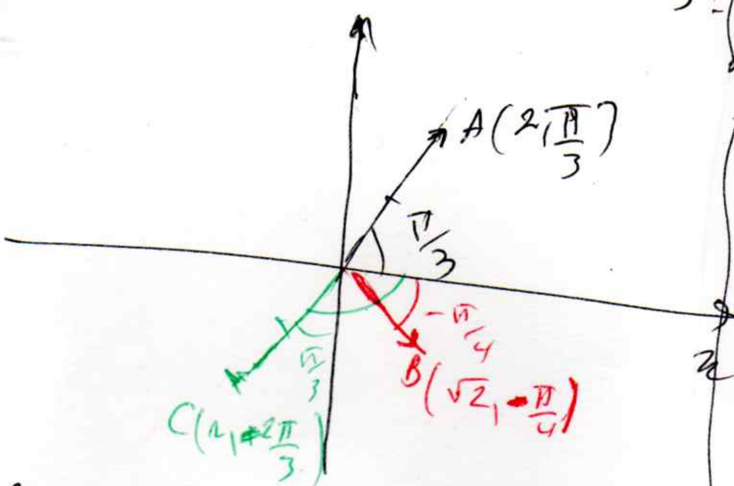
Exo3 :

$A(2, \frac{\pi}{3})$, $B(\sqrt{2}, -\frac{\pi}{4})$, $C(2, -\frac{2\pi}{3})$



Les coordonnées cartésiennes :

$A \begin{cases} r = 2 \\ \theta = \frac{\pi}{3} \end{cases} \Rightarrow \begin{array}{l} x = r\cos\theta = 2\cos\frac{\pi}{3} = 1 \\ y = r\sin\theta = 2\sin\frac{\pi}{3} = \sqrt{3} \end{array}$

$(\cos\frac{\pi}{3} = \frac{1}{2}, \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2})$

$A(1, \sqrt{3})$

$B \begin{cases} r = \sqrt{2} \\ \theta = -\frac{\pi}{4} \end{cases} \begin{cases} x = \sqrt{2}\cos(-\frac{\pi}{4}) = 1 \\ y = \sqrt{2}\sin(-\frac{\pi}{4}) = -1 \end{cases}$

$B(1, -1)$

$C(2, -\frac{2\pi}{3}) \begin{cases} x = 2\cos(-\frac{2\pi}{3}) = -1 \\ y = 2\sin(-\frac{2\pi}{3}) = \end{cases}$

$\cos(\frac{2\pi}{3}) = \cos(-\frac{\pi}{3})$

$x = 2\cos(-\frac{2\pi}{3}) = 2\times(-\frac{1}{2}) = -1$

$\sin(-2\frac{\pi}{3}) = \sin(-\frac{\pi}{3})$

$y = 2\sin(-\frac{2\pi}{3}) = 2\times(-\frac{\sqrt{3}}{2}) = \boxed{-\sqrt{3}}$

$C(-1, -\sqrt{3})$

Exo4

$M_1(1,1,1)$, $M_2(2,2,1)$, $M_3(2,1,0)$

L'angle formé par les vecteurs $\overrightarrow{M_2M_1}$ et $\overrightarrow{M_2M_3}$

$\overrightarrow{M_2M_1}\begin{pmatrix} 1-2 \\ 1-2 \\ 1-1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow |\overrightarrow{M_2M_1}| = \sqrt{2}$

$\overrightarrow{M_2M_3} = \begin{pmatrix} 2-2 \\ 1-2 \\ 0-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \Rightarrow |\overrightarrow{M_2M_3}| = \sqrt{2}$

$\overrightarrow{M_2M_1} \cdot \overrightarrow{M_2M_3} = |\overrightarrow{M_2M_1}| |\overrightarrow{M_2M_3}| \cos\beta$

$\cos\beta = \frac{\overrightarrow{M_2M_1} \cdot \overrightarrow{M_2M_3}}{|\overrightarrow{M_2M_1}| |\overrightarrow{M_2M_3}|}$

$\overrightarrow{M_2M_1} \cdot \overrightarrow{M_2M_3} = (-1)\times(0) + (-1)\times(-1) + 0\times(-1) = 1$

Donc : $\cos\beta = \frac{2}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \beta = 60°$

b) $\vec{i}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{j}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\vec{k}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

- $\vec{i} \wedge \vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0\vec{i} - 0\vec{j} + \vec{k}$

$\Rightarrow \vec{i} \wedge \vec{j} = \vec{k}$

- $\vec{j} \wedge \vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} - 0\vec{j} + 0\vec{k}$

$\vec{i} \wedge \vec{j} = \vec{k}$