

$$f'(z) = \frac{-1}{(z-1)^2} \left(\frac{1}{2(1+\frac{z-1}{2})} \right)'$$

$$= \frac{-1}{(z-1)^2} \left[\frac{1}{2} \sum_{n \geq 0} (-1)^n \left(\frac{z-1}{2} \right)^n \right]'$$

$$= \frac{-1}{(z-1)^2} \cdot \frac{1}{2} \sum_{n \geq 0} (-1)^n n \frac{1}{2} \left(\frac{z-1}{2} \right)^{n-1}$$

$$= \sum_{n \geq 0} (-1)^{n+1} \frac{n}{(z-1)^2} \cdot \frac{(z-1)^{n-1}}{2^{n-1+2}}$$

$$= \sum_{n \geq 0} (-1)^{n+1} \frac{n}{(z-1)^2} \frac{(z-1)^{n-1}}{2^{n+1}}$$

$$= \sum_{n \geq 0} (-1)^{n+1} n \frac{1}{2^{n+1}} \cdot (z-1)^{n-3}$$

$$= \frac{1}{4} (z-1)^{-2} - \frac{2}{2^3} (z-1)^{-1} + \frac{3}{2^4} (z-1)^0$$

$$+ \frac{-4}{2^5} (z-1)^1 + \dots$$