Equation de Diffusion 2D

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EXAMEN

Détermination de la distribution de temperature $T(x, y)$ à travers une plaque de largeur L, de hauteur H, d'éapisseur e et de conductivité thermique k soumise aux (C. L.) soit de Dirichler soit de Neumann..

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x}, \mathrm{y})\right) + \frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x}, \mathrm{y})\right) = 0$$

$$\begin{aligned} T(0, y) &= T_w \quad \textit{ou bien} \quad q(0, y) = q_w, \\ T(L, y) &= T_e \quad \textit{ou bien} \quad q(L, y) = q_e, \\ T(x, 0) &= T_s \quad \textit{ou bien} \quad q(x, 0) = q_s, \\ T(x, H) &= T_n \quad \textit{ou bien} \quad q(x, H) = q_n, \end{aligned}$$

Solution

> $Restart : Digits := 4 :$

Données:

> $L := 1.0;\ H := 0.4;\ e := 0.001;\ k := 1000;\ \delta x := 0.2;\ \delta y := 0.1;$

$$L := 1.0$$
$$H := 0.4$$
$$e := 0.001$$
$$k := 1000$$
$$\delta x := 0.2$$
$$\delta y := 0.1 \qquad (1.1)$$

Calcul du nombre de divisions:

> $ndx := \mathrm{trunc}\left(\frac{L}{\delta x}\right);\ ndy := \mathrm{trunc}\left(\frac{H}{\delta y}\right);$

$$ndx := 5$$
$$ndy := 4 \qquad (1.2)$$

Calcul des surfaces:

> $Aw := \delta y \cdot e;$
> $Ae := \delta y \cdot e;$
> $As := \delta x \cdot e;$
> $An := \delta x \cdot e;$

$$Aw := 0.0001$$
$$Ae := 0.0001$$
$$As := 0.0002$$
$$An := 0.0002 \qquad (1.3)$$

> $i_{max} := \mathrm{round}(ndx);\ j_{max} := \mathrm{round}(ndy);$

$$i_{max} := 5$$
$$j_{max} := 4 \qquad (1.4)$$

Nombre d'équations:

> $Ne := i_{max} \cdot j_{max}$

$$Ne := 20 \qquad (1.5)$$

Conditions aux Limites:

> $T_w := 100 : T_e := 100 : T_s := 0 : T_n := 0 :$

> $q_w := 0 : q_e := 0 : q_s := 6 \cdot 10^5 : q_n := 6 \cdot 10^5 :$

Noeuds internes:

> **for** j **from** 2 **to** $j_{max} - 1$ **do**
> **for** i **from** 2 **to** $i_{max} - 1$ **do**
> $Su[i, j] := 0;$
> $a_W[i, j] := \frac{k \cdot Aw}{\delta x};$
> $a_E[i, j] := \frac{k \cdot Ae}{\delta x};$

$a_S[i, j] := \frac{k \cdot As}{\delta y};$

$a_N[i, j] := \frac{k \cdot An}{\delta y};$

$a_P[i, j] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

$Sp[i, j] := a_W[i, j] + a_E[i, j] + a_S[i, j] + a_N[i, j] - a_P[i, j];$

end do:

end do:

Noeuds Ouest:

\> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Su[1, j] := q_w \cdot Aw + \frac{2 \cdot k \cdot Aw}{\delta x} \cdot T_w;$

$a_W[1, j] := 0;$

$a_E[1, j] := \frac{k \cdot Ae}{\delta x};$

$a_S[1, j] := \frac{k \cdot As}{\delta y};$

$a_N[1, j] := \frac{k \cdot An}{\delta y};$

$a_P[1, j] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

if $T_w = 0$ **then** $a_P[1, j] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**;

$Sp[1, j] := a_W[1, j] + a_E[1, j] + a_S[1, j] + a_N[1, j] - a_P[1, j];$

end do:

Noeuds Est:

\> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Su[i_{\max}, j] := q_e \cdot Ae + \frac{2 \cdot k \cdot Ae}{\delta x} \cdot T_e;$

$a_W[i_{\max}, j] := \frac{k \cdot Aw}{\delta x};$

$a_E[i_{\max}, j] := 0;$

$a_S[i_{\max}, j] := \frac{k \cdot As}{\delta y};$

$a_N[i_{\max}, j] := \frac{k \cdot An}{\delta y};$

$a_P[i_{\max}, j] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

$$\textbf{if } T_e = 0 \textbf{ then } a_P[i_{\max}, j] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y} \textbf{ end if;}$$

$$Sp[i_{\max}, j] := a_W[i_{\max}, j] + a_E[i_{\max}, j] + a_S[i_{\max}, j] + a_N[i_{\max}, j] - a_P[i_{\max}, j];$$

end do:

