# Equation d'onde 1D

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Dr. Laid MESSAOUDI

Département de Mécanique

Université de Batna

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Détermination du déplacement $u(x, t)$ d'une corde dont les extrémités sont maintenues.

$$\frac{\partial^2}{\partial t^2} u(x, t) = c^2 \cdot \frac{\partial^2}{\partial x^2} u(x, t)$$

Conditions aux limites et initiale:

$$u(0, t) = 0,$$
$$u(L, t) = 0,$$
$$u(x, 0) = 2 \cdot \sin\left(\frac{\pi}{L} x\right),$$
$$\frac{\partial}{\partial t}(u(x, 0)) = -\sin\left(\frac{2 \cdot \pi}{L} x\right),$$

## Solution analytique:

```
> Restart:
> Eq := diff(u(x,t), t$2) = c^2 * diff(u(x,t), x$2);
```

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} u(x, t)\right) = c^2 \left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} u(x, t)\right)\right) \quad \textbf{(1.1)}$$

```
> Eq1 := subs(u(x, t) = X(x)·T(t), Eq);
```

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}(X(x)\, T(t))\right) = c^2 \left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(X(x)\, T(t))\right)\right) \quad \textbf{(1.2)}$$

```
> expand(Eq1);
```

$$X(x)\left(\frac{d}{dt}\left(\frac{d}{dt} T(t)\right)\right) = c^2 \left(\frac{d}{dx}\left(\frac{d}{dx} X(x)\right)\right) T(t) \quad \textbf{(1.3)}$$

> $Eq2 := \dfrac{(\%)}{c^2 \cdot X(x) \cdot T(t)};$

$$\frac{\frac{d}{dt}\left(\frac{d}{dt} T(t)\right)}{c^2\, T(t)} = \frac{\frac{d}{dx}\left(\frac{d}{dx} X(x)\right)}{X(x)} \quad (1.4)$$

> $Eq3 := rhs(Eq2) = -\lambda^2;$

$$\frac{\frac{d}{dx}\left(\frac{d}{dx} X(x)\right)}{X(x)} = -\lambda^2 \quad (1.5)$$

> $dsolve(Eq3);$

$$X(x) = \_C1 \sin(\lambda x) + \_C2 \cos(\lambda x) \quad (1.6)$$

> $dsolve(\{Eq3, X(0) = 0, X(\mathrm{L}) = 0\});$

$$X(x) = 0 \quad (1.7)$$

Solution triviale.

> $dsolve(\{Eq3, X(0) = 0\});$

$$X(x) = \_C1 \sin(\lambda x) \quad (1.8)$$

> $X := unapply(\, rhs(\%), x);$

$$x \rightarrow \_C1 \sin(\lambda x) \quad (1.9)$$

> $\lambda[n] := \dfrac{n * \pi}{L};$

$$\frac{n\,\pi}{L} \quad (1.10)$$

> $X[n](x) := subs(\lambda = \lambda[n], X(x));$

$$\_C1 \sin\left(\frac{n\,\pi\,x}{L}\right) \quad (1.11)$$

> $Eq4 := lhs(Eq2) = -\lambda^2;$

$$\frac{\frac{d}{dt}\left(\frac{d}{dt} T(t)\right)}{c^2\, T(t)} = -\lambda^2 \quad (1.12)$$

> $dsolve(Eq4);$

$$T(t) = \_C1 \sin(c\,\lambda\,t) + \_C2 \cos(c\,\lambda\,t) \quad (1.13)$$

> $T := unapply(\, rhs(\%), t);$

$$t \rightarrow \_C1 \sin(c\,\lambda\,t) + \_C2 \cos(c\,\lambda\,t) \quad (1.14)$$

> $T[n](\mathrm{t}) := subs(\lambda = \lambda[n], T(\mathrm{t}));$

$$\_C1 \sin\left(\frac{c\,n\,\pi\,t}{L}\right) + \_C2 \cos\left(\frac{c\,n\,\pi\,t}{L}\right) \quad (1.15)$$

> $\mathrm{u}[n](x, \mathrm{t}) := \dfrac{X[n](x) \cdot T[n](\mathrm{t})}{\_C1};$

$$\sin\left(\frac{n\,\pi\,x}{L}\right)\left(\_C1 \sin\left(\frac{c\,n\,\pi\,t}{L}\right) + \_C2 \cos\left(\frac{c\,n\,\pi\,t}{L}\right)\right) \quad (1.16)$$

> $u(x, t) := Sum(\mathrm{u}[n](x, \mathrm{t}), n = 1 .. \infty);$

$$\sum_{n=1}^{\infty} \sin\left(\frac{n\,\pi\,x}{L}\right)\left(\_C1 \sin\left(\frac{c\,n\,\pi\,t}{L}\right) + \_C2 \cos\left(\frac{c\,n\,\pi\,t}{L}\right)\right) \quad (1.17)$$

> $u(x, t) := expand(u(x, t));$

$$\sum_{n=1}^{\infty} \left(\sin\left(\frac{n\,\pi\,x}{L}\right) \_C1 \sin\left(\frac{c\,n\,\pi\,t}{L}\right) + \sin\left(\frac{n\,\pi\,x}{L}\right) \_C2 \cos\left(\frac{c\,n\,\pi\,t}{L}\right)\right) \quad (1.18)$$

> $CI1 := 2\sin\left(\frac{\pi x}{L}\right);\ CI2 := -\sin\left(\frac{2\cdot\pi x}{L}\right);$

$$2\sin\left(\frac{\pi x}{L}\right)$$
$$-\sin\left(\frac{2\pi x}{L}\right) \quad (1.19)$$

> $p1 := \sin\left(\frac{\pi x}{L}\right)\_C1\sin\left(\frac{c\pi t}{L}\right)+\sin\left(\frac{\pi x}{L}\right)\_C2\cos\left(\frac{c\pi t}{L}\right);$

$$\sin\left(\frac{\pi x}{L}\right)\_C1\sin\left(\frac{c\pi t}{L}\right)+\sin\left(\frac{\pi x}{L}\right)\_C2\cos\left(\frac{c\pi t}{L}\right) \quad (1.20)$$

> $p2 := \sin\left(\frac{2\pi x}{L}\right)\_C1\sin\left(\frac{2\cdot c\pi t}{L}\right)+\sin\left(\frac{2\pi x}{L}\right)\_C2\cos\left(\frac{2\cdot c\pi t}{L}\right);$

$$\sin\left(\frac{2\pi x}{L}\right)\_C1\sin\left(\frac{2c\pi t}{L}\right)+\sin\left(\frac{2\pi x}{L}\right)\_C2\cos\left(\frac{2c\pi t}{L}\right) \quad (1.21)$$

> $q := eval(subs(t=0, p1));$

$$\sin\left(\frac{\pi x}{L}\right)\_C2 \quad (1.22)$$

> $solve(q = CI1, \{\_C2\});$

$$\{\_C2 = 2\} \quad (1.23)$$

> $pp := p1 + p2;$

$$\sin\left(\frac{\pi x}{L}\right)\_C1\sin\left(\frac{c\pi t}{L}\right)+\sin\left(\frac{\pi x}{L}\right)\_C2\cos\left(\frac{c\pi t}{L}\right)+\sin\left(\frac{2\pi x}{L}\right)\_C1\sin\left(\frac{2c\pi t}{L}\right)+\sin\left(\frac{2\pi x}{L}\right)\_C2\cos\left(\frac{2c\pi t}{L}\right) \quad (1.24)$$

