

Equation de diffusion (chaleur) 1D instationnaire

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Détermination de la température $T(x, t)$ à travers l'épaisseur d'une plaque dont les extrémités sont maintenues à des températures constantes.

$$\frac{\partial}{\partial t} T(x, t) = \frac{\partial^2}{\partial x^2} T(x, t)$$

Conditions aux limites et initiale:

$$\begin{aligned} T(0, t) &= 0, \\ T(1, t) &= 0, \\ T(x, 0) &= 1 \end{aligned}$$

Solution discrétisée:

> *Restart* :

> $\Delta x := 0.2$; $\Delta t := 0.005$;

$$\begin{aligned} \Delta x &:= 0.2 \\ \Delta t &:= 0.005 \end{aligned}$$

(1.1)

> $\lambda := \frac{\Delta t}{\Delta x^2}$;

$$\lambda := 0.1250000000$$

(1.2)

> $i_{\max} := 11$;

$$i_{\max} := 11$$

(1.3)

> $n_{\max} := 15$;

$$n_{\max} := 15$$

(1.4)

> $\alpha := 0$; $\beta := 0$; $\sigma := 1$;

$$\begin{aligned} \alpha &:= 0 \\ \beta &:= 0 \end{aligned}$$

$\sigma := 1$ (1.5)
> for i from 2 to $i_{\max} - 1$ do $T[i, 0] := \sigma$ end do;

$T_{2,0} := 1$

$T_{3,0} := 1$

$T_{4,0} := 1$

$T_{5,0} := 1$

$T_{6,0} := 1$

$T_{7,0} := 1$

$T_{8,0} := 1$

$T_{9,0} := 1$

$T_{10,0} := 1$ (1.6)

> for n from 0 to n_{\max} do $T[1, n] := \alpha$ end do;

$T_{1,0} := 0$

$T_{1,1} := 0$

$T_{1,2} := 0$

$T_{1,3} := 0$

$T_{1,4} := 0$

$T_{1,5} := 0$

$T_{1,6} := 0$

$T_{1,7} := 0$

$T_{1,8} := 0$

$T_{1,9} := 0$

$T_{1,10} := 0$

$T_{1,11} := 0$

$T_{1,12} := 0$

$T_{1,13} := 0$

$T_{1,14} := 0$

$T_{1,15} := 0$ (1.7)

> for n from 0 to n_{\max} do $T[i_{\max}, n] := \beta$ end do;

$T_{11,0} := 0$

$T_{11,1} := 0$

$T_{11,2} := 0$

$T_{11,3} := 0$

$T_{11,4} := 0$

$T_{11,5} := 0$

$T_{11,6} := 0$

$T_{11,7} := 0$

$T_{11,8} := 0$

$T_{11,9} := 0$

$T_{11,10} := 0$

$T_{11,11} := 0$

$T_{11,12} := 0$

$$\begin{aligned} T_{11, 13} &:= 0 \\ T_{11, 14} &:= 0 \\ T_{11, 15} &:= 0 \end{aligned}$$

(1.8)

Schéma explicite:

```
> for n from 0 to n_max do
  for i from 2 to i_max - 1 do
    T[i, n + 1] := λ · T[i - 1, n] + (1 - 2 · λ) · T[i, n] + λ · T[i + 1, n]
  end do;
end do;
```

```
> for i from 2 to i_max - 1 do T[i, n_max] end do;
0.3892606696
0.6900617869
0.8704018496
0.9533285673
0.9756458209
0.9533285673
0.8704018496
0.6900617868
0.3892606696
```