Noeuds Sud:

> **for** i **from** 2 **to** $i_{\max} - 1$ **do**

$$Su[i, 1] := q_s \cdot As + \frac{2 \cdot k \cdot As}{\delta y} \cdot T_s;$$

$$a_W[i, 1] := \frac{k \cdot Aw}{\delta x};$$

$$a_E[i, 1] := \frac{k \cdot Ae}{\delta x};$$

$$a_S[i, 1] := 0;$$

$$a_N[i, 1] := \frac{k \cdot An}{\delta y};$$

$$a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$$

$$\textbf{if } T_s = 0 \textbf{ then } a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y} \textbf{ end if;}$$

$$Sp[i, 1] := a_W[i, 1] + a_E[i, 1] + a_S[i, 1] + a_N[i, 1] - a_P[i, 1];$$

end do:

Noeuds Nord:

> **for** i **from** 2 **to** $i_{\max} - 1$ **do**

$$Su[i, j_{\max}] := q_n \cdot An + \frac{2 \cdot k \cdot An}{\delta y} \cdot T_n;$$

$$a_W[i, j_{\max}] := \frac{k \cdot Aw}{\delta x};$$

$$a_E[i, j_{\max}] := \frac{k \cdot Ae}{\delta x};$$

$$a_S[i, j_{\max}] := \frac{k \cdot As}{\delta y};$$

$$a_N[i, j_{\max}] := 0;$$

$$a_P[i, j_{\max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y};$$

$$\textbf{if } T_n = 0 \textbf{ then } a_P[i, j_{\max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} \textbf{ end if;}$$

$Sp[i, j_{max}] := a_W[i, j_{max}] + a_E[i, j_{max}] + a_S[i, j_{max}] + a_N[i, j_{max}] - a_P[i, j_{max}];$

end do:

Noeud (1,1):

> $Su[1, 1] := Su[1, 2] + Su[2, 1]:$

$a_W[1, 1] := 0:$

$a_E[1, 1] := \frac{k \cdot Ae}{\delta x}:$

$a_S[1, 1] := 0:$

$a_N[1, 1] := \frac{k \cdot An}{\delta y}:$

if $(T_w = 0$ **and** $T_s = 0)$ **then** $a_P[1, 1] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_w = 0$ **and** $T_s \neq 0)$ **then** $a_P[1, 1] := \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if** :

if $(T_w \neq 0$ **and** $T_s = 0)$ **then** $a_P[1, 1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_w \neq 0$ **and** $T_s \neq 0)$ **then** $a_P[1, 1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if:

$Sp[1, 1] := a_W[1, 1] + a_E[1, 1] + a_S[1, 1] + a_N[1, 1] - a_P[1, 1]:$

Noeud (imax,1):

> $Su[i_{max}, 1] := Su[i_{max}, 2] + Su[2, 1]:$

$a_W[i_{max}, 1] := \frac{k \cdot Aw}{\delta x}:$

$a_E[i_{max}, 1] := 0:$

$a_S[i_{max}, 1] := 0:$

$a_N[i_{max}, 1] := \frac{k \cdot An}{\delta y}:$

if $(T_e = 0$ **and** $T_s = 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e = 0$ **and** $T_s \neq 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_s = 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_s \neq 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if:

$Sp[i_{max}, 1] := a_W[i_{max}, 1] + a_E[i_{max}, 1] + a_S[i_{max}, 1] + a_N[i_{max}, 1] - a_P[i_{max},$

$1]:$

Noeud (1,jmax):

$$> Su[1, j_{max}] := Su[1, 2] + Su[2, j_{max}]:$$

$$a_W[1, j_{max}] := 0:$$

$$a_E[1, j_{max}] := \frac{k \cdot Ae}{\delta x}:$$

$$a_S[1, j_{max}] := \frac{k \cdot As}{\delta y}:$$

$$a_N[1, j_{max}] := 0:$$

$$\textbf{if } (T_w = 0 \textbf{ and } T_n = 0) \textbf{ then } a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_w = 0 \textbf{ and } T_n \neq 0) \textbf{ then } a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_w \neq 0 \textbf{ and } T_n = 0) \textbf{ then } a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_w \neq 0 \textbf{ and } T_n \neq 0) \textbf{ then } a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y} \textbf{ end if:}$$

$$Sp[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - a_P[1, j_{max}]:$$

Noeud (imax,jmax):

$$> Su[i_{max}, j_{max}] := Su[i_{max}, 2] + Su[2, j_{max}]:$$

$$a_W[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x}:$$

$$a_E[i_{max}, j_{max}] := 0:$$

$$a_S[i_{max}, j_{max}] := \frac{k \cdot As}{\delta y}:$$

$$a_N[i_{max}, j_{max}] := 0:$$

$$\textbf{if } (T_e = 0 \textbf{ and } T_n = 0) \textbf{ then } a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_e = 0 \textbf{ and } T_n \neq 0) \textbf{ then } a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_e \neq 0 \textbf{ and } T_n = 0) \textbf{ then } a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} \textbf{ end if:}$$

$$\textbf{if } (T_e \neq 0 \textbf{ and } T_n \neq 0) \textbf{ then } a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y} \textbf{ end if:}$$

$$Sp[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}]$$

$- a_P[i_{max}, j_{max}]:$

Equations:

