De plus $f(t_{n+1}, y(t_{n+1})) = y'(t_{n+1})$.

$$y'(t_{n+1}) = y'(t_n + h) = y'(t_n) + h y''(t_n) + \frac{h^2}{2} y^{(3)}(t_n) + \frac{h^3}{3!} y^{(4)}(t_n) + O_3(h^4).$$

donc $\tau_n = y(t_{n+1}) + \alpha_0 y(t_n) + \alpha_1 y(t_{n-1}) - h\beta f(t_{n+1}, y(t_{n+1}))$

$$= y(t_n) + h y'(t_n) + \frac{h^2}{2} y''(t_n) + \frac{h^3}{3!} y^{(3)}(t_n) + O_1(h^4) +$$

$$\alpha_0 y(t_n) + \alpha_1 \left(y(t_n) - h y'(t_n) + \frac{h^2}{2} y''(t_n) - \frac{h^3}{3!} y^{(3)}(t_n) + O_2(h^4)\right)$$

$$- h\beta \left(y'(t_n) + h y''(t_n) + \frac{h^2}{2} y^{(3)}(t_n) + \frac{h^3}{3!} y^{(4)}(t_n) + O_3(h^4)\right)$$

$$= y(t_n)(1 + \alpha_0 + \alpha_1) + y'(t_n)(h - \alpha_1 h - \beta h) + y''(t_n)\left(\frac{h^2}{2} + \alpha_1 \frac{h^2}{2} - h^2\beta\right) + y^{(3)}(t_n)\left(\frac{h^3}{3!} - \alpha_1 \frac{h^3}{3!} - \beta \frac{h^3}{2}\right) + O(h^4)$$

$$\tau_n = O(h^3) \iff \begin{cases} 1 + \alpha_0 + \alpha_1 = 0 \iff \alpha_0 - \alpha_1 = -1 & — (1) \\ 1 - \alpha_1 - \beta = 0 \quad \alpha_1 + \beta = 1 & — (2) \\ \frac{1}{2} + \frac{\alpha_1}{2} - \beta = 0 \quad -\alpha_1 + 2\beta = 1. & — (3) \end{cases}$$

$(2)+(3) \iff 3\beta = 2 \iff \beta = \frac{2}{3}$, $(2) \iff \alpha_1 = 1 - \beta = 1 - \frac{2}{3} = \frac{1}{3}$

$(1) \iff \alpha_0 = +1 + \alpha_1 = +1 + \frac{1}{3} = \frac{4}{3}$.

Donc le schéma est $y_{n+1} + \frac{4}{3} y_n + \frac{1}{3} y_{n-1} = \frac{2h}{3} f_{n+1}$