> $dpp := \frac{d}{dt}pp;$

$$\frac{\sin\left(\frac{\pi x}{L}\right)\_C1\cos\left(\frac{c\pi t}{L}\right)c\pi}{L}-\frac{\sin\left(\frac{\pi x}{L}\right)\_C2\sin\left(\frac{c\pi t}{L}\right)c\pi}{L}+\frac{2\sin\left(\frac{2\pi x}{L}\right)\_C1\cos\left(\frac{2c\pi t}{L}\right)c\pi}{L}-\frac{2\sin\left(\frac{2\pi x}{L}\right)\_C2\sin\left(\frac{2c\pi t}{L}\right)c\pi}{L} \quad (1.25)$$

> $qq := eval(subs(t=0, dpp));$

$$\frac{\sin\left(\frac{\pi x}{L}\right)\_C1\,c\pi}{L}+\frac{2\sin\left(\frac{2\pi x}{L}\right)\_C1\,c\pi}{L} \quad (1.26)$$

> $solve(qq = CI2, \{\_C1\});$

$$\left\{\_C1 = -\frac{\sin\left(\frac{2\pi x}{L}\right)L}{\pi c\left(\sin\left(\frac{\pi x}{L}\right)+2\sin\left(\frac{2\pi x}{L}\right)\right)}\right\} \quad (1.27)$$

> $qqq := op(2, qq);$

$$\frac{2\sin\left(\frac{2\pi x}{L}\right)\_C1\,c\pi}{L} \quad (1.28)$$

> $solve(qqq = CI2, \{\_C1\});$

(1.29)

$$\left\{\_C1 = -\frac{1}{2}\,\frac{L}{c\,\pi}\right\} \quad \textbf{(1.29)}$$

> $u(x,t) := eval\left(subs\left(\left\{\_C1 = -\frac{1}{2}\,\frac{L}{c\,\pi}, \_C2 = 2\right\}, \sin\left(\frac{2\cdot\pi\,x}{L}\right)\_C1\,\sin\left(\frac{2\cdot c\,\pi\,t}{L}\right) + \sin\left(\frac{\pi\,x}{L}\right)\_C2\,\cos\left(\frac{c\,\pi\,t}{L}\right)\right)\right);$

$$-\frac{1}{2}\,\frac{\sin\left(\frac{2\,\pi\,x}{L}\right) L \sin\left(\frac{2\,c\,\pi\,t}{L}\right)}{c\,\pi} + 2\,\sin\left(\frac{\pi\,x}{L}\right)\cos\left(\frac{c\,\pi\,t}{L}\right) \quad \textbf{(1.30)}$$

> *restart*;

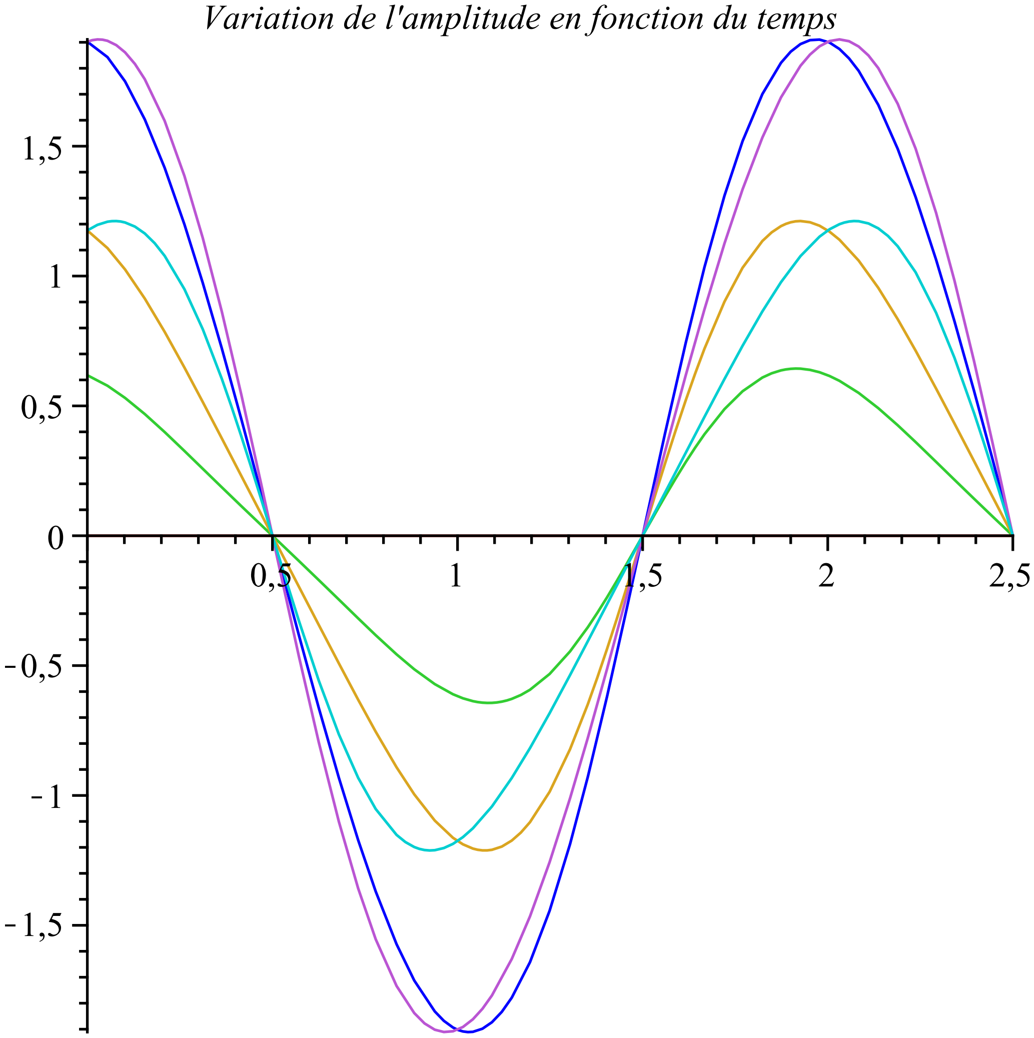
> $u(x,t) := 2\,\sin\left(\frac{\pi\,x}{L}\right)\cos\left(\frac{c\,\pi\,t}{L}\right) - \frac{\mathrm{L}}{2\,\pi\,c}\sin\left(\frac{2\,\pi\,x}{L}\right)\,\sin\left(\frac{2\,c\,\pi\,t}{L}\right);$

$$(x,t) \rightarrow 2\,\sin\left(\frac{\pi\,x}{L}\right)\cos\left(\frac{c\,\pi\,t}{L}\right) - \frac{1}{2}\,\frac{L\,\sin\left(\frac{2\,\pi\,x}{L}\right)\sin\left(\frac{2\,c\,\pi\,t}{L}\right)}{c\,\pi} \quad \textbf{(1.31)}$$

