Equation de diffusion (chaleur) 1D instationnaire

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LMD : Energétique

Matière : Outils Numériques

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Détermination de la temperature $T(x, t)$ à travers l'épaisseur d'une plaque dont les extrémités sont maintenues à des températures constantes.

$$\frac{\partial}{\partial t} T(x, t) = \frac{\partial^2}{\partial x^2} T(x, t)$$

Conditions aux limites et initiale:

$$T(0, t) = 0,$$
$$T(1, t) = 0,$$
$$T(x, 0) = 1$$

Solution discrétisée:

```
> Restart :
> Δx := 0.2 ; Δt := 0.005;
```

$$\Delta x := 0.2$$
$$\Delta t := 0.005 \quad \textbf{(1.1)}$$

```
> λ := Δt / Δx^2 ;
```

$$\lambda := 0.1250000000 \quad \textbf{(1.2)}$$

```
> i_max := 11;
```

$$i_{max} := 11 \quad \textbf{(1.3)}$$

```
> n_max := 15;
```

$$n_{max} := 15 \quad \textbf{(1.4)}$$

```
> α := 0; β := 0; σ := 1;
```

$$\alpha := 0$$
$$\beta := 0$$

$$\sigma := 1 \tag{1.5}$$

```
> for i from 2 to i_max − 1 do T[i, 0] := σ end do;
```

$$T_{2,0} := 1$$
$$T_{3,0} := 1$$
$$T_{4,0} := 1$$
$$T_{5,0} := 1$$
$$T_{6,0} := 1$$
$$T_{7,0} := 1$$
$$T_{8,0} := 1$$
$$T_{9,0} := 1$$
$$T_{10,0} := 1 \tag{1.6}$$

```
> for n from 0 to n_max do T[1, n] := α end do;
```

$$T_{1,0} := 0$$
$$T_{1,1} := 0$$
$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0$$
$$T_{1,5} := 0$$
$$T_{1,6} := 0$$
$$T_{1,7} := 0$$
$$T_{1,8} := 0$$
$$T_{1,9} := 0$$
$$T_{1,10} := 0$$
$$T_{1,11} := 0$$
$$T_{1,12} := 0$$
$$T_{1,13} := 0$$
$$T_{1,14} := 0$$
$$T_{1,15} := 0 \tag{1.7}$$

```
> for n from 0 to n_max do T[i_max, n] := β end do;
```

$$T_{11,0} := 0$$
$$T_{11,1} := 0$$
$$T_{11,2} := 0$$
$$T_{11,3} := 0$$
$$T_{11,4} := 0$$
$$T_{11,5} := 0$$
$$T_{11,6} := 0$$
$$T_{11,7} := 0$$
$$T_{11,8} := 0$$
$$T_{11,9} := 0$$
$$T_{11,10} := 0$$
$$T_{11,11} := 0$$
$$T_{11,12} := 0$$

$$T_{11,\,13} := 0$$
$$T_{11,\,14} := 0$$
$$T_{11,\,15} := 0 \qquad \textbf{(1.8)}$$

Schéma implicite:

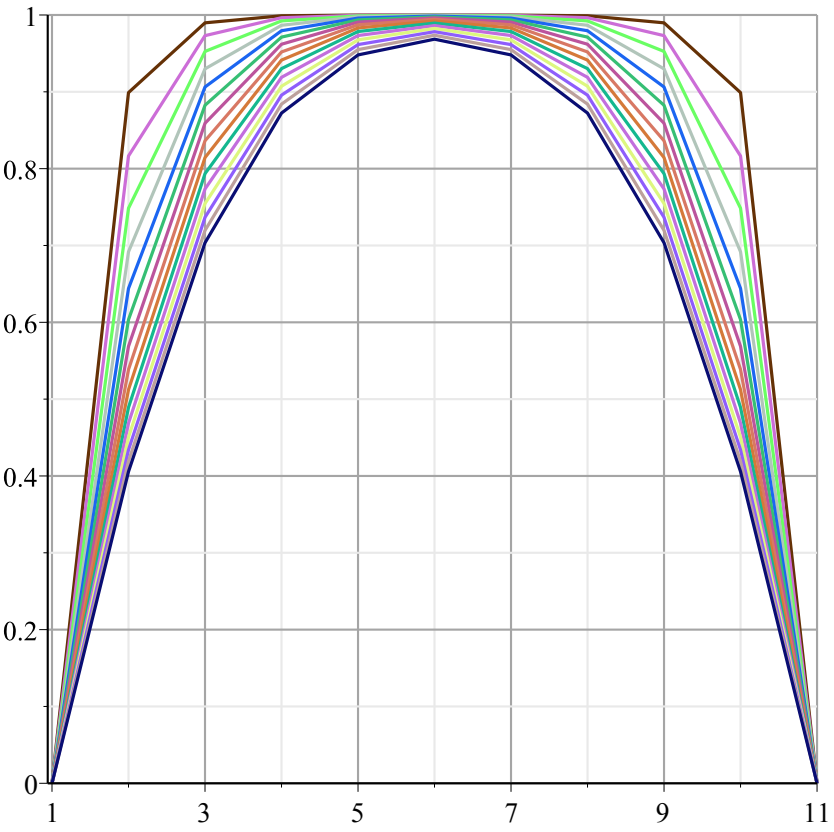
```
> with(plots) :
> for n from 0 to n_max do
    k := 1 :
    for i from 2 to i_max − 1 do
        Eq[k] := T[i, n] = -λ·T[i − 1, n + 1] + (1 + 2·λ)·T[i, n + 1] - λ·T[i + 1, n
      + 1] ;
        k := k + 1 :
    end do:
    for k from 1 to i_max − 2 do Eq[k] end do:
    Eqs := {seq(Eq[k], k = 1 .. i_max − 2)} :
    Temps := [seq(T[i, n + 1], i = 2 .. i_max − 1)] :
    SolTemp := solve(Eqs, Temps) :
    for i from 2 to i_max − 1 do T[i, n + 1] := rhs(SolTemp_{1, i − 1}) end do:
  end do:

> for n from 1 to n_max do
    liste[n] := [α, seq(T[i, n], i = 2 .. i_max − 1), β]
  end do
liste_1 := [0, 0.8989794845, 0.9897948448, 0.9989689637, 0.9998947922,
    0.9999789584, 0.9998947922, 0.9989689637, 0.9897948448, 0.8989794845, 0]
liste_2 := [0, 0.8164965719, 0.9731298431, 0.9964431002, 0.9995494494,
    0.9998930566, 0.9995494494, 0.9964431002, 0.9731298431, 0.8164965719, 0]
liste_3 := [0, 0.7484551577, 0.9525790019, 0.9922961166, 0.9988373630,
    0.9996819179, 0.9988373630, 0.9922961166, 0.9525790019, 0.7484551577, 0]
liste_4 := [0, 0.6917539090, 0.9298978282, 0.9865923577, 0.9976568159,
    0.9992768975, 0.9976568159, 0.9865923577, 0.9298978282, 0.6917539090, 0]
liste_5 := [0, 0.6440302008, 0.9062707364, 0.9794945372, 0.9959357737,
    0.9986086727, 0.9959357737, 0.9794945372, 0.9062707364, 0.6440302008, 0]
liste_6 := [0, 0.6034726172, 0.8824845659, 0.9712071504, 0.9936306406,
    0.9976130663, 0.9936306406, 0.9712071504, 0.8824845659, 0.6034726172, 0]
liste_7 := [0, 0.5686831200, 0.8590502624, 0.9619429767, 0.9907223015,
    0.9962349133, 0.9907223015, 0.9619429767, 0.8590502624, 0.5686831200, 0]
liste_8 := [0, 0.5385753134, 0.8362881741, 0.9519043281, 0.9872112929,
    0.9944301892, 0.9872112929, 0.9519043281, 0.8362881741, 0.5385753134, 0]
liste_9 := [0, 0.5122990304, 0.8143877972, 0.9412735488, 0.9831130656,
    0.9921667645, 0.9831130656, 0.9412735488, 0.8143877972, 0.5122990304, 0]
liste_10 := [0, 0.4891841817, 0.7934495734, 0.9302091751, 0.9784537869,
    0.9894241690, 0.9784537869, 0.9302091751, 0.7934495734, 0.4891841817, 0]
liste_11 := [0, 0.4686987531, 0.7735140772, 0.9188454321, 0.9732668431,
    0.9861927038, 0.9732668431, 0.9188454321, 0.7735140772, 0.4686987531, 0]
liste_12 := [0, 0.4504172369, 0.7545823440, 0.9072935851, 0.9675900503,
    0.9824721731, 0.9675900503, 0.9072935851, 0.7545823440, 0.4504172369, 0]
liste_13 := [0, 0.4339967871, 0.7366299755, 0.8956442162, 0.9614635058,
```

