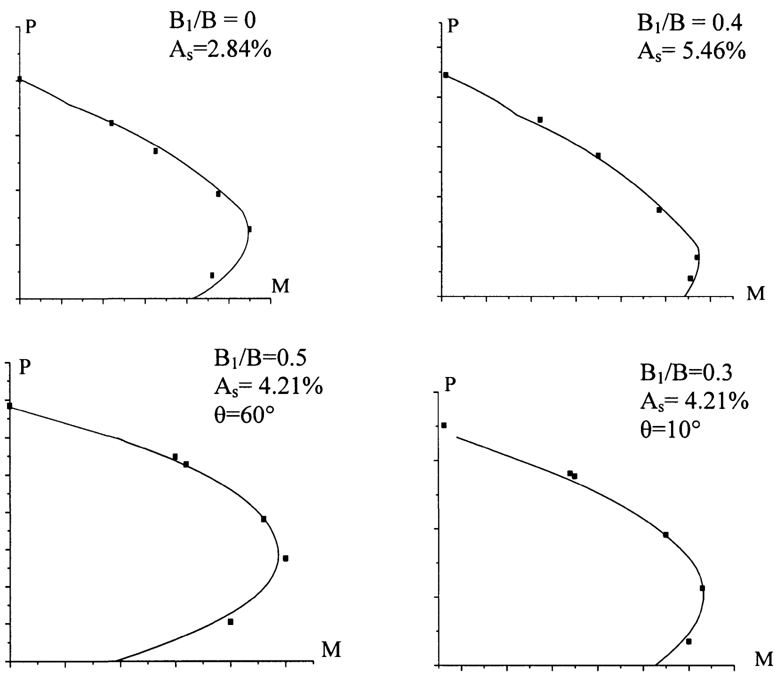

Figure 3 Comparison with PCACOL results for Biaxial loading

CONCLUSIONS

An analytical model by which the theoretical strength of a short column of a square an L shape cross section subjected to axial force compression and biaxial moments can be determined is presented. The proposed algorithm allows the designer to (1) determine the interaction curves of a column with a given cross section for design or analyse. Numerical results indicate that the proposed algorithm is in good agreement with theoretical and experimental data. The effects of creep can be investigated by varying the value of ε_o, the curve became more opened, particularly in the region of balanced point.

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