Equation de diffusion (chaleur) 2D instationnaire

Dr. Lad MESSAOUDI

Département de Mécanique

Université de Batna

==========

LMD : Energétique

Matire : Outils Numériques

==========

2014/2015

Détermination de la temperature $T(x, y, t)$ travers une plaque dont les extrémités sont maintenues des températures constantes.

$$\frac{\partial}{\partial t} T(x, y, t) = \alpha \frac{\partial^2}{\partial x^2} T(x, y, t) + \alpha \frac{\partial^2}{\partial y^2} T(x, y, t)$$

Conditions aux limites et initiale:

$$T(0, y, t) = \alpha 1 = 0,$$
$$T(1, y, t) = \beta 1 = 0,$$
$$T(x, 0, t) = \alpha 2 = 0,$$
$$T(x, 1, t) = \beta 2 = 0,$$
$$T(x, y, 0) = \sigma = x \cdot (1 - x) \cdot y \cdot (1 - y)$$

Solution discrétisée:

```
> Restart:
```

Définition de la condition initiale:

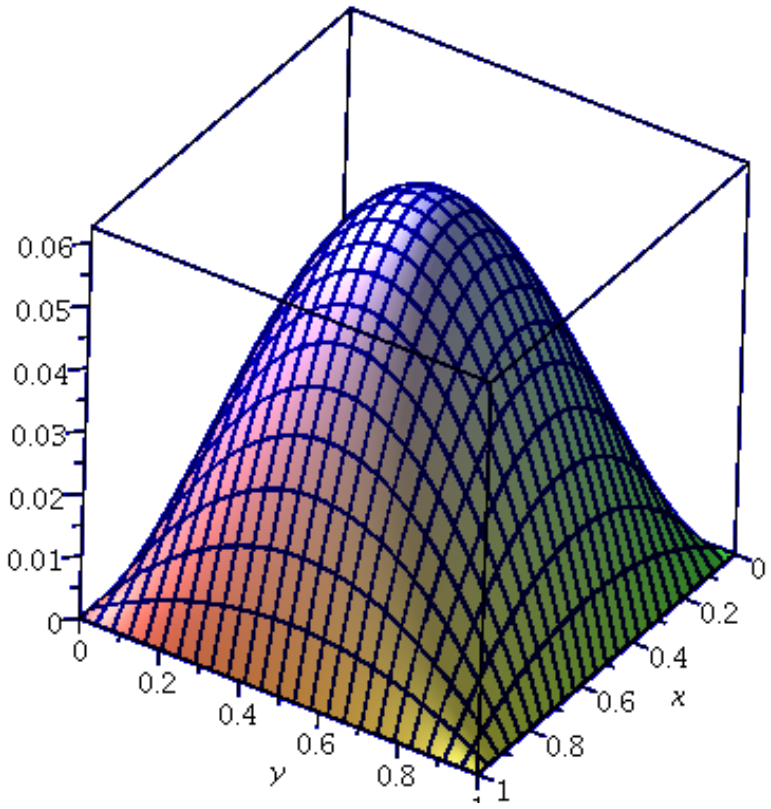
```
> f := evalf(x·(1 − x)·y·(1 − y))
```

$$f := x\,(1. - 1.\,x)\,y\,(1. - 1.\,y)$$

```
> Cin := unapply(f, (x, y));
```

$$Cin := (x, y) \rightarrow x\,(1. - 1.\,x)\,y\,(1. - 1.\,y)$$

```
> plot3d(Cin(x, y), x = 0..1, y = 0..1)
```



> $\Delta x := 0.1 ; \Delta y := 0.1 ; \Delta t := 0.005; \alpha := \frac{1.0}{80}$

$$\Delta x := 0.1$$
$$\Delta y := 0.1$$
$$\Delta t := 0.005$$
$$\alpha := 0.01250000000$$

