Equation de diffusion (chaleur) 2D instationnaire

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Détermination de la temperature $T(x, y, t)$ travers une plaque dont les extrémités sont maintenues des températures constantes.

$$\frac{\partial}{\partial t} T(x, y, t) = \alpha \frac{\partial^2}{\partial x^2} T(x, y, t) + \alpha \frac{\partial^2}{\partial y^2} T(x, y, t)$$

Conditions aux limites et initiale:

$$T(0, y, t) = \alpha 1 = 0,$$
$$T(1, y, t) = \beta 1 = 0,$$
$$T(x, 0, t) = \alpha 2 = 0,$$
$$T(x, 1, t) = \beta 2 = 0,$$
$$T(x, y, 0) = \sigma = 1$$

Solution discrétisée:

```
> Restart:
> Δx := 0.2 ; Δy := 0.2 ; Δt := 0.005; α := 1.0/80
```

$$\Delta x := 0.2$$
$$\Delta y := 0.2$$
$$\Delta t := 0.005$$
$$\alpha := 0.01250000000$$

```
> λ1 := α·Δt/Δx^2; λ2 := α·Δt/Δy^2;
```

$$\lambda_1 := 0.001562500000$$
$$\lambda_2 := 0.001562500000$$

```
> imax := 11; jmax := 11;
```

$$i_{max} := 11$$
$$j_{max} := 11$$

```
> nmax := 15;
```

$$n_{max} := 15$$

```
> α1 := 0; β1 := 0; α2 := 0; β2 := 0; σ := 1;
```

$$\alpha 1 := 0$$
$$\beta 1 := 0$$
$$\alpha 2 := 0$$
$$\beta 2 := 0$$
$$\sigma := 1$$

Condition Initiale:

```
> for i from 2 to imax − 1 do
      for j from 2 to jmax − 1 do
          T[i, j, 0] := σ
      end do
  end do;
```

Conditions aux limites:

```
> for n from 0 to nmax do
      for i from 1 to imax do
          T[i, 1, n] := α2;
          T[i, jmax, n] := β2;
      end do;

      for j from 1 to jmax do
          T[1, j, n] := α1;
          T[imax, j, n] := β1;
      end do
  end do;
```

Schéma explicite:

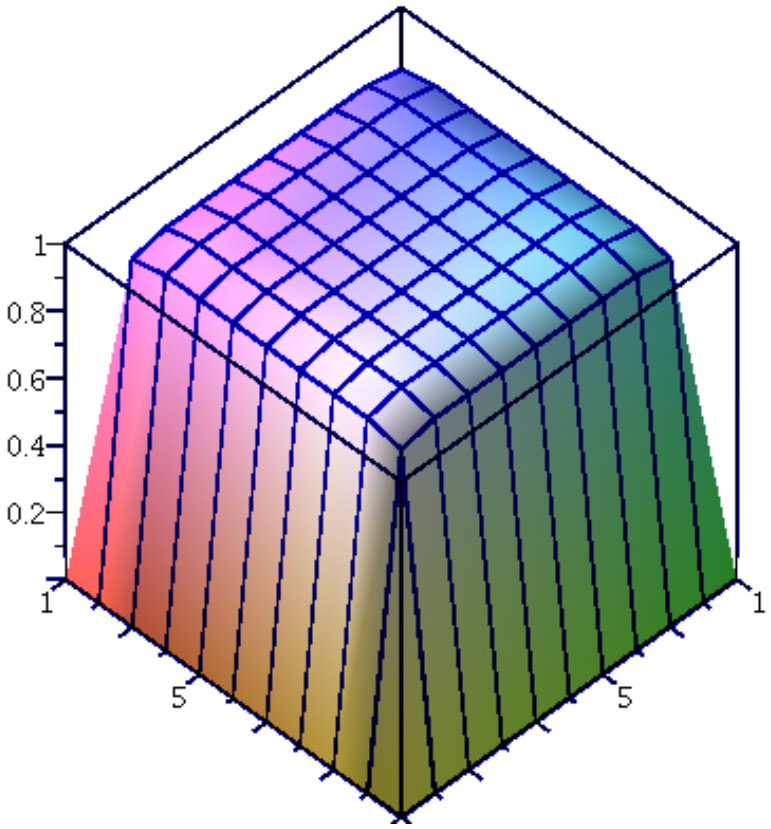
```
> for n from 0 to nmax do
      for j from 2 to jmax − 1 do
          for i from 2 to imax − 1 do
          T[i, j, n + 1] := (1 − 2·λ1 − 2·λ2)·T[i, j, n] + λ1·(T[i − 1, j, n] + T[i
      + 1, j, n]) + λ2·(T[i, j − 1, n] + T[i, j + 1, n]);
          end do;
```

```
    end do;
  end do;
```

```
> for i from 2 to i_max - 1 do
      for j from 2 to j_max - 1 do
          T[i, j, n_max];
      end do
  end do;
```

Tracé des contours de températures:

```
> with(plots):
> listplot3d([seq([seq(T[i, j, n_max], i = 1 .. i_max)], j = 1 .. j_max)])
```



Création d'une liste de points:

```
> Liste := [seq([seq(T[i, j, n_max], i = 1 .. i_max)], j = 1 .. j_max)]
```

Liste := [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0.9546256373, 0.9768220970, 0.9770649732, 0.9770666219, 0.9770666294, 0.9770666219, 0.9770649732, 0.9768220970, 0.9546256373, 0], [0, 0.9768220970, 0.9995010117, 0.9997487953, 0.9997504745, 0.9997504825, 0.9997504745, 0.9997487953, 0.9995010117, 0.9768220970, 0], [0,

```
0.9770649732, 0.9997487953, 0.9999966249, 0.9999983046,
0.9999983127, 0.9999983046, 0.9999966249, 0.9997487953,
0.9770649732, 0], [0, 0.9770666219, 0.9997504745, 0.9999983046,
0.9999999842, 0.9999999920, 0.9999999842, 0.9999983046,
0.9997504745, 0.9770666219, 0], [0, 0.9770666294, 0.9997504825,
0.9999983127, 0.9999999920, 1.000000000, 0.9999999920, 0.9999983127,
0.9997504825, 0.9770666294, 0], [0, 0.9770666219, 0.9997504745,
0.9999983046, 0.9999999842, 0.9999999920, 0.9999999842,
0.9999983046, 0.9997504745, 0.9770666219, 0], [0, 0.9770649732,
0.9997487953, 0.9999966249, 0.9999983046, 0.9999983127,
0.9999983046, 0.9999966249, 0.9997487953, 0.9770649732, 0], [0,
0.9768220970, 0.9995010117, 0.9997487953, 0.9997504745,
0.9997504825, 0.9997504745, 0.9997487953, 0.9995010117,
0.9768220970, 0], [0, 0.9546256373, 0.9768220970, 0.9770649732,
0.9770666219, 0.9770666294, 0.9770666219, 0.9770649732,
0.9768220970, 0.9546256373, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]
```

Enregistrement dans un fichier:

```
> writedata("Diff-2D-Explicite-m.txt", Liste)
> ?
                                                    `?`
>
```