(1.1.1)

```
> with(plots) :
> for n from 1 to n_max do
  liste[n] := [α, seq(T[i, n], i = 2 .. i_max - 1), β]
end do
liste_1 := [0, 0.8750000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000,
1.0000000000, 1.0000000000, 1.0000000000, 0.8750000000, 0]
liste_2 := [0, 0.7812500000, 0.9843750000, 1.0000000000, 1.0000000000, 1.0000000000,
1.0000000000, 1.0000000000, 0.9843750000, 0.7812500000, 0]
liste_3 := [0, 0.7089843750, 0.9609375000, 0.9980468750, 1.0000000000,
1.0000000000, 1.0000000000, 0.9980468750, 0.9609375000, 0.7089843750, 0]
liste_4 := [0, 0.6518554687, 0.9340820313, 0.9936523437, 0.9997558594,
1.0000000000, 0.9997558594, 0.9936523437, 0.9340820313, 0.6518554687, 0]
liste_5 := [0, 0.6056518554, 0.9062500001, 0.9869689941, 0.9990234376,
0.9999389648, 0.9990234376, 0.9869689941, 0.9062500001, 0.6056518554, 0]
liste_6 := [0, 0.5675201416, 0.8787651063, 0.9783859253, 0.9976310731,
0.9997100830, 0.9976310731, 0.9783859253, 0.8787651063, 0.5675201416, 0]
liste_7 := [0, 0.5354857445, 0.8523120881, 0.9683389664, 0.9954853059,
0.9991903304, 0.9954853059, 0.9683389664, 0.8523120881, 0.5354857445, 0]
liste_8 := [0, 0.5081533194, 0.8272121550, 0.9572288990, 0.9925551415,
