Equation de Laplace 2D

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D termination de la temperature $T(x, y)$ travers la surface d'une plaque rectangulaire (a x b) dont les extr mit s sont soumises des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = \frac{\partial}{\partial x} T(x, y)$$

Conditions aux limites (C.L):

$$T(x, 0) = 40,$$
$$T(x, b) = 2.x + 30,$$
$$T(0, y) = 100,$$
$$T(a, y) = 3 \cdot y + 10.$$

Solution discr tis e (formulation en 5 points):

```
> Restart:
> a := 12; b := 8; ndx := 3; ndy := 4
```

$$a := 12$$
$$b := 8$$
$$ndx := 3$$
$$ndy := 4$$

(1.1)

> $\Delta x := \frac{a}{ndx}$; $\Delta y := \frac{b}{ndy}$; $\beta := \frac{\Delta x}{\Delta y}$;

$$\Delta x := 4$$
$$\Delta y := 2$$
$$\beta := 2 \quad \textbf{(1.2)}$$

> $i_{max} := ndx + 1$; $j_{max} := ndy + 1$;

$$i_{max} := 4$$
$$j_{max} := 5 \quad \textbf{(1.3)}$$

Nombre d' quations:

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 6 \quad \textbf{(1.4)}$$

Maillage:

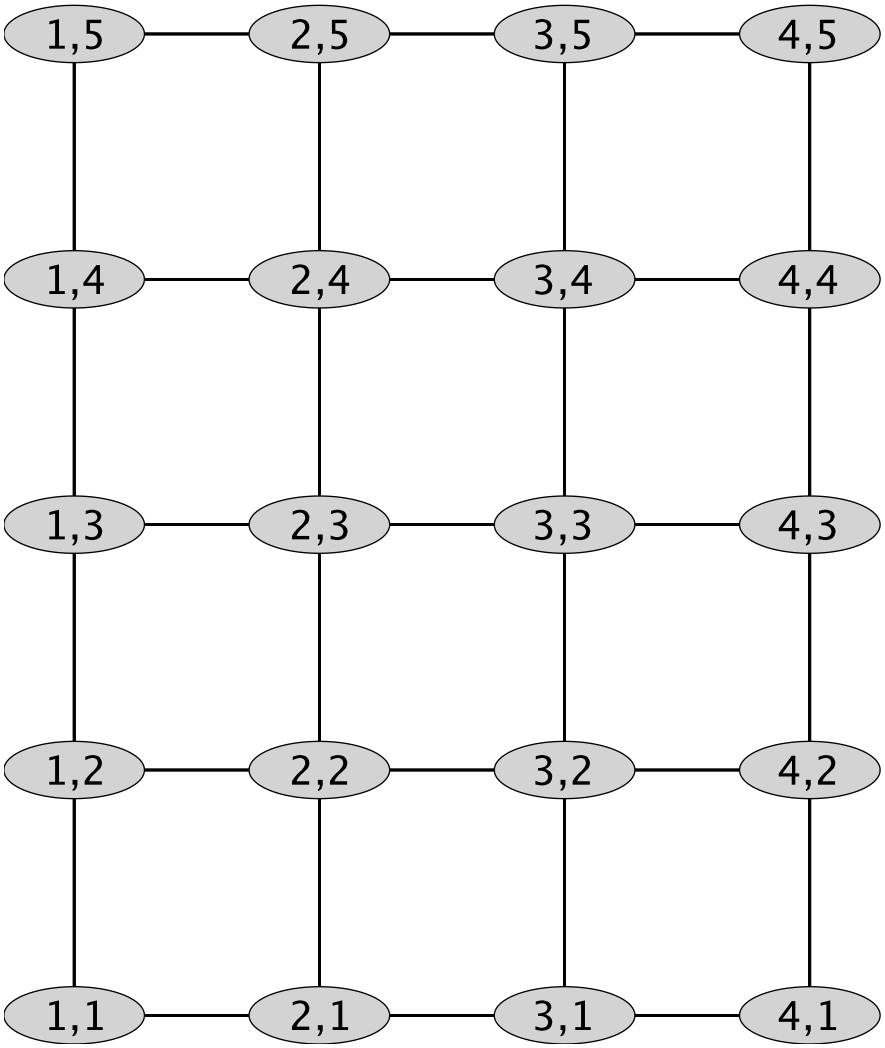
> *with*(*GraphTheory*) : *with*(*SpecialGraphs*) :

> $G :=$ *GridGraph*(i_{max}, j_{max})

$G :=$ **(1.5)**

Graph 1: an undirected unweighted graph with 20 vertices and 31 edge (s)

> *DrawGraph*(*G*)