0.9782704396, 0.9614635058, 0.8956442162, 0.7366299755, 0.4339967871, 0]

$liste_{14}$:= [0, 0.4191591175, 0.7196168781, 0.8839698593, 0.9549279854,
0.9736019488, 0.9549279854, 0.8839698593, 0.7196168781, 0.4191591175, 0]

$liste_{15}$:= [0, 0.4056766877, 0.7034939372, 0.8723276598, 0.9480237859, **(1.1.1)**
0.9684863162, 0.9480237859, 0.8723276598, 0.7034939372, 0.4056766877, 0]

```
> multiple( listplot, seq( [ liste[n], color = COLOR( RGB, rand( ) / 10^12, rand( ) / 10^12,
    rand( ) / 10^12 ) ], n = 1 .. n_max ), gridlines = true );
```



Comparaison:

```
> writedata( diffusionImp, liste[ n_max ] )
```

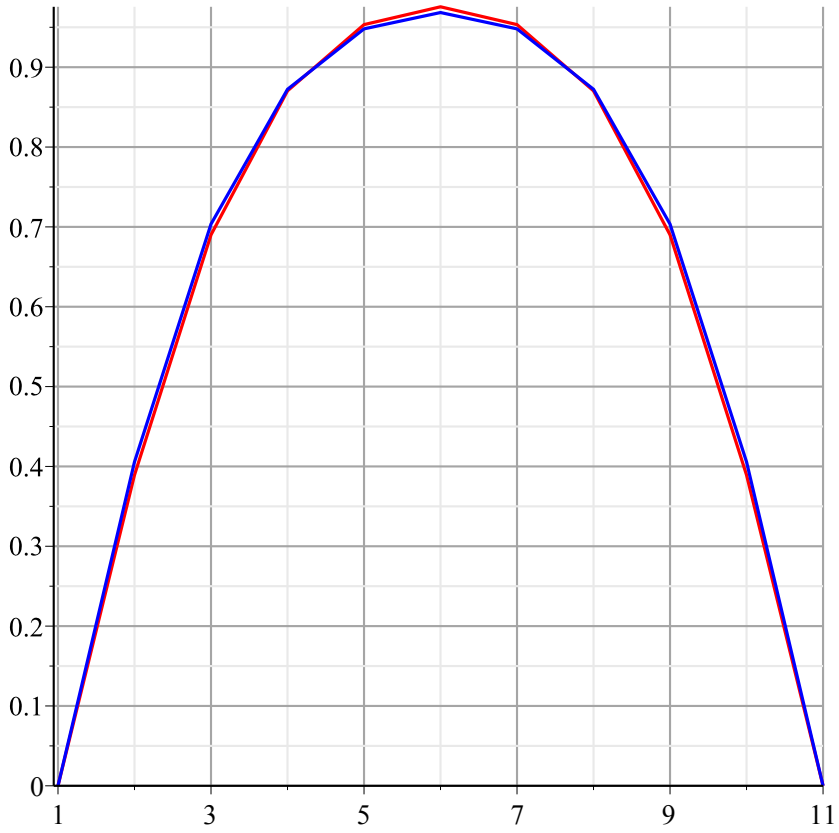
```
> SolExplicite := readdata( diffusionExp )
```

SolExplicite := [0., 0.3892606696, 0.6900617869, 0.8704018496, 0.9533285673, **(1.1.2)**
0.9756458209, 0.9533285673, 0.8704018496, 0.6900617868, 0.3892606696, 0.]

```
> SolImplicite := readdata( diffusionImp )
```