0.9982640742, 0.9925551415, 0.9572288990, 0.8272121550, 0.5081533194, 0]
liste_9 := [0, 0.4845165090, 0.8035818935, 0.9453925863, 0.9888529778,
0.9968368410, 0.9888529778, 0.9453925863, 0.8035818935, 0.4845165090, 0]
liste_10 := [0, 0.4638351185, 0.7814250570, 0.9330987986, 0.9844184118,
0.9948408752, 0.9844184118, 0.9330987986, 0.7814250570, 0.4638351185, 0]
liste_11 := [0, 0.4455544710, 0.7606855324, 0.9205545326, 0.9793062680,
0.9922352594, 0.9793062680, 0.9205545326, 0.7606855324, 0.4455544710, 0]
liste_12 := [0, 0.4292515448, 0.7412777748, 0.9079148745, 0.9735784250,
0.9890030116, 0.9735784250, 0.9079148744, 0.7412777748, 0.4292515448, 0]
```

```

liste13 := [0, 0.4145983804, 0.7231041335, 0.8952931809, 0.9672985545,
0.9851468649, 0.9672985545, 0.8952931808, 0.7231041335, 0.4145983804, 0]
liste14 := [0, 0.4013368020, 0.7060645452, 0.8827702217, 0.9605289216,
0.9806847873, 0.9605289216, 0.8827702216, 0.7060645452, 0.4013368020, 0]
liste15 := [0, 0.3892606696, 0.6900617869, 0.8704018496, 0.9533285673,
0.9756458209, 0.9533285673, 0.8704018496, 0.6900617868, 0.3892606696, 0]

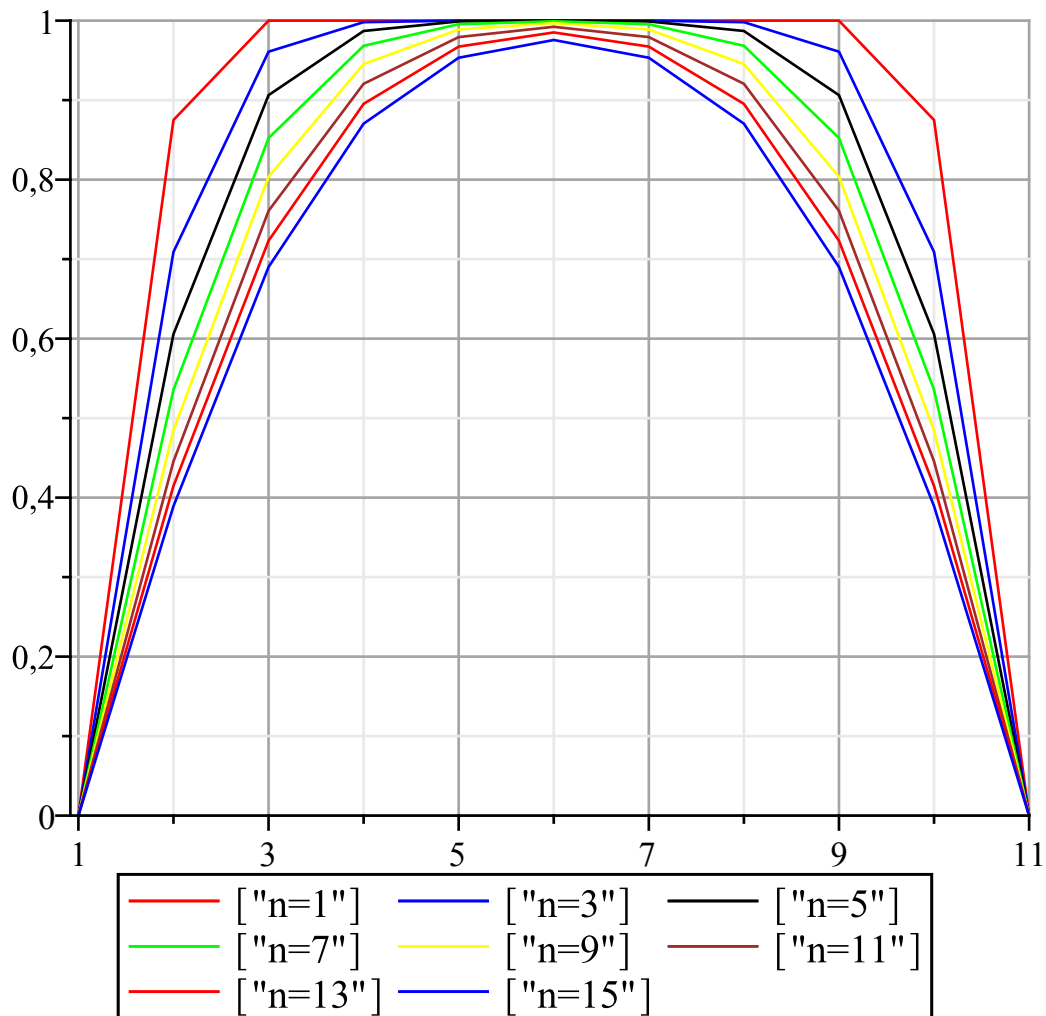
```