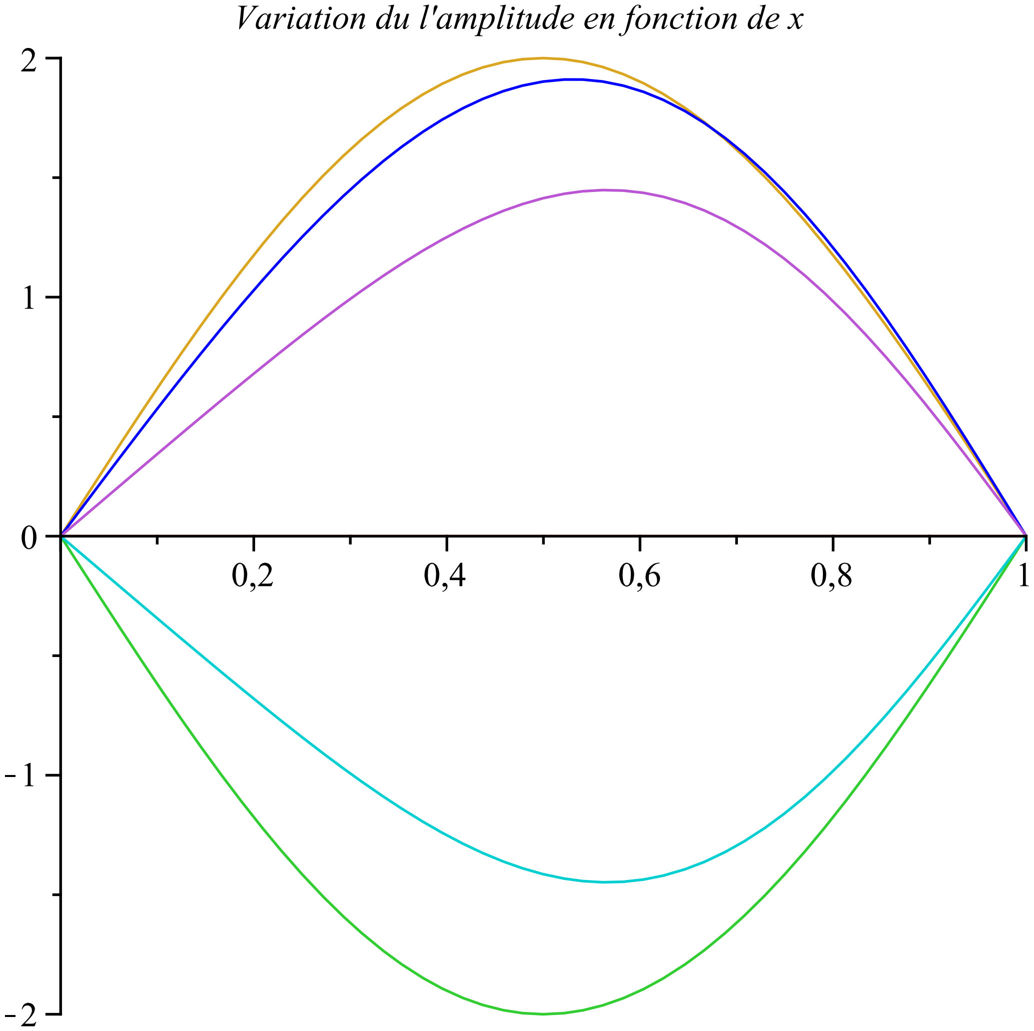
> $L := 1;\ c := 1;$

$$1$$
$$1 \quad \textbf{(1.32)}$$

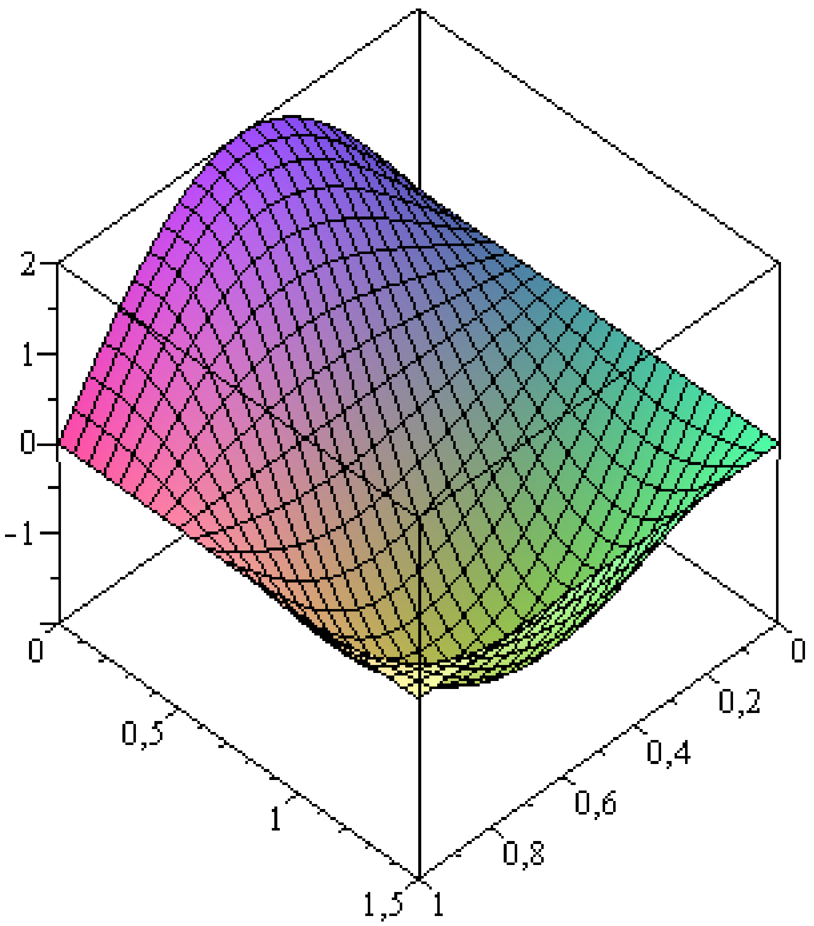
> *plot*( {*u*(0.1, t), *u*(0.2, t), *u*(0.4, t), *u*(0.6, t), *u*(0.8, t), *u*(1, t) }, t = 0 ..2.5, title = `Variation de l'amplitude en fonction du temps`, *labels* = [ `*t*`, `*u*`]);



> *plot*( {*u*(x, 0.1), *u*(x, 0.25), *u*(x, 0.75), *u*(x, 1.0), *u*(x, 1.5), *u*(x, 2.0)}, x = 0 ..1, title = `Variation du l'amplitude en fonction de x`, *labels* = [`*x*`, `*u*`]);




> *plot3d*(*u*(*x*, *t*), *x* = 0 ..1, *t* = 0 ..1.5, *axes* = *boxed*);




# Solution numérique:

```
> Restart:with(plots):
```

$> Eq := \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} u(x,t)\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} u(x,t)\right);$

$$Eq := \frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t) \quad (2.1)$$

```
> CL:={u(0,t) = 0, u(1,t) = 0, u(x,0) = 2*sin(Pi*x/L), (D[2]
  (u))(x, 0) = -sin(2*Pi*x/L)};
```

$$CL := \left\{u(0,t) = 0, u(1,t) = 0, u(x,0) = 2\sin\left(\frac{\pi x}{L}\right), D_2(u)(x,0) = -\sin\left(\frac{2\pi x}{L}\right)\right\} \quad (2.2)$$