SolImplicite := [0., 0.4056766877, 0.7034939372, 0.8723276598, 0.9480237859, **(1.1.3)**
0.9684863162, 0.9480237859, 0.8723276598, 0.7034939372, 0.4056766877, 0.]

```
> multiple( listplot, [ SolExplicite, color = red ], [ SolImplicite, color = blue ], gridlines = true )
```



Méthode matricielle

```
> with(LinearAlgebra) :
> a := 1 + 2·λ; b := -λ;
```

$$a := 1.250000000$$
$$b := -0.1250000000 \qquad \textbf{(1.2.1)}$$

```
> M := BandMatrix( [b, a, b], 1, i_max - 2, outputoptions = [storage = rectangular])
```

$$M := \begin{bmatrix} 1.250000000 & -0.1250000000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1250000000 & 1.250000000 & -0.1250000000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1250000000 & 1.250000000 & -0.1250000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1250000000 & 1.250000000 & -0.1250000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1250000000 & 1.250000000 & -0.1250000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1250000000 & 1.250000000 & -0.1250000000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1250000000 & 1.250000000 & -0.1250000000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1250000000 & 1.250000000 & -0.1250000000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1250000000 & 1.250000000 \end{bmatrix} \qquad \textbf{(1.2.2)}$$

```
> R := Matrix( i_max - 2, 1, [ T[2, 0] + λ·α, seq( T[i, 0], i = 3 .. i_max - 2), T[ i_max - 1, 0 ]
      + λ·β ])
```

$$R := \begin{bmatrix} 1. \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1. \end{bmatrix} \qquad \textbf{(1.2.3)}$$

```
> for n from 0 to n_max do
      T[n+1] := LinearSolve(M, R, method='Cholesky');
      R := Matrix(i_max - 2, 1, [T[n+1](1) + λ·α, seq(T[n+1](i), i = 2..i_max - 3), T[n+1](i_max - 2) + λ·β])
  end do:
> for n from 1 to n_max do
    liste[n] := [α, seq(T[n](i), i = 1 .. i_max - 2), β]
  end do
```

$liste_1 := [0, 0.898979484481852, 0.989794844818516, 0.998968963703314, 0.999894792214624, 0.999978958442925, 0.999894792214624, 0.998968963703314, 0.989794844818516, 0.898979484481852, 0]$

$liste_2 := [0, 0.816496571892033, 0.973129843065514, 0.996443100214982, 0.999549449457787, 0.999893056645897, 0.999549449457787, 0.996443100214981, 0.973129843065514, 0.816496571892033, 0]$

$liste_3 := [0, 0.748455157702526, 0.952579001889000, 0.992296116663358, 0.998837363024732, 0.999681917921664, 0.998837363024732, 0.992296116663358, 0.952579001889000, 0.748455157702526, 0]$

$liste_4 := [0, 0.691753908980382, 0.929897828183611, 0.986592357743731, 0.997656815946828, 0.999276897526696, 0.997656815946828, 0.986592357743730, 0.929897828183611, 0.691753908980382, 0]$

$liste_5 := [0, 0.644030200819241, 0.906270736349357, 0.979494537205445, 0.995935773755248, 0.998608672772406, 0.995935773755247, 0.979494537205445, 0.906270736349357, 0.644030200819242, 0]$

$liste_6 := [0, 0.603472617239911, 0.882484565845177, 0.971207150417006, 0.993630640681318, 0.997613066354188, 0.993630640681317, 0.971207150417005, 0.882484565845177, 0.603472617239911, 0]$

$liste_7 := [0, 0.568683120027058, 0.859050262351291, 0.961942976724431, 0.990722301556972, 0.996234913394745, 0.990722301556971, 0.961942976724430, 0.859050262351291, 0.568683120027058, 0]$

$liste_8 := [0, 0.538575313424809, 0.836288174031634, 0.951904328081206, 0.987211292984977, 0.994430189312791, 0.987211292984977, 0.951904328081205, 0.836288174031634, 0.538575313424810, 0]$