Conditions aux Limites:

```
> for i from 2 to i_max - 1 do T[i, 1] := 40 end do;
```

$$T_{2,1} := 40$$
$$T_{3,1} := 40 \quad \textbf{(1.6)}$$

```
> for j from 2 to j_max - 1 do T[1, j] := 100 end do;
```

$$T_{1,2} := 100$$
$$T_{1,3} := 100$$
$$T_{1,4} := 100 \quad \textbf{(1.7)}$$

```
> for j from 2 to j_max - 1 do T[i_max, j] := (3.((j - 1)·Δy)) + 10 end do;
```

$$T_{4,2} := 16.$$
$$T_{4,3} := 22.$$
$$T_{4,4} := 28. \quad \textbf{(1.8)}$$

```
> for i from 2 to i_max - 1 do T[i, j_max] := (2.((i - 1)·Δx)) + 30 end do;
```

$$T_{2,5} := 38.$$
$$T_{3,5} := 46. \quad \textbf{(1.9)}$$

```
> k := 1
```

$$k := 1 \tag{1.1.1}$$

R solution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Eq[k] := (1 + Δx/2)·T[i - 1, j] + β^2·(T[i, j - 1] + T[i, j + 1]) - 2·(1
      + β^2)·T[i, j] + (1 - Δx/2)·T[i + 1, j] = 0;
      Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Ecriture du syst me d' quations:

```
> for k from 1 to N do Eq[k] end do;
```

$$\begin{aligned}
460 + 4\,T_{2,3} - 10\,T_{2,2} - T_{3,2} &= 0\\
3\,T_{2,2} + 144. + 4\,T_{3,3} - 10\,T_{3,2} &= 0\\
300 + 4\,T_{2,2} + 4\,T_{2,4} - 10\,T_{2,3} - T_{3,3} &= 0\\
3\,T_{2,3} + 4\,T_{3,2} + 4\,T_{3,4} - 10\,T_{3,3} - 22. &= 0\\
452. + 4\,T_{2,3} - 10\,T_{2,4} - T_{3,4} &= 0\\
3\,T_{2,4} + 4\,T_{3,3} + 156. - 10\,T_{3,4} &= 0
\end{aligned} \tag{1.1.2}$$

```
> Eqs := {seq(Eq[i], i = 1..N)}:
> Tmps := [seq(Temps[i], i = 1..N)];
```

$$Tmps := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}, T_{2,4}, T_{3,4}] \tag{1.1.3}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 71.09544987,\ T_{3,2} = 65.42522482,\ T_{2,3} = 79.09493088,\ T_{3,3} = 74.24147465,\ T_{2,4} = 70.20224598,\ T_{3,4} = 66.35726366]] \tag{1.1.4}$$

```
> Solution := evalf(SolT);
```

$$Solution := [[T_{2,2} = 71.09544987,\ T_{3,2} = 65.42522482,\ T_{2,3} = 79.09493088,\ T_{3,3} = 74.24147465,\ T_{2,4} = 70.20224598,\ T_{3,4} = 66.35726366]] \tag{1.1.5}$$

```
> Sys := [seq(Eq[i], i = 1..N)];
```

$$Sys := [460 + 4\,T_{2,3} - 10\,T_{2,2} - T_{3,2} = 0,\ 3\,T_{2,2} + 144. + 4\,T_{3,3} - 10\,T_{3,2} = 0,\ 300 + 4\,T_{2,2} + 4\,T_{2,4} - 10\,T_{2,3} - T_{3,3} = 0,\ 3\,T_{2,3} + 4\,T_{3,2} + 4\,T_{3,4} - 10\,T_{3,3} - 22. = 0,\ 452. + 4\,T_{2,3} - 10\,T_{2,4} - T_{3,4} = 0,\ 3\,T_{2,4} + 4\,T_{3,3} + 156. - 10\,T_{3,4} = 0] \tag{1.1.6}$$

```
> Var := [seq(Temps[i], i = 1..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}, T_{2,4}, T_{3,4}] \tag{1.1.7}$$

```
> with(LinearAlgebra):
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := \begin{bmatrix} -10 & -1 & 4 & 0 & 0 & 0 \\ 3 & -10 & 0 & 4 & 0 & 0 \\ 4 & 0 & -10 & -1 & 4 & 0 \\ 0 & 4 & 3 & -10 & 0 & 4 \\ 0 & 0 & 4 & 0 & -10 & -1 \\ 0 & 0 & 0 & 4 & 3 & -10 \end{bmatrix}, \begin{bmatrix} -460 \\ -144. \\ -300 \\ 22. \\ -452. \\ -156. \end{bmatrix} \quad \textbf{(1.1.8)}$$