D'après Q3) on a :

$$\frac{\partial}{\partial h}\phi(t,y,h) = \frac{2b}{3}\left[\frac{\partial f}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) + \right.$$

$$\left.\frac{\partial f}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) \times f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right]$$

Comme $b = \frac{3}{4}$, on trouve

$$\frac{\partial}{\partial h}\phi(t,y,h) = \frac{1}{2}\left[\frac{\partial f}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) + \right.$$

$$\left.\frac{\partial f}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) \times f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right]$$

$$= \frac{1}{2}\left[\frac{\partial f}{\partial t}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) + \frac{\partial f}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\right.$$

$$\left.\times f\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\right] +$$

$$\frac{1}{2}\frac{\partial f}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\left[f\left(t+\frac{h}{3}, y+hf(t,y)\right) - \right.$$

$$\left. f\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\right] =$$

$$\frac{1}{2}f^{(1)}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right) +$$

$$\frac{1}{2}\frac{\partial f}{\partial y}\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\left[f\left(t+\frac{h}{3}, y+hf(t,y)\right) - \right.$$

$$\left. f\left(t+\frac{2h}{3}, y+\frac{2h}{3}f\left(t+\frac{h}{3}, y+\frac{h}{3}f(t,y)\right)\right)\right] = I_1 + I_2$$