

Corrigé Examen Final TS

Exo 1:

1°/ Classification phénoménologique:

Signal déterministe périodique, car le spectre est discret,
sa fréquence = $\frac{3}{2}$ Hz (D'après le spectre) donc sa période = $\frac{2}{3}$ s.

Classification morphologique:

Signal Analogique, car le temps est continu et l'amplitude aussi.

2°/ Les coefficients de Fourier:

$A_0 = 2$ $A_1 = \frac{4}{\pi}$ $A_3 = \frac{4}{3\pi}$ $A_7 = \frac{4}{7\pi}$

$\varphi_0 = \pi$ $\varphi_1 = \varphi_3 = \varphi_7 = -\frac{\pi}{2}$

$f = \frac{3}{2}$ Hz $T = \frac{2}{3}$ s

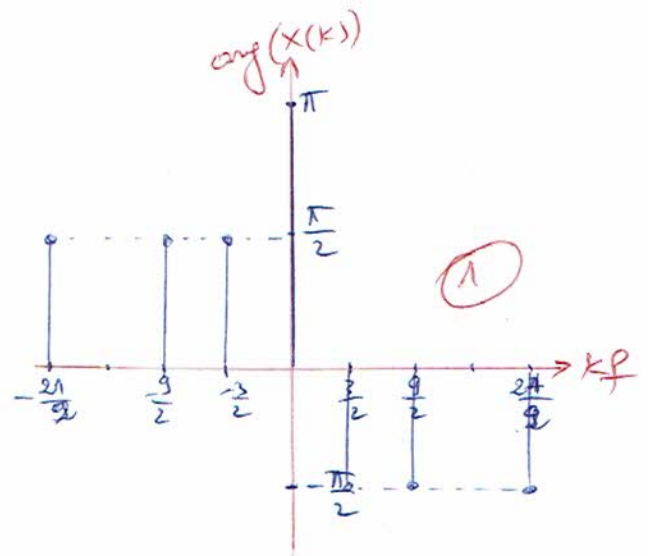
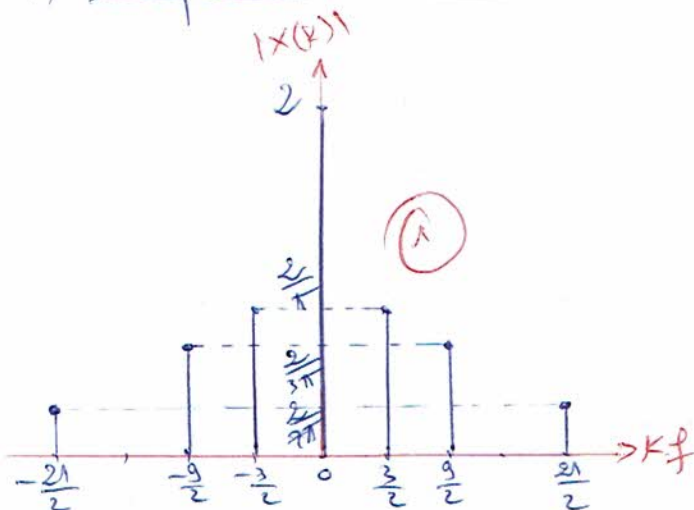
3°/ Les coefficients Complexes de Fourier:

$$\begin{cases} X(k) = \frac{A_k}{2} e^{j\varphi_k} & : k > 0 \\ X(k) = \frac{A_k}{2} e^{-j\varphi_k} & : k < 0 \end{cases}$$

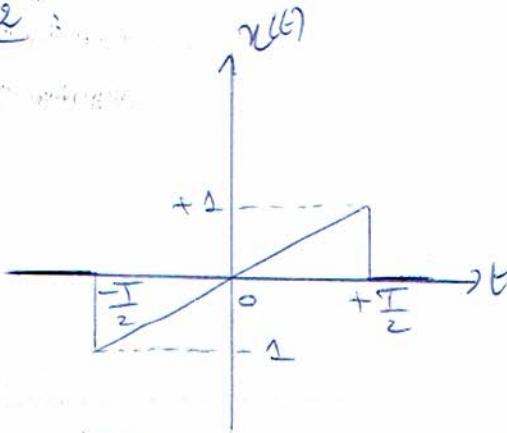
$\rightarrow X(0) = 2 e^{j\pi}$
 $X(1) = \frac{2}{\pi} e^{-j\frac{\pi}{2}}$
 $X(3) = \frac{2}{3\pi} e^{-j\frac{\pi}{2}}$
 $X(7) = \frac{2}{7\pi} e^{-j\frac{\pi}{2}}$

$X(-1) = \frac{2}{\pi} e^{+j\frac{\pi}{2}}$
 $X(-3) = \frac{2}{3\pi} e^{+j\frac{\pi}{2}}$
 $X(-7) = \frac{2}{7\pi} e^{+j\frac{\pi}{2}}$

4°/ les spectres bilatéraux:



Exo 2:



$$x(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt = \int_{-T/2}^{+T/2} \frac{2}{T} t e^{-j2\pi f t} dt \quad (0.5)$$

$$u(t) = t \Rightarrow u'(t) = 1$$

$$v'(t) = e^{-j2\pi f t} \Rightarrow v(t) = \frac{e^{-j2\pi f t}}{-j2\pi f}$$

$$\Rightarrow x(f) = \frac{2}{T} \left[t \frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-T/2}^{+T/2} - \frac{2}{T} \int_{-T/2}^{+T/2} \frac{e^{-j2\pi f t}}{-j2\pi f} dt$$

$$= \frac{2}{T} \left[\frac{T}{2} \frac{e^{-j\pi f T}}{(-j2\pi f)} + \frac{T}{2} \frac{e^{j\pi f T}}{(-j2\pi f)} \right] - \frac{2}{T} \frac{e^{-j\pi f T} - e^{j\pi f T}}{(-j2\pi f)^2}$$

$$= -\frac{1}{j\pi f} \frac{e^{j\pi f T} + e^{-j\pi f T}}{2} + \frac{2}{T} \frac{e^{-j\pi f T} - e^{j\pi f T}}{4\pi^2 f^2}$$

$$= \frac{j}{\pi f} \cos(\pi f T) - \frac{j}{T\pi^2 f^2} \sin(\pi f T) = \frac{j}{\pi f} [\cos(\pi f T) - \sin(\pi f T)]$$

Exo 3:

$$1 \Rightarrow H(p) = \frac{Y(p)}{X(p)} \Rightarrow Y(p) = H(p) \cdot X(p) \quad X(p) = 1 \Rightarrow Y(p) = \frac{p}{p^2 - p - 2}$$

$$x(f) \Rightarrow Y(p) = \frac{p}{(p+1)(p-2)} = \frac{A}{p+1} + \frac{B}{p-2} \quad (0.5)$$

$$A = \lim_{p \rightarrow -1} \left[(p+1) \frac{p}{(p+1)(p-2)} \right] = \frac{1}{3} \quad (1)$$

$$\Rightarrow Y(p) = \frac{1/3}{p+1} + \frac{2/3}{p-2} \quad (0.5)$$

$$B = \lim_{p \rightarrow 2} \left[(p-2) \frac{p}{(p+1)(p-2)} \right] = \frac{2}{3} \quad (1)$$

$$\Rightarrow y(t) = \frac{1}{3} e^{-t} + \frac{2}{3} e^{2t} \quad (1)$$