(1.1.2)

```

> multiple(listplot, [liste[1], color = red, legend = ["n=1"]], [liste[3], color = blue,
legend = ["n=3"]], [liste[5], color = black, legend = ["n=5"]], [liste[7], color
= green, legend = ["n=7"]], [liste[9], color = yellow, legend = ["n=9"]],
[liste[11], color = brown, legend = ["n=11"]], [liste[13], color = red, legend
= ["n=13"]], [liste[15], color = blue, legend = ["n=15"]], gridlines = true);

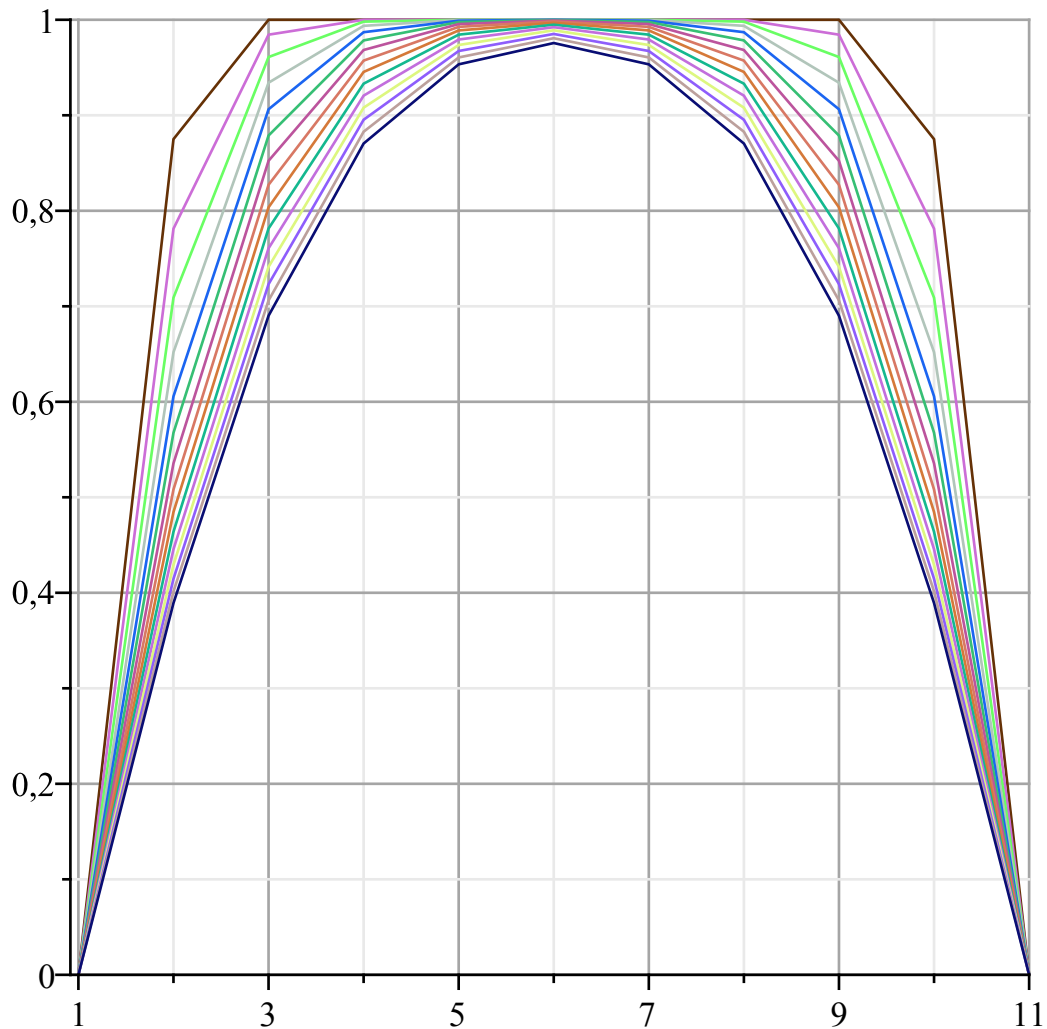
```



```

> multiple(listplot, seq([liste[n], color = COLOR(RGB, rand(), rand(),
rand()/1012)]), n = 1 ..nmax), gridlines = true);