$> L := 1;$

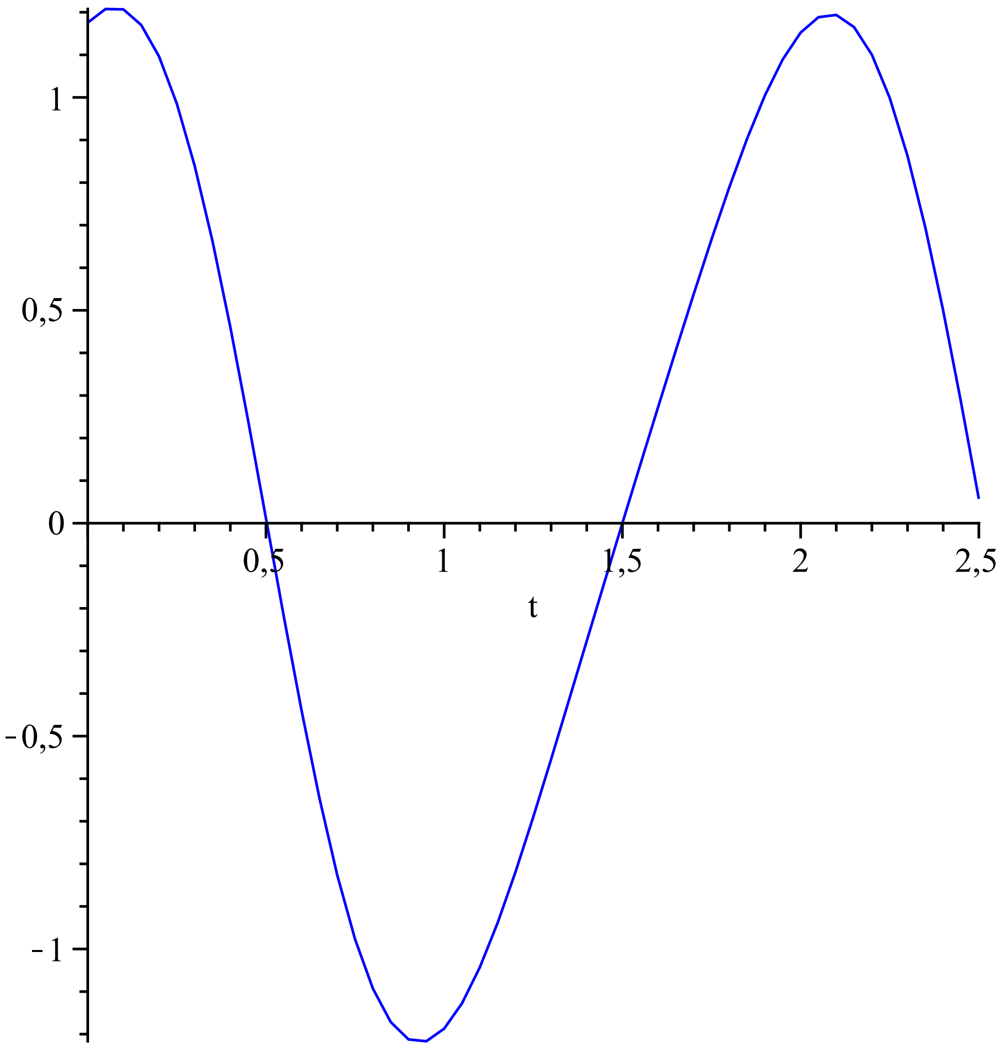
$$L := 1 \quad (2.3)$$

Résolution et création d'un module de solutions comprenant les focntions: plot, plot3d, value, ... etc.

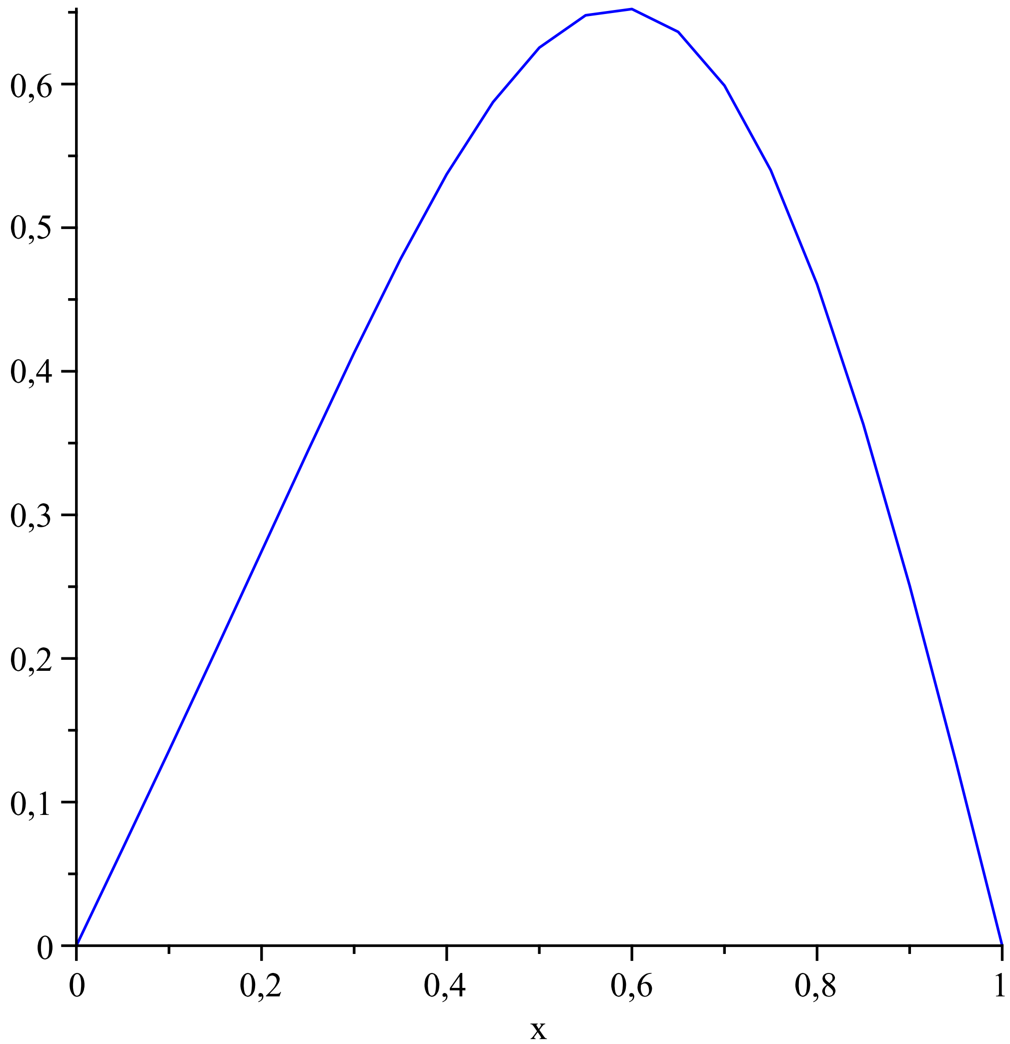
```
> Sol := pdsolve( Eq, CL, numeric, time = t, range = 0 .. 1);
```

*Sol* := **module**( ) **export** *plot, plot3d, animate, value, settings*; ... **end module** (2.4)

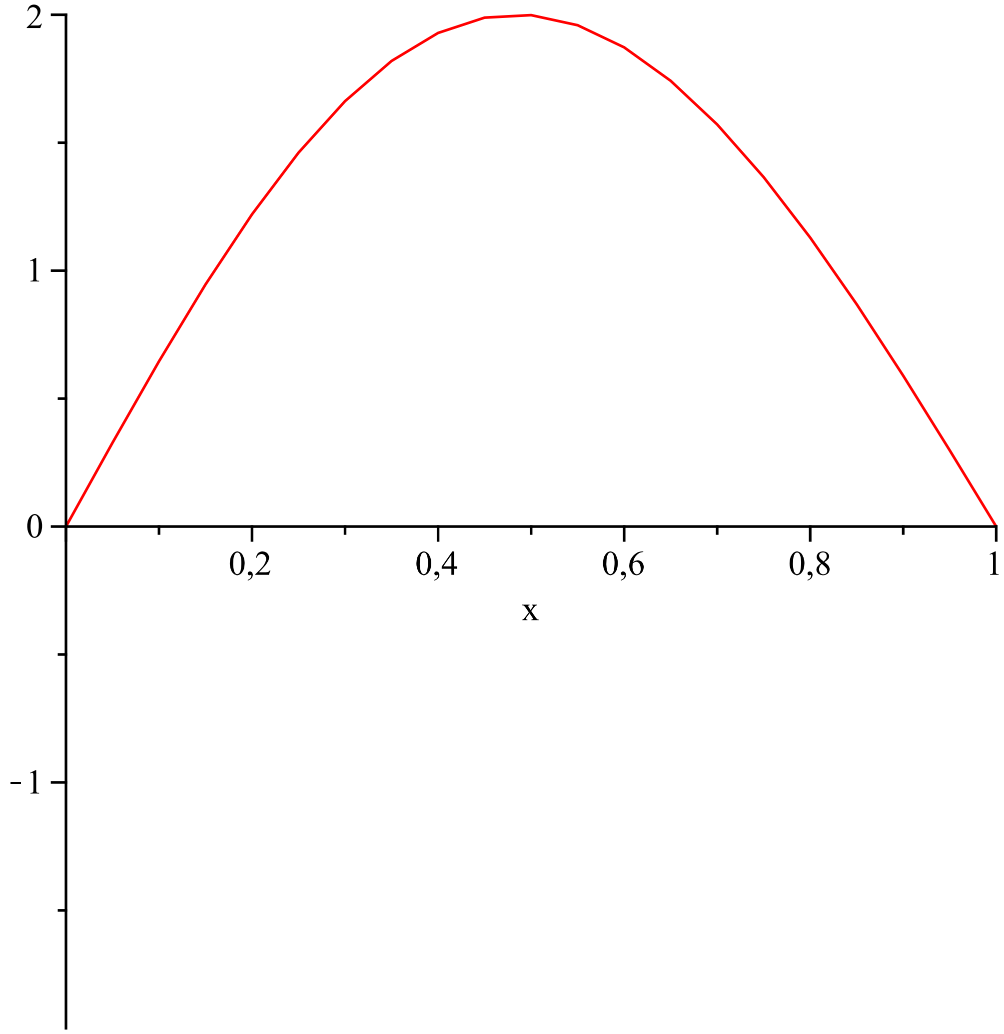
$> Sol :- plot(u(x,t), x = 0.8, t = 0 ..2.5, color = blue);$