$liste_9 := [0, 0.512299030454441, 0.814387797145933, 0.941273548751822, 0.983113065722640, 0.992166764594760, 0.983113065722639, 0.941273548751822, 0.814387797145933, 0.512299030454441, 0]$

$liste_{10} := [0, 0.489184181702649, 0.793449573390961, 0.930209175039497, 0.978453786989431, 0.989424169073694, 0.978453786989431,$

0.930209175039497, 0.793449573390961, 0.489184181702649, 0]

$liste_{11}$:= [0, 0.468698753084979, 0.773514077228598, 0.918845432073312, 0.973266843188543, 0.986192703896664, 0.973266843188542, 0.918845432073311, 0.773514077228598, 0.468698753084979, 0]

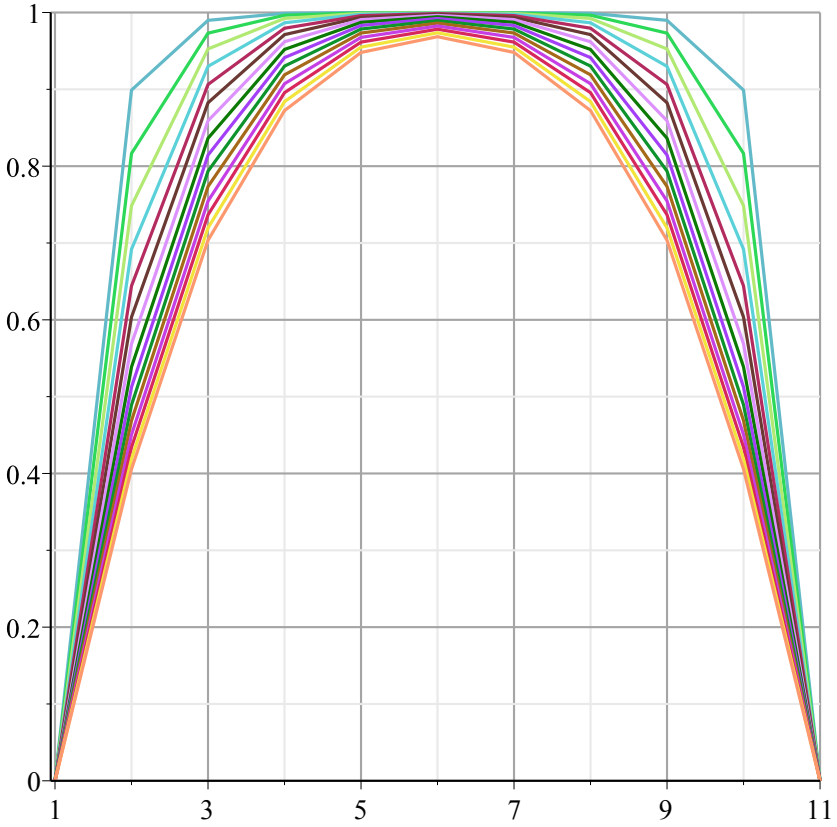
$liste_{12}$:= [0, 0.450417236865874, 0.754582343978916, 0.907293585094502, 0.967590050379610, 0.982472173193253, 0.967590050379609, 0.907293585094501, 0.754582343978916, 0.450417236865875, 0]

$liste_{13}$:= [0, 0.433996787043613, 0.736629975509133, 0.895644216216391, 0.961463505898762, 0.978270439734354, 0.961463505898761, 0.895644216216390, 0.736629975509133, 0.433996787043613, 0]

$liste_{14}$:= [0, 0.419159117443413, 0.719616878085233, 0.883969859335855, 0.954927985542187, 0.973601948895921, 0.954927985542186, 0.883969859335854, 0.719616878085233, 0.419159117443414, 0]

$liste_{15}$:= [0, 0.405676687676260, 0.703493937215292, 0.872327659794795, 0.948023786045819, 0.968486316325900, 0.948023786045818, 0.872327659794794, 0.703493937215292, 0.405676687676260, 0] **(1.2.4)**

```
> multiple(listplot, seq([liste[n], color = COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12)], n = 1 ..n_max), gridlines = true);
```




```
> ?
```

`?` **(1.2.5)**

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