```



Comparison:

```
> writedata(diffusionExp, liste[n_max])
```

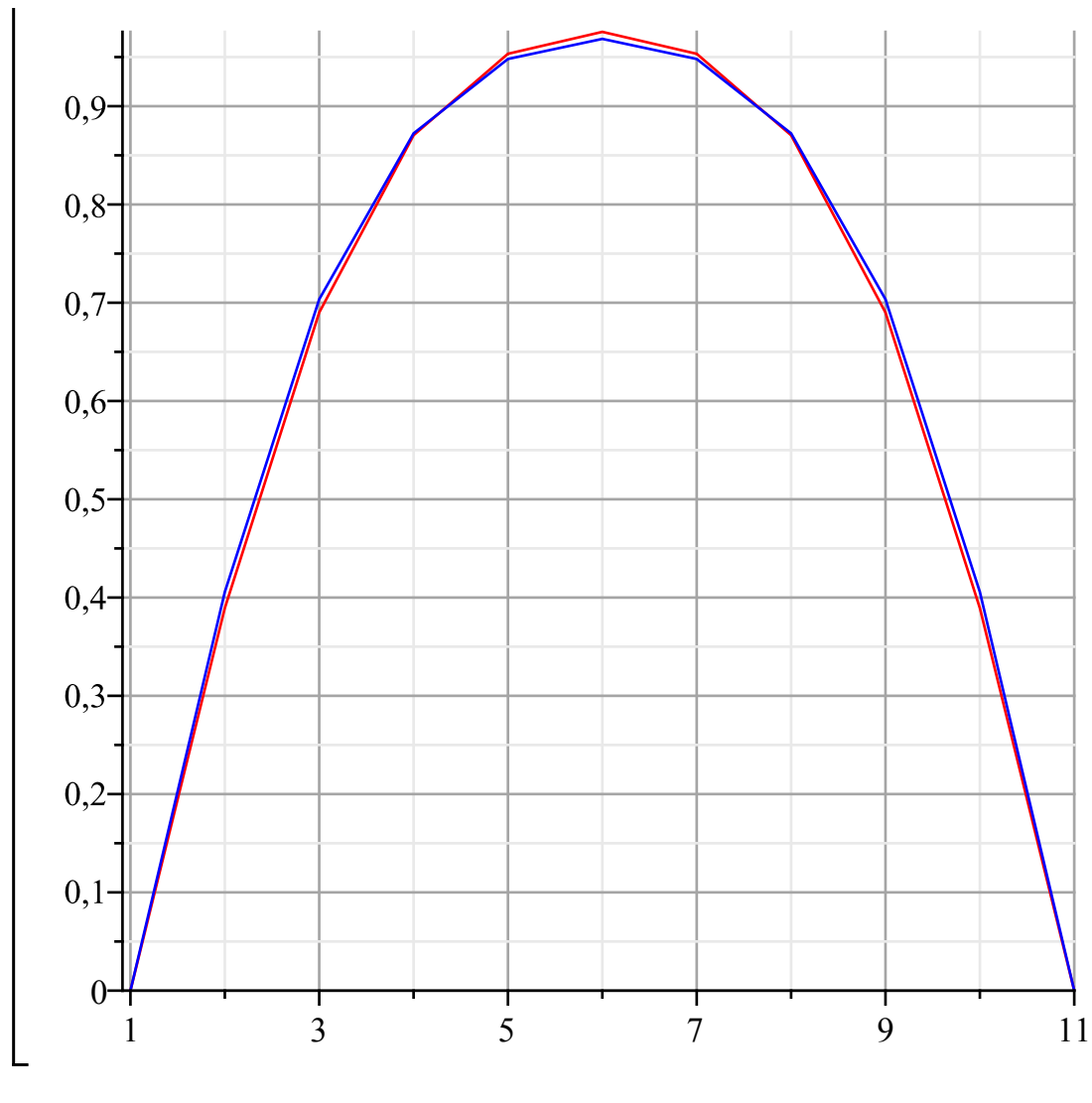
```
> SolExplicite := readdata(diffusionExp)
```

```
SolExplicite := [0., 0.3892606696, 0.6900617869, 0.8704018496, 0.9533285673, 0.9756458209, 0.9533285673, 0.8704018496, 0.6900617868, 0.3892606696, 0.] (1.1.3)
```

```
> SolImplicite := readdata(diffusionImp)
```

```
SolImplicite := [0., 0.4056766877, 0.7034939372, 0.8723276598, 0.9480237859, 0.9684863162, 0.9480237859, 0.8723276598, 0.7034939372, 0.4056766877, 0.] (1.1.4)
```

```
> multiple(listplot, [SolExplicite, color = red], [SolImplicite, color = blue], gridlines = true)
```



Méthode matricielle

```
> with(LinearAlgebra) :
```

```
> a := 1 - 2·λ; b := λ;
```

```
a := 0.7500000000
```

```
b := 0.1250000000
```

(1.2.1)

```
> M := BandMatrix([b, a, b], 1, i_max - 2, outputoptions = [storage = rectangular])
```

```
M := [[0.7500000000, 0.1250000000, 0, 0, 0, 0, 0, 0, 0, 0],
```

(1.2.2)

```
[0.1250000000, 0.7500000000, 0.1250000000, 0, 0, 0, 0, 0, 0, 0],
```

```
[0, 0.1250000000, 0.7500000000, 0.1250000000, 0, 0, 0, 0, 0, 0],
```

```
[0, 0, 0.1250000000, 0.7500000000, 0.1250000000, 0, 0, 0, 0, 0],
```

```
[0, 0, 0, 0.1250000000, 0.7500000000, 0.1250000000, 0, 0, 0, 0],
```

```
[0, 0, 0, 0, 0.1250000000, 0.7500000000, 0.1250000000, 0, 0, 0],
```

```
[0, 0, 0, 0, 0, 0.1250000000, 0.7500000000, 0.1250000000, 0, 0],
```

```
[0, 0, 0, 0, 0, 0, 0.1250000000, 0.7500000000, 0.1250000000, 0],
```

```
[0, 0, 0, 0, 0, 0, 0, 0.1250000000, 0.7500000000]]
```

```
> R := Vector(i_max - 2, [T[2, 0] + λ·α, seq(T[i, 0], i = 3 .. i_max - 2), T[i_max - 1, 0] + λ·β])
```

$$R := \begin{bmatrix} 1. \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1. \end{bmatrix}$$

