

Equation de Laplace 2D

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LMD : G nie Energ tique

Mati re : Outils Num riques

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D termination de la temperature $T(x, y)$ travers la surface d'une plaque rectangulaire ($a \times b$) dont les extr mit s sont soumises des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = \frac{\partial}{\partial x} T(x, y)$$

Conditions aux limites (C.L.):

$$\begin{aligned} T(x, 0) &= 40, \\ T(x, b) &= 2 \cdot x + 30, \\ T(0, y) &= 100, \\ T(a, y) &= 3 \cdot y + 10. \end{aligned}$$

▼ Solution discr tis e (formulation en 5 points):

[> *Restart:*

[> $a := 6; b := 4; ndx := 3; ndy := 4$

$a := 6$

$b := 4$

$ndx := 3$

$ndy := 4$

(1.1)

```
>  $\Delta x := \frac{a}{ndx}$ ;  $\Delta y := \frac{b}{ndy}$ ;  $\beta := \frac{\Delta x}{\Delta y}$ ;
```

```
 $\Delta x := 2$ 
```

```
 $\Delta y := 1$ 
```

```
 $\beta := 2$ 
```

(1.2)

```
>  $i_{\max} := ndx + 1$ ;  $j_{\max} := ndy + 1$ ;
```

```
 $i_{\max} := 4$ 
```

```
 $j_{\max} := 5$ 
```

(1.3)

Nombre d' equations:

```
>  $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$ 
```

```
 $N := 6$ 
```

(1.4)

Maillage:

```
> with(GraphTheory) : with(SpecialGraphs) :
```

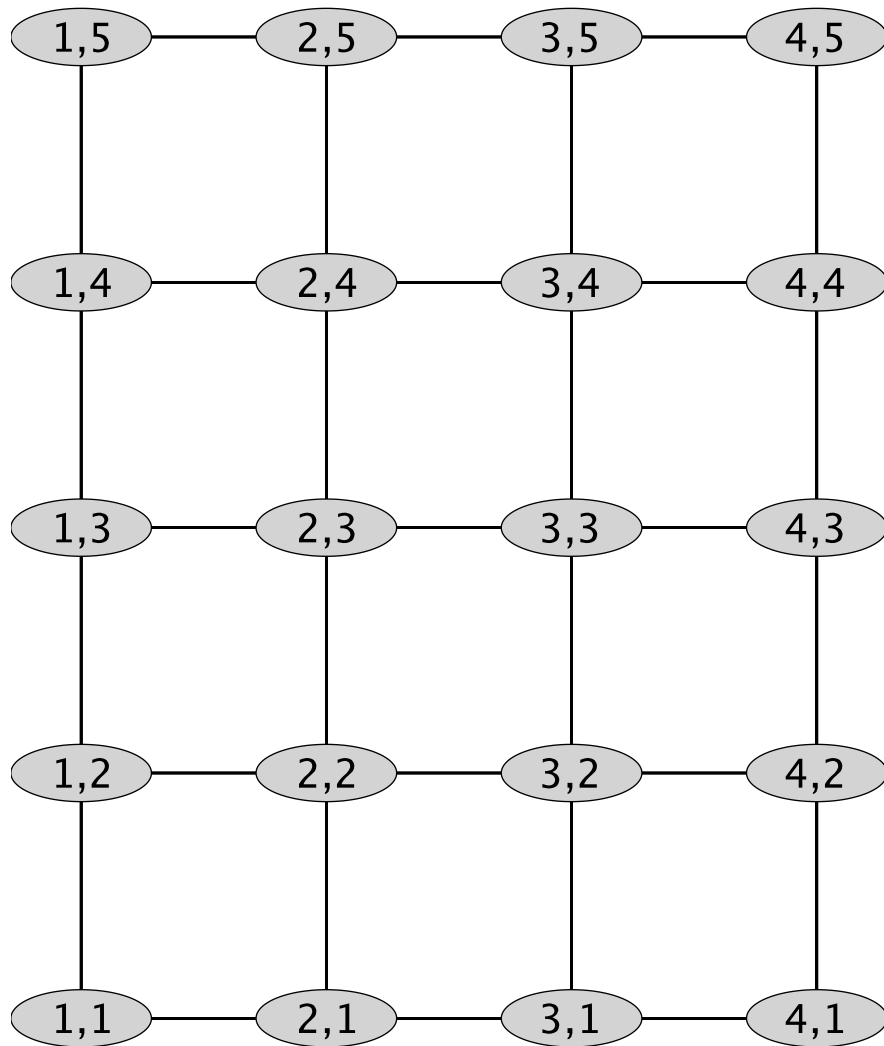
```
>  $G := \text{GridGraph}(i_{\max}, j_{\max})$ 
```

```
 $G :=$ 
```

```
Graph 1: an undirected unweighted graph with 20 vertices and 31 edge(s)
```

(1.5)

```
> DrawGraph(G)
```



Conditions aux Limites:

> **for** i **from** 2 **to** $i_{\max} - 1$ **do** $T[i, 1] := 40$ **end do**;

$$T_{2,1} := 40$$

$$T_{3,1} := 40$$

(1.6)

> **for** j **from** 2 **to** $j_{\max} - 1$ **do** $T[1, j] := 100$ **end do**;

$$T_{1,2} := 100$$

$$T_{1,3} := 100$$

$$T_{1,4} := 100$$

(1.7)

> **for** j **from** 2 **to** $j_{\max} - 1$ **do** $T[i_{\max}, j] := (3 \cdot ((j - 1) \cdot \Delta y)) + 10$ **end do**;

$$T_{4,2} := 13.$$

$$T_{4,3} := 16.$$

$$T_{4,4} := 19.$$

(1.8)

> **for** i **from** 2 **to** $i_{\max} - 1$ **do** $T[i, j_{\max}] := (2 \cdot ((i - 1) \cdot \Delta x)) + 30$ **end do**;

$$T_{2,5} := 34.$$

$$T_{3,5} := 38.$$

(1.9)

```
> k := 1
k := 1 (1.1.1)
```

R solution pour les noeuds internes:

```
> for j from 2 to jmax - 1 do
  for i from 2 to imax - 1 do
    Eq[k] :=  