

2021-2022

Physique 2 / TD 01

Exercice 1 :

$$T(x, y) = x^2 + y^2$$

$$\vec{v} = \langle 2xy; x^2; x^3 + y^3 \rangle$$

$$\begin{aligned} \text{grad } T &= \frac{\partial T}{\partial x} \vec{i} + \frac{\partial T}{\partial y} \vec{j} + \frac{\partial T}{\partial z} \vec{k} \\ &= 2x \vec{i} + 2y \vec{j} \end{aligned}$$

$$\begin{aligned} \text{div } \vec{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\ &= 2y + 0 + 0 = 2y \end{aligned}$$

$$\text{rot } \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} =$$

$$\begin{aligned} & \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \vec{i} - \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} \right) \vec{j} + \\ & \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \vec{k} \end{aligned}$$

$$\begin{aligned} & (3y^2 - 0) \vec{i} - (3x^2 - 0) \vec{j} + (2x - 2x) \vec{k} \\ & = 3y^2 \vec{i} - 3x^2 \vec{j} \end{aligned}$$

Exo 2 :

les dérivées partielles d'ordre 2
des fonctions suivantes :

1) $f(x, y) = x^2(x+y)$
 $= x^3 + yx^2$

2) $f(x, y) = \cos(xy)$

les dérivées partielles d'ordre 1

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (x^3 + yx^2)$$

$$\frac{\partial f}{\partial x} = 3x^2 + 2yx$$

$$\frac{\partial f}{\partial y} = x^2$$

les dérivées partielles d'ordre 2

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial x} (3x^2 + 2yx) \\ &= 6x + 2y \end{aligned}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} (x^2) = 0$$

$$\begin{aligned} \frac{\partial^2 f}{\partial y \partial x} &= \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} (3x^2 + 2yx) \\ &= 2x \end{aligned}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} (x^2) = 2x$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

1) $f(x, y) = \cos(xy)$

les dérivées partielles d'ordre 1

$$\frac{\partial f}{\partial x} = -y \sin(xy)$$

$$\frac{\partial f}{\partial y} = -x \sin(xy)$$