Pour I_1, on a :

$$I_1 = \frac{\partial}{\partial h}\frac{1}{2} f^{(1)}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right) =$$

$$\frac{1}{2}\left[\frac{\partial f^{(1)}}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right)\cdot\frac{2}{3} + \frac{\partial f^{(1)}}{\partial y}\left(t+\frac{2h}{3},\right.\right.$$

$$\left.y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right) \times \frac{2}{3} \frac{\partial}{\partial h}\left(h f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right)$$

$$\frac{1}{3}\left[\cancel{\frac{2}{2}}\frac{\partial f^{(1)}}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right) + \frac{\partial f^{(1)}}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\right.\right.\right.$$

$$\left.\left.\frac{h}{3} f(t,y)\right)\right)\cancel{\frac{2}{2}}\left(1\cdot f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right) + h\left(\frac{\partial f}{\partial t}\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\cdot\frac{1}{3} + \right.\right.$$

$$\left.\left.\frac{\partial f}{\partial y}\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\cdot f(t,y)\frac{1}{3}\right)\right).$$

$$= \frac{1}{3}\left(\frac{\partial f^{(1)}}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right) + \frac{\partial f^{(1)}}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3} f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right)\right.$$

$$\times\left(f\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right) + h\left(\frac{\partial f}{\partial t}\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\times\frac{1}{3} + \frac{\partial f}{\partial y}\left(t+\frac{h}{3}, y+\frac{h}{3} f(t,y)\right)\right.\right.$$

$$\left.\left.\left.\times\frac{1}{3} f(t,y)\right)\right)\right)$$

Donc $\frac{\partial}{\partial h} I_1(t,y,0) = \frac{1}{3}\left(\frac{\partial f^{(1)}}{\partial t}(t,y) + \frac{\partial f^{(1)}}{\partial y}(t,y) f(t,y)\right)$

$$= \frac{1}{3} f^{(2)}(t,y)$$

Donc pour que le schéma (S3) soit exactement d'ordre (3)