> $\lambda_1 := \alpha \cdot \frac{\Delta t}{\Delta x^2}; \lambda_2 := \alpha \cdot \frac{\Delta t}{\Delta y^2};$

$$\lambda_1 := 0.006250000000$$
$$\lambda_2 := 0.006250000000$$

> $i_{max} := 11; j_{max} := 11;$

$$i_{max} := 11$$
$$j_{max} := 11$$

> $n_{max} := 15;$

$$n_{max} := 15$$

```
> α1 := 0; β1 := 0; α2 := 0; β2 := 0; σ := Cin;
```

$$\alpha 1 := 0$$
$$\beta 1 := 0$$
$$\alpha 2 := 0$$
$$\beta 2 := 0$$
$$\sigma := Cin$$

Condition Initiale:

```
> for i from 2 to i_max − 1 do
      for j from 2 to j_max − 1 do
          T[i, j, 0] := σ((i − 1)·Δx, (j − 1)·Δy);
      end do
  end do;
```

Conditions aux limites:

```
> for n from 0 to n_max do
      for i from 1 to i_max do
          T[i, 1, n] := α2;
          T[i, j_max, n] := β2;
      end do;

      for j from 1 to j_max do
          T[1, j, n] := α1;
          T[i_max, j, n] := β1;
      end do
  end do;
```

Schéma explicite:

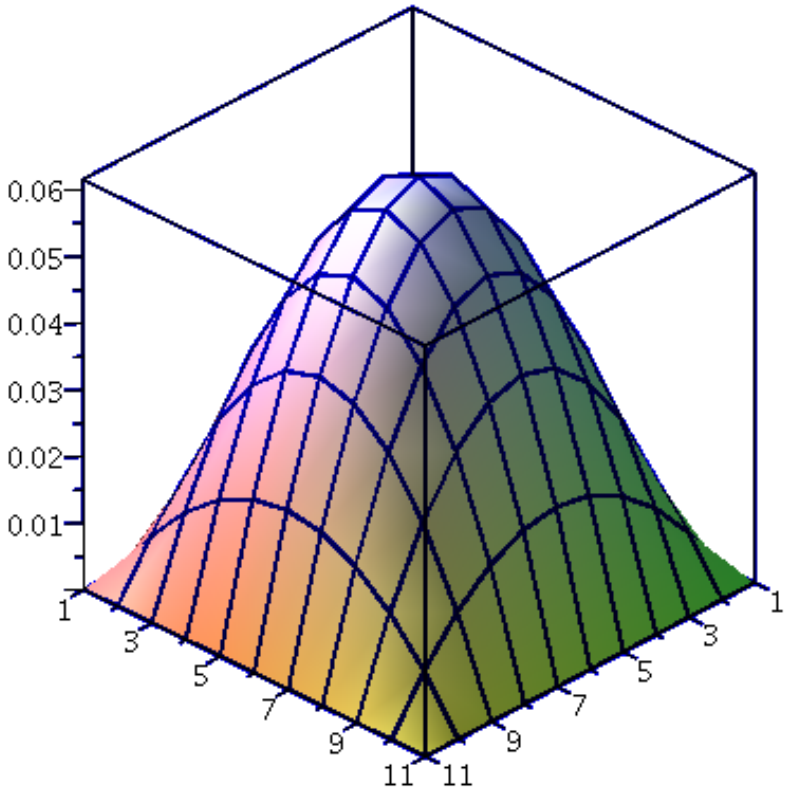
```
> for n from 0 to n_max do
    for j from 2 to j_max-1 do
        for i from 2 to i_max-1 do
          T[i, j, n + 1] := (1 − 2·λ1 − 2·λ2)·T[i, j, n] + λ1·(T[i − 1, j, n] + T[i
      + 1, j, n]) + λ2·(T[i, j − 1, n] + T[i, j + 1, n]);
        end do;
    end do;
  end do;
```

```
> for i from 2 to i_max − 1 do
      for j from 2 to j_max-1 do
          T[i, j, n_max];
      end do
  end do;
```

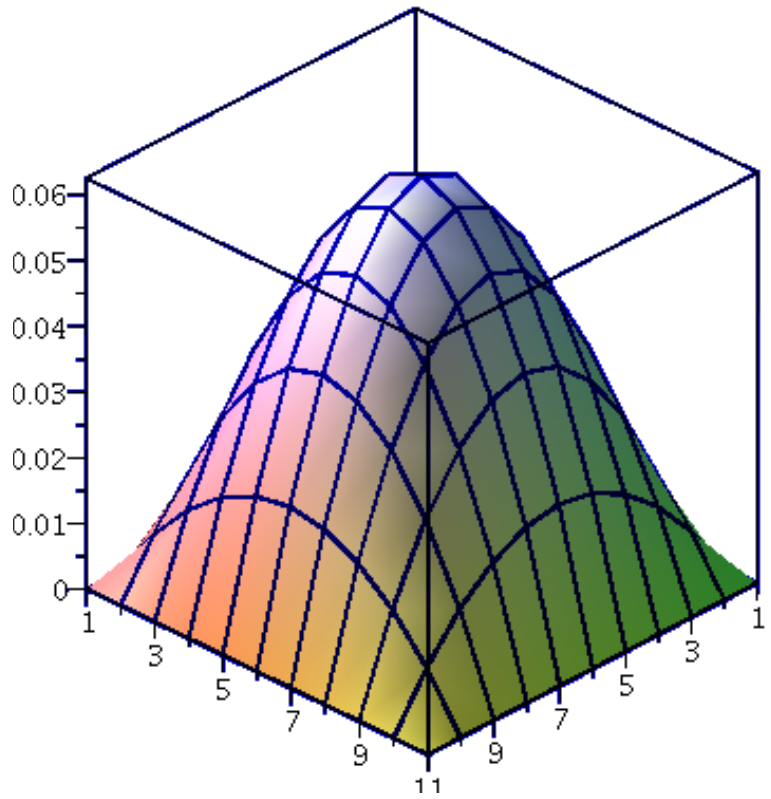
Tracé des contours de températures:

```
> with(plots):
```

> listplot3d([seq([seq(T[i, j, n_{max}], $i = 1 .. i_{max}$)], $j = 1 .. j_{max}$)])



> listplot3d([seq([seq(T[i, j, 0], $i = 1 .. i_{max}$)], $j = 1 .. j_{max}$)])



Création d'une liste de points:

> $Liste := [seq([seq(T[i, j, n_{max}], i = 1..i_{max})], j = 1..j_{max})]$

$Liste :=$ [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0.007779533907, 0.01394703315, 0.01835699330, 0.02100307359, 0.02188510143, 0.02100307359, 0.01835699330, 0.01394703315, 0.007779533907, 0], [0, 0.01394703315, 0.02500393511, 0.03290996741, 0.03765377330, 0.03923504380, 0.03765377330, 0.03290996741, 0.02500393511, 0.01394703315, 0], [0, 0.01835699330, 0.03290996741, 0.04331579726, 0.04955954050, 0.05164079080, 0.04955954050, 0.04331579726, 0.03290996741, 0.01835699330, 0], [0, 0.02100307359, 0.03765377330, 0.04955954050, 0.05670328148, 0.05908453136, 0.05670328148, 0.04955954050, 0.03765377330, 0.02100307359, 0], [0, 0.02188510142, 0.03923504380, 0.05164079080, 0.05908453136, 0.06156578126, 0.05908453136, 0.05164079080, 0.03923504380, 0.02188510142, 0], [0, 0.02100307359, 0.03765377330, 0.04955954050, 0.05670328148, 0.05908453136, 0.05670328148, 0.04955954050, 0.03765377330, 0.02100307359, 0], [0, 0.01835699330, 0.03290996741, 0.04331579726, 0.04955954050, 0.05164079080, 0.04955954050, 0.04331579726, 0.03290996741, 0.01835699330, 0], [0, 0.01394703315, 0.02500393511, 0.03290996741,

```
    0.03765377330, 0.03923504380, 0.03765377330, 0.03290996741,
    0.02500393511, 0.01394703315, 0], [0, 0.007779533907, 0.01394703315,
    0.01835699330, 0.02100307359, 0.02188510143, 0.02100307359,
    0.01835699330, 0.01394703315, 0.007779533907, 0], [0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0]]
```

Enregistrement dans un fichier:

```
> writedata("Diff-2D-Explicite-mm.txt", Liste)
>
```