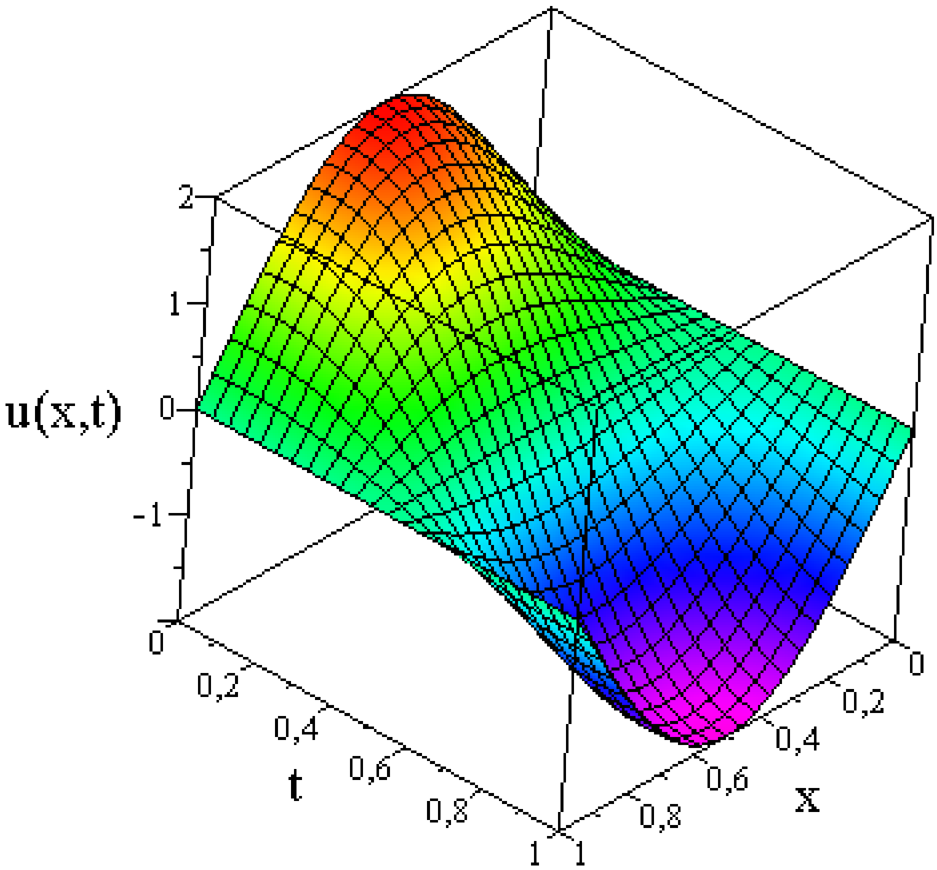

> *Sol* :- *plot*( *u*( *x*, *t*) , *x* = 0 ..1, *t* = 0.4, *color* = *blue*) ;




> $Sol:\text{-}animate(u(x,t),\ t=0\,..\,4)$




> *Sol* :- *plot3d*(*u*(*x*, *t*), *t* = 0 ..1, *shading* = *zhue*, *axes* = *boxed*, *labels* = ["x", "t", "u(x,t)"], *labelfont* = [*TIMES*, *ROMAN*, 16]);




> ValU := Sol :- value();

$ValU :=$ **proc**( ) ... **end proc** **(2.5)**

La veleur de la temperature en $x = 0.1, t = 0.01$ *est* :

> ValU( 0.5, 0.1 );

$[x = 0.5, t = 0.1, u(x, t) = 1.902708234636595 11]$ **(2.6)**

>

# Solution discrétisée:

> *Restart* :

> $\Delta x := 0.2$ ; $\Delta t := 0.1$; $L := 1$;

$$\Delta x := 0.2$$
$$\Delta t := 0.1$$
$$L := 1$$ **(3.1)**

> $\lambda := \left(\frac{\Delta t}{\Delta x}\right)^2$;

$$\lambda := 0.2500000000$$ **(3.2)**

> $i_{max} := 11$;

$$i_{max} := 11$$ **(3.3)**

> $n_{max} := 15$;

**(3.4)**

$$n_{max} := 15 \quad (3.4)$$

> α := 0; β := 0;

$$\alpha := 0$$
$$\beta := 0 \quad (3.5)$$

> $f(x) := 2 \cdot \sin\left(\frac{\pi \cdot x}{L}\right)$

$$f := x \rightarrow 2 \sin\left(\frac{\pi x}{L}\right) \quad (3.6)$$

> $g(x) := -\sin\left(\frac{2 \cdot \pi \cdot x}{L}\right)$

$$g := x \rightarrow -\sin\left(\frac{2 \pi x}{L}\right) \quad (3.7)$$

> **for** i **from** 1 **to** $i_{max}$ **do** $u[i, 0] := evalf(f((i-1) \cdot \Delta x))$ **end do**;

$$u_{1,0} := 0.$$
$$u_{2,0} := 1.175570505$$
$$u_{3,0} := 1.902113033$$
$$u_{4,0} := 1.902113033$$
$$u_{5,0} := 1.175570504$$
$$u_{6,0} := 0.$$
$$u_{7,0} := -1.175570506$$
$$u_{8,0} := -1.902113033$$
$$u_{9,0} := -1.902113032$$
$$u_{10,0} := -1.175570504$$
$$u_{11,0} := 0. \quad (3.8)$$

> **for** n **from** 1 **to** $n_{max}$ **do** $u[1, n] := \alpha$ **end do**;

$$u_{1,1} := 0$$
$$u_{1,2} := 0$$
$$u_{1,3} := 0$$
$$u_{1,4} := 0$$
$$u_{1,5} := 0$$
$$u_{1,6} := 0$$
$$u_{1,7} := 0$$
$$u_{1,8} := 0$$
$$u_{1,9} := 0$$
$$u_{1,10} := 0$$
$$u_{1,11} := 0$$
$$u_{1,12} := 0$$
$$u_{1,13} := 0$$
$$u_{1,14} := 0$$
$$u_{1,15} := 0 \quad (3.9)$$