> $k := 1:$

Résolution pour les noeuds internes:

> **for** j **from** 1 **to** j_{max} **do**
> **for** i **from** 1 **to** i_{max} **do**
> $Eq[k] := a_P[i, j] \cdot T[i, j] = a_W[i, j] \cdot T[i-1, j] + a_E[i, j] \cdot T[i+1, j] + a_S[i, j] \cdot T[i, j-1] + a_N[i, j] \cdot T[i, j+1] + Su[i, j];$
> $Var[k] := T[i, j];$
> $k := k+1;$
> **end do;**
> **end do;**

Ecriture du système d'équations:

> **for** k **from** 1 **to** Ne **do** $Eq[k]$ **end do;**

$$3.500\, T_{1,1} = 2.000\, T_{1,2} + 0.5000\, T_{2,1} + 220.0$$
$$3.000\, T_{2,1} = 2.000\, T_{2,2} + 0.5000\, T_{1,1} + 0.5000\, T_{3,1} + 120.0$$
$$3.000\, T_{3,1} = 2.000\, T_{3,2} + 0.5000\, T_{2,1} + 0.5000\, T_{4,1} + 120.0$$
$$3.000\, T_{4,1} = 2.000\, T_{4,2} + 0.5000\, T_{3,1} + 0.5000\, T_{5,1} + 120.0$$
$$3.500\, T_{5,1} = 2.000\, T_{5,2} + 0.5000\, T_{4,1} + 220.0$$
$$5.500\, T_{1,2} = 2.000\, T_{1,1} + 2.000\, T_{1,3} + 0.5000\, T_{2,2} + 100.0$$
$$5.000\, T_{2,2} = 2.000\, T_{2,1} + 2.000\, T_{2,3} + 0.5000\, T_{1,2} + 0.5000\, T_{3,2}$$
$$5.000\, T_{3,2} = 2.000\, T_{3,1} + 2.000\, T_{3,3} + 0.5000\, T_{2,2} + 0.5000\, T_{4,2}$$
$$5.000\, T_{4,2} = 2.000\, T_{4,1} + 2.000\, T_{4,3} + 0.5000\, T_{3,2} + 0.5000\, T_{5,2}$$
$$5.500\, T_{5,2} = 2.000\, T_{5,1} + 2.000\, T_{5,3} + 0.5000\, T_{4,2} + 100.0$$
$$5.500\, T_{1,3} = 2.000\, T_{1,2} + 2.000\, T_{1,4} + 0.5000\, T_{2,3} + 100.0$$
$$5.000\, T_{2,3} = 2.000\, T_{2,2} + 2.000\, T_{2,4} + 0.5000\, T_{1,3} + 0.5000\, T_{3,3}$$
$$5.000\, T_{3,3} = 2.000\, T_{3,2} + 2.000\, T_{3,4} + 0.5000\, T_{2,3} + 0.5000\, T_{4,3}$$
$$5.000\, T_{4,3} = 2.000\, T_{4,2} + 2.000\, T_{4,4} + 0.5000\, T_{3,3} + 0.5000\, T_{5,3}$$
$$5.500\, T_{5,3} = 2.000\, T_{5,2} + 2.000\, T_{5,4} + 0.5000\, T_{4,3} + 100.0$$
$$3.500\, T_{1,4} = 2.000\, T_{1,3} + 0.5000\, T_{2,4} + 220.0$$
$$3.000\, T_{2,4} = 2.000\, T_{2,3} + 0.5000\, T_{1,4} + 0.5000\, T_{3,4} + 120.0$$
$$3.000\, T_{3,4} = 2.000\, T_{3,3} + 0.5000\, T_{2,4} + 0.5000\, T_{4,4} + 120.0$$
$$3.000\, T_{4,4} = 2.000\, T_{4,3} + 0.5000\, T_{3,4} + 0.5000\, T_{5,4} + 120.0$$
$$3.500\, T_{5,4} = 2.000\, T_{5,3} + 0.5000\, T_{4,4} + 220.0 \qquad \textbf{(1.1.1)}$$

Système d'équations:

> $Eqs := [seq(Eq[k], k = 1..Ne)]:$

Variables:

> $Vars := [seq(Var[k], k = 1..Ne)]:$

Résolution du système d'équations pour les variables:

> $SolT := solve(Eqs, Vars);$

(1.1.2)

$$SolT := [[T_{1,1} = 262.2, T_{2,1} = 444.7, T_{3,1} = 504.9, T_{4,1} = 444.7, T_{5,1} = 262.2, T_{1,2} = 237.8, T_{2,2} = 415.3, T_{3,2} = 475.1, T_{4,2} = 415.3, T_{5,2} = 237.8, T_{1,3} = 237.8, T_{2,3} = 415.3, T_{3,3} = 475.1, T_{4,3} = 415.3, T_{5,3} = 237.8, T_{1,4} = 262.2, T_{2,4} = 444.7, T_{3,4} = 504.9, T_{4,4} = 444.7, T_{5,4} = 262.2]] \qquad \textbf{(1.1.2)}$$

> $with(LinearAlgebra):$

Forme matricielle:

> $A, b := GenerateMatrix(Eqs, Vars)$

$$A, b := \begin{bmatrix} 3.500 & -0.5000 & 0 & 0 & 0 & -2.000 & 0 & 0 \\ -0.5000 & 3.000 & -0.5000 & 0 & 0 & 0 & -2.000 & 0 \\ 0 & -0.5000 & 3.000 & -0.5000 & 0 & 0 & 0 & -2.0 \\ 0 & 0 & -0.5000 & 3.000 & -0.5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5000 & 3.500 & 0 & 0 & 0 \\ -2.000 & 0 & 0 & 0 & 0 & 5.500 & -0.5000 & 0 \\ 0 & -2.000 & 0 & 0 & 0 & -0.5000 & 5.000 & -0.50 \\ 0 & 0 & -2.000 & 0 & 0 & 0 & -0.5000 & 5.00 \\ 0 & 0 & 0 & -2.000 & 0 & 0 & 0 & -0.50 \\ 0 & 0 & 0 & 0 & -2.000 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$, \begin{bmatrix} 220.0 \\ 120.0 \\ 120.0 \\ 120.0 \\ 220.0 \\ 100.0 \\ 0 \\ 0 \\ 0 \\ 100.0 \\ \vdots \end{bmatrix}$$

20 element Vector[column]

Affichage du second membre:

> $seq(b[i], i = 1..Ne)$

$220.0, 120.0, 120.0, 120.0, 220.0, 100.0, 0, 0, 0, 100.0, 100.0, 0, 0, 0,$ **(1.1.4)**

100.0, 220.0, 120.0, 120.0, 120.0, 220.0

Affichage de la diagonale:

> $seq(A[i, i], i = 1..Ne)$

3.500, 3.000, 3.000, 3.000, 3.500, 5.500, 5.000, 5.000, 5.000, 5.500, 5.500, 5.000, 5.000, 5.000, 5.500, 3.500, 3.000, 3.000, 3.000, 3.500 **(1.1.5)**

Récapitulation: Affichage de tous les coefficients:

> $seq(seq(a_W[i, j], i = 1..i_{max}), j = 1..j_{max})$

0, 0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000 **(1.1.6)**

> $seq(seq(a_E[i, j], i = 1..i_{max}), j = 1..j_{max})$

0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000, 0, 0.5000, 0.5000, 0.5000, 0.5000, 0 **(1.1.7)**

> $seq(seq(a_S[i, j], i = 1..i_{max}), j = 1..j_{max})$

0, 0, 0, 0, 0, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000 **(1.1.8)**

> $seq(seq(a_N[i, j], i = 1..i_{max}), j = 1..j_{max})$

2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 2.000, 0, 0, 0, 0, 0 **(1.1.9)**

> $seq(seq(Su[i, j], i = 1..i_{max}), j = 1..j_{max})$

220.0, 120.0, 120.0, 120.0, 220.0, 100.0, 0, 0, 0, 100.0, 100.0, 0, 0, 0, 100.0, 220.0, 120.0, 120.0, 120.0, 220.0 **(1.1.10)**

> $seq(seq(a_P[i, j], i = 1..i_{max}), j = 1..j_{max})$

3.500, 3.000, 3.000, 3.000, 3.500, 5.500, 5.000, 5.000, 5.000, 5.500, 5.500, 5.000, 5.000, 5.000, 5.500, 3.500, 3.000, 3.000, 3.000, 3.500 **(1.1.11)**

> $seq(seq(Sp[i, j], i = 1..i_{max}), j = 1..j_{max})$

−1.000, 0., 0., 0., −1.000, −1.000, 0., 0., 0., −1.000, −1.000, 0., 0., 0., −1.000, −1.000, 0., 0., 0., −1.000 **(1.1.12)**

>