

$$= \frac{3 + i\sqrt{3}}{-2 + i2\sqrt{3} + 4 + 2 + i2\sqrt{3}}$$

$$= \frac{3 + i\sqrt{3}}{4 + i4\sqrt{3}} = \frac{1}{4} \left(\frac{3 + i\sqrt{3}}{1 + i\sqrt{3}} \right)$$

$$= \frac{1}{4} \frac{3 + i\sqrt{3}}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$$

$$= \frac{1}{4} \frac{3 - i3\sqrt{3} + i\sqrt{3} + 3}{4}$$

$$= \frac{1}{16} (6 - i2\sqrt{3})$$

Donc $e^{i\frac{\pi}{3}} = \frac{z^3 + 8}{z^4 + 4z^2 + 16} = \frac{1}{16} (6 - i2\sqrt{3})$

Exercice 2

soit $z \in \mathbb{C}^*$:

$$f(z) = \frac{1}{2} \left(z + \frac{1}{z} \right)$$

$$= \frac{1}{2} \left(z + \frac{\bar{z}}{z\bar{z}} \right)$$

$$= \frac{1}{2} \left(z + \frac{\bar{z}}{r^2} \right)$$