> **for** n **from** 1 **to** $n_{max}$ **do** $u[i_{max}, n] := \beta$ **end do**;

$$u_{11,1} := 0$$

$$u_{11,2} := 0$$
$$u_{11,3} := 0$$
$$u_{11,4} := 0$$
$$u_{11,5} := 0$$
$$u_{11,6} := 0$$
$$u_{11,7} := 0$$
$$u_{11,8} := 0$$
$$u_{11,9} := 0$$
$$u_{11,10} := 0$$
$$u_{11,11} := 0$$
$$u_{11,12} := 0$$
$$u_{11,13} := 0$$
$$u_{11,14} := 0$$
$$u_{11,15} := 0 \qquad \textbf{(3.10)}$$

Schéma explicite:

> **for** *i* **from** 1 **to** $i_{max}$ **do**

$$u[\mathrm{i}, 1] := (1-\lambda)\cdot evalf(f((i-1)\cdot\Delta x)) + \frac{\lambda}{2}\cdot(evalf(f(i\cdot\Delta x)) + evalf(f((i-2)\cdot\Delta x))) + \Delta t\cdot evalf(g((i-1)\cdot\Delta x));$$

**end do**;

$$u_{1,1} := 0.$$
$$u_{2,1} := 1.024336356$$
$$u_{3,1} := 1.752516692$$
$$u_{4,1} := 1.870073742$$
$$u_{5,1} := 1.214547659$$
$$u_{6,1} := -2.500000000\ 10^{-10}$$
$$u_{7,1} := -1.214547661$$
$$u_{8,1} := -1.870073742$$
$$u_{9,1} := -1.752516691$$
$$u_{10,1} := -1.024336355$$
$$u_{11,1} := 2.500000000\ 10^{-10} \qquad \textbf{(3.1.1)}$$

> **for** n **from** 1 **to** $n_{max}$ **do**
 **for** i **from** 2 **to** $i_{max} - 1$ **do**

$$u[\mathrm{i}, \mathrm{n}+1] := 2\cdot\lambda\cdot u[\mathrm{i}, \mathrm{n}] + \lambda\cdot(u[\mathrm{i}+1, \mathrm{n}] + u[\mathrm{i}-1, \mathrm{n}]) - u[\mathrm{i}, \mathrm{n}-1]$$

 **end do**;
**end do**;

> **for** i **from** 2 **to** $i_{max} - 1$ **do** $u[\mathrm{i}, n_{max}]$ **end do**;

$$-1.257397365$$
$$-2.063152841$$
$$-2.098555267$$
$$-1.314679696$$
$$1.501922610\ 10^{-10}$$
$$1.314679697$$

2.098555267
2.063152842
1.257397367 **(3.1.2)**

> with(plots) :

> **for** n **from** 1 **to** $n_{max}$ **do**

$liste[n] := [[0.0, u[1, n]], [0.1, u[2, n]], [0.2, u[3, n]], [0.3, u[4, n]], [0.4, u[5, n]], [0.5, u[6, n]], [0.6, u[7, n]], [0.7, u[8, n]], [0.8, u[9, n]], [0.9, u[10, n]], [1.0, u[11, n]]]$

**end do**

$liste_1 := [[0., 0.], [0.1, 1.024336356], [0.2, 1.752516692], [0.3, 1.870073742], [0.4, 1.214547659], [0.5, -2.500000000\ 10^{-10}], [0.6, -1.214547661], [0.7, -1.870073742], [0.8, -1.752516691], [0.9, -1.024336355], [1.0, 2.500000000\ 10^{-10}]]$

$liste_2 := [[0., 0], [0.1, -0.2252731540], [0.2, -0.302252163], [0.3, -0.225310074], [0.4, -0.100778239], [0.5, -6.250000000\ 10^{-10}], [0.6, 0.100778240], [0.7, 0.225310074], [0.8, 0.302252162], [0.9, 0.2252731536], [1.0, 0]]$

$liste_3 := [[0., 0], [0.1, -1.212535974], [0.2, -2.016288580], [0.3, -2.083486380], [0.4, -1.321264297], [0.5, 1.875000000\ 10^{-10}], [0.6, 1.321264299], [0.7, 2.083486380], [0.8, 2.016288579], [0.9, 1.212535972], [1.0, 0]]$

$liste_4 := [[0., 0], [0.1, -0.8850669780], [0.2, -1.529897715], [0.3, -1.650821335], [0.4, -1.080725504], [0.5, 1.218750000\ 10^{-9}], [0.6, 1.080725505], [0.7, 1.650821336], [0.8, 1.529897716], [0.9, 0.8850669774], [1.0, 0]]$

$liste_5 := [[0., 0], [0.1, 0.3875280562], [0.2, 0.617367644], [0.3, 0.605419908], [0.4, 0.3681962115], [0.5, 6.718750000\ 10^{-10}], [0.6, -0.3681962124], [0.7, -0.605419907], [0.8, -0.617367643], [0.9, -0.3875280542], [1.0, 0]]$

$liste_6 := [[0., 0], [0.1, 1.233172917], [0.2, 2.086818528], [0.3, 2.199922253], [0.4, 1.416178587], [0.5, -1.107812500\ 10^{-9}], [0.6, -1.416178588], [0.7, -2.199922253], [0.8, -2.086818528], [0.9, -1.233172915], [1.0, 0]]$

$liste_7 := [[0., 0], [0.1, 0.7507630338], [0.2, 1.284315412], [0.3, 1.370290497], [0.4, 0.8898736455], [0.5, -1.475781250\ 10^{-9}], [0.6, -0.8898736456], [0.7, -1.370290498], [0.8, -1.284315413], [0.9, -0.7507630358], [1.0, 0]]$

$liste_8 := [[0., 0], [0.1, -0.5367125472], [0.2, -0.914397439], [0.3, -0.971229740], [0.4, -0.6286691402], [0.5, 3.449218750\ 10^{-10}], [0.6, 0.6286691404], [0.7, 0.971229739], [0.8, 0.914397438], [0.9, 0.5367125438], [1.0, 0]]$

$liste_9 := [[0., 0], [0.1, -1.247718667], [0.2, -2.118499703], [0.3, -2.241672012], [0.4, -1.447015650], [0.5, 1.698242188\ 10^{-9}], [0.6, 1.447015651], [0.7, 2.241672012], [0.8, 2.118499703], [0.9, 1.247718667], [1.0, 0]]$

$liste_{10} := [[0., 0], [0.1, -0.6167717118], [0.2, -1.017200083], [0.3, -1.040985104], [0.4, -0.6552566878], [0.5, 7.541992190\ 10^{-10}], [0.6, 0.6552566886], [0.7, 1.040985105], [0.8, 1.017200084], [0.9, 0.6167717152], [1.0, 0]]$

$liste_{11} := [[0., 0], [0.1, 0.6850327902], [0.2, 1.195460457], [0.3, 1.303065267], [0.4, 0.8591410302], [0.5, -1.121142578\ 10^{-9}], [0.6, -0.8591410301], [0.7, -1.303065266], [0.8, -1.195460456], [0.9, -0.6850327884], [1.0, 0]]$