(1.2.3)

```

> for n from 0 to n_max do
    T[n + 1] := MatrixVectorMultiply(M, R);
    R := Vector(i_max - 2, [T[n + 1](1) + λ·α, seq(T[n + 1](i), i = 2 .. i_max
- 3), T[n + 1](i_max - 2) + λ·β])
end do;
> for n from 1 to n_max do
    liste[n] := [α, seq(T[n](i), i = 1 .. i_max - 2), β]
end do
liste_1 := [0, 0.875000000000000000, 1., 1., 1., 1., 1., 1., 1., 0.875000000000000000,
0]
liste_2 := [0, 0.781250000000000000, 0.984375000000000000, 1., 1., 1., 1., 1.,
0.984375000000000000, 0.781250000000000000, 0]
liste_3 := [0, 0.708984375000000000, 0.960937500000000000,
0.998046875000000000, 1., 1., 1., 0.998046875000000000,
0.960937500000000000, 0.708984375000000000, 0]
liste_4 := [0, 0.651855468750000000, 0.934082031250000000,
0.993652343750000000, 0.999755859375000000, 1., 0.999755859375000000,
0.993652343750000000, 0.934082031250000000, 0.651855468750000000, 0]
liste_5 := [0, 0.605651855506250002, 0.906250000006250000,
0.986968994140625000, 0.999023437500000000, 0.999938964843750000,
0.999023437500000000, 0.986968994140625000, 0.906250000006250000,
0.605651855506250002, 0]
liste_6 := [0, 0.567520141625781238, 0.878765106209765666,
0.978385925293750014, 0.997631072998046876, 0.999710083007812500,
0.997631072998046876, 0.978385925293750014, 0.878765106209765666,
0.567520141625781238, 0]
liste_7 := [0, 0.535485744476220660, 0.852312088019043013,
0.968338966371289134, 0.995485305786230512, 0.999190330505371094,
0.995485305786230512, 0.968338966371289134, 0.852312088019043013,
0.535485744476220660, 0]
liste_8 := [0, 0.508153319377380396, 0.827212154873193440,
0.957228899004126000, 0.992555141449255385, 0.998264074325585948,
0.992555141449255385, 0.957228899004126000, 0.827212154873193440,
0.508153319377380396, 0]
liste_9 := [0, 0.484516508909149158, 0.803581893455410867,
0.945392586293400616, 0.988852977753155504, 0.996836841106503280,
0.988852977753155504, 0.945392586293400616, 0.803581893455410867,

```

```

0.484516508909149158, 0]
liste10 := [0, 0.463835118356926368, 0.781425056990733280,
0.933098798621121217, 0.984418411739854671, 0.994840875268166336,
0.984418411739854671, 0.933098798621121217, 0.781425056990733280,
0.463835118356926368, 0]
liste11 := [0, 0.445554470923841706, 0.760685532370690143,
0.920554532557164462, 0.979306268041051920, 0.992235259386088364,
0.979306268041051920, 0.920554532557164462, 0.760685532370690143,
0.445554470923841706, 0]
liste12 := [0, 0.429251544721336252, 0.74127774710163206,
0.907914874469341050, 0.973578425023695515, 0.989003011549829280,
0.973578425023695515, 0.907914874469341050, 0.74127774710163206,
0.429251544721336252, 0]
liste13 := [0, 0.414598380363770392, 0.723104133428789986,
0.895293180818738098, 0.967298554520167886, 0.985146864918295840,
0.967298554520167886, 0.895293180818738098, 0.723104133428789986,
0.414598380363770392, 0]
liste14 := [0, 0.401336801978598778, 0.706064545223934804,
0.882770221607673267, 0.960528921607255115, 0.980684787318763851,
0.960528921607255115, 0.882770221607673267, 0.706064545223934804,
0.401336801978598778, 0]
liste15 := [0, 0.389260669652991830, 0.690061786868910287,
0.870401849559653717, 0.953328567321245935, 0.975645820890886694,
0.953328567321245935, 0.870401849559653717, 0.690061786868910287,
0.389260669652991830, 0]

```

(1.2.4)

```

> multiple( listplot, seq( [ liste[n], color = COLOR( RGB,  $\frac{rand()}{10^{12}}$ ,  $\frac{rand()}{10^{12}}$ ,
 $\frac{rand()}{10^{12}}$  ) ], n = 1 ..nmax ), gridlines = true )

```