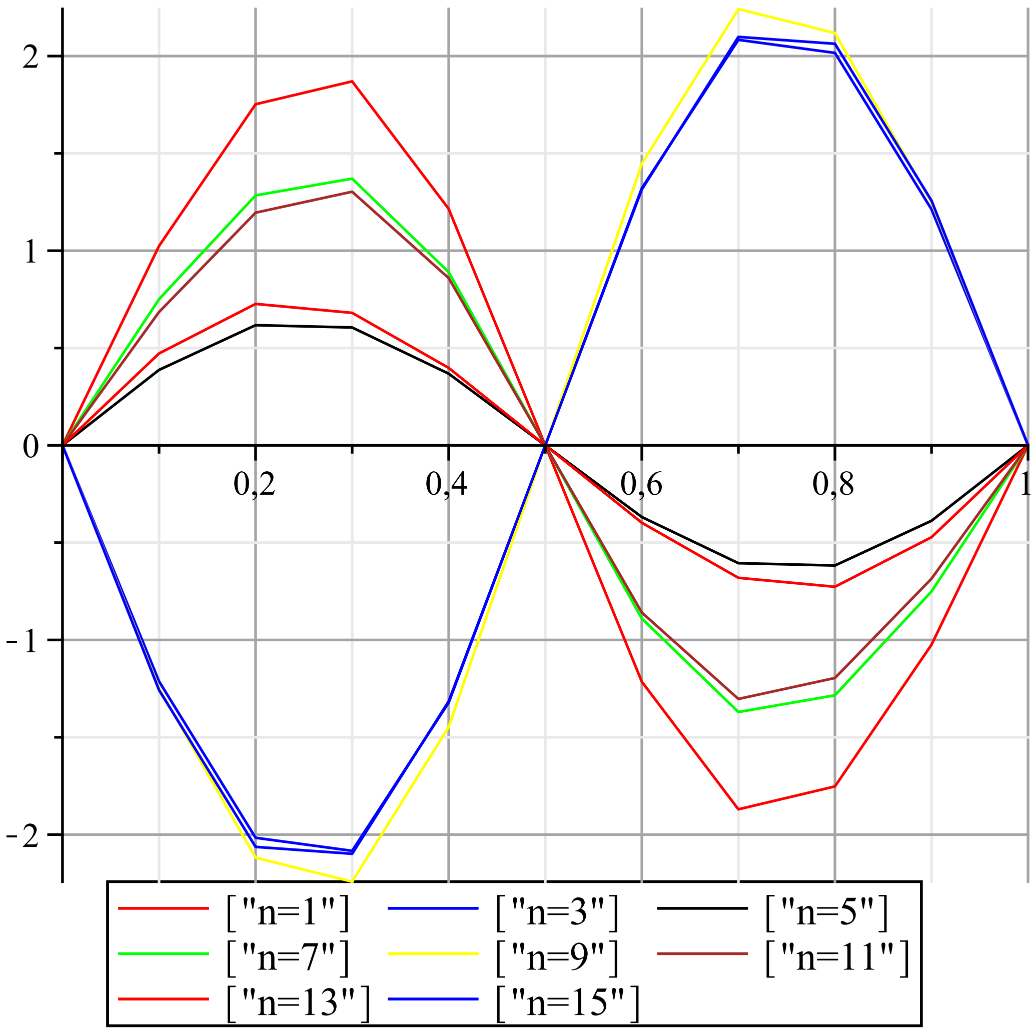
$liste_{12} := [[0., 0], [0.1, 1.258153221], [0.2, 2.111954826], [0.3, 2.206168109], [0.4, 1.410593520], [0.5, -1.289770508\ 10^{-9}], [0.6, -1.410593520], [0.7, -2.206168109], [0.8, -2.111954826], [0.9, -1.258153223], [1.0, 0]]$

$liste_{13} := [[0., 0], [0.1, 0.4720325268], [0.2, 0.726597289], [0.3, 0.680655873], [0.4, 0.3976977568], [0.5, 4.762573240\ 10^{-10}], [0.6, -0.3976977579], [0.7, -0.680655874], [0.8, -0.726597290], [0.9, -0.4720325296], [1.0, 0]]$

$liste_{14} := [[0., 0], [0.1, -0.8404876354], [0.2, -1.460484082], [0.3, -1.584766411], [0.4, -1.041580673], [0.5, 1.252899170\ 10^{-9}], [0.6, 1.041580673], [0.7, 1.584766410], [0.8, 1.460484080], [0.9, 0.8404876357], [1.0, 0]]$

$liste_{15} := [[0., 0], [0.1, -1.257397365], [0.2, -2.063152841], [0.3, -2.098555267], [0.4, -1.314679696], [0.5, 1.501922610\ 10^{-10}], [0.6, 1.314679697], [0.7, 2.098555267], [0.8, 2.063152842], [0.9, 1.257397367], [1.0, 0]]$ **(3.1.3)**

```
> multiple(listplot, [liste[1], color = red, legend = ["n=1"]], [liste[3], color = blue,
    legend = ["n=3"]], [liste[5], color = black, legend = ["n=5"]], [liste[7], color
    = green, legend = ["n=7"]], [liste[9], color = yellow, legend = ["n=9"]],
    [liste[11], color = brown, legend = ["n=11"]], [liste[13], color = red, legend
    = ["n=13"]], [liste[15], color = blue, legend = ["n=15"]], gridlines = true);
```




> **for** n **from** 1 **to** $n_{max}$ **do**

$liste[n] := [\alpha, seq(u[i, n], i = 2 .. i_{max} - 1), \beta]$

**end do**

$liste_1 := [0, 1.024336356, 1.752516692, 1.870073742, 1.214547659, -2.500000000\ 10^{-10}, -1.214547661, -1.870073742, -1.752516691, -1.024336355, 0]$

$liste_2 := [0, -0.2252731540, -0.302252163, -0.225310074, -0.100778239, -6.250000000\ 10^{-10}, 0.100778240, 0.225310074, 0.302252162, 0.2252731536, 0]$

$liste_3 := [0, -1.212535974, -2.016288580, -2.083486380, -1.321264297, 1.875000000\ 10^{-10}, 1.321264299, 2.083486380, 2.016288579, 1.212535972, 0]$

$liste_4 := [0, -0.8850669780, -1.529897715, -1.650821335, -1.080725504, 1.218750000\ 10^{-9}, 1.080725505, 1.650821336, 1.529897716, 0.8850669774, 0]$

$liste_5 := [0, 0.3875280562, 0.617367644, 0.605419908, 0.3681962115, 6.718750000\ 10^{-10}, -0.3681962124, -0.605419907, -0.617367643, -0.3875280542, 0]$

$liste_6 := [0, 1.233172917, 2.086818528, 2.199922253, 1.416178587, -1.107812500\ 10^{-9}, -1.416178588, -2.199922253, -2.086818528, -1.233172915, 0]$

$liste_7 := [0, 0.7507630338, 1.284315412, 1.370290497, 0.8898736455, -1.475781250\ 10^{-9}, -0.8898736456, -1.370290498, -1.284315413, -0.7507630358, 0]$

$liste_8 := [0, -0.5367125472, -0.914397439, -0.971229740, -0.6286691402, 3.449218750\ 10^{-10}, 0.6286691404, 0.971229739, 0.914397438, 0.5367125438, 0]$

$liste_9 := [0, -1.247718667, -2.118499703, -2.241672012, -1.447015650, 1.698242188\ 10^{-9}, 1.447015651, 2.241672012, 2.118499703, 1.247718667, 0]$

$liste_{10} := [0, -0.6167717118, -1.017200083, -1.040985104, -0.6552566878, 7.541992190\ 10^{-10}, 0.6552566886, 1.040985105, 1.017200084, 0.6167717152, 0]$

$liste_{11} := [0, 0.6850327902, 1.195460457, 1.303065267, 0.8591410302, -1.121142578\ 10^{-9}, -0.8591410301, -1.303065266, -1.195460456, -0.6850327884, 0]$

$liste_{12} := [0, 1.258153221, 2.111954826, 2.206168109, 1.410593520, -1.289770508\ 10^{-9}, -1.410593520, -2.206168109, -2.111954826, -1.258153223, 0]$

$liste_{13} := [0, 0.4720325268, 0.726597289, 0.680655873, 0.3976977568, 4.762573240\ 10^{-10}, -0.3976977579, -0.680655874, -0.726597290, -0.4720325296, 0]$

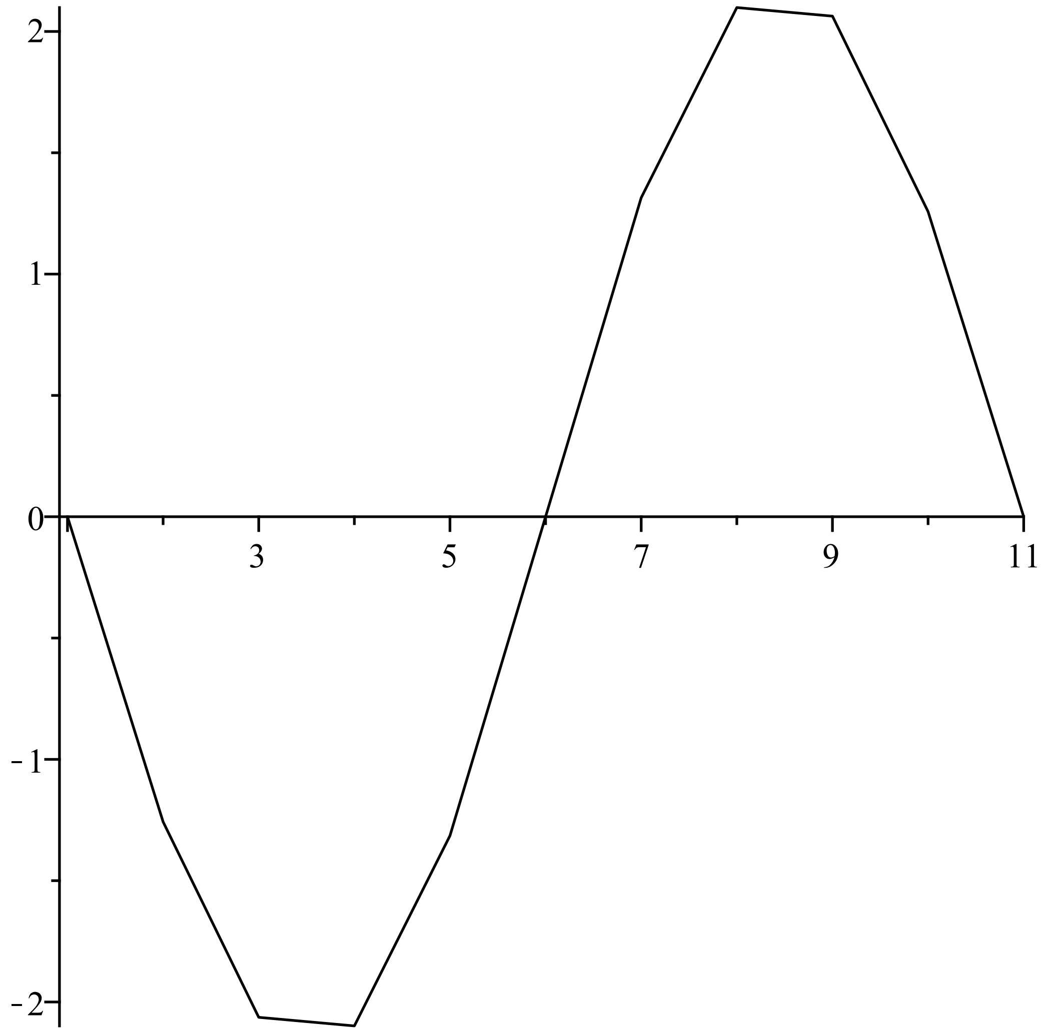
$liste_{14} := [0, -0.8404876354, -1.460484082, -1.584766411, -1.041580673, 1.252899170\ 10^{-9}, 1.041580673, 1.584766410, 1.460484080, 0.8404876357, 0]$

$liste_{15} := [0, -1.257397365, -2.063152841, -2.098555267, -1.314679696, 1.501922610\ 10^{-10}, 1.314679697, 2.098555267, 2.063152842, 1.257397367, 0]$ **(3.1.4)**

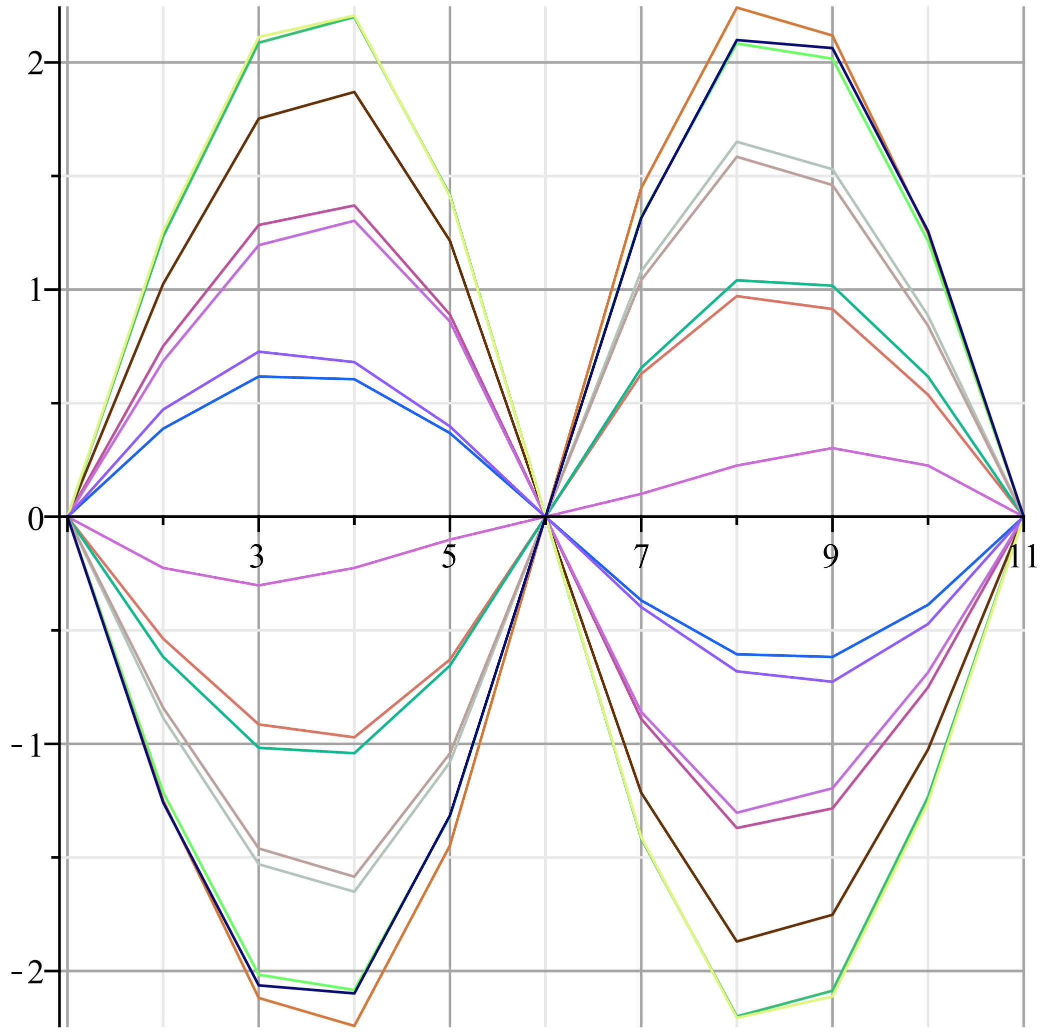
> SolExplicite := $liste[n_{max}]$

[0, 0.4013368020, 0.7060645452, 0.8827702217, 0.9605289216, 0.9806847873, 0.9605289216, 0.8827702216, 0.7060645452, 0.4013368020, 0] **(3.1.5)**

> *listplot*( *liste*[15])




$$> multiple\left(listplot, seq\left(\left[liste[i], color = COLOR\left(RGB, \frac{rand(\ )}{10^{12}}, \frac{rand(\ )}{10^{12}}, \frac{rand(\ )}{10^{12}}\right)\right], i = 1 ..n_{\max}\right), gridlines = true\right);$$




>

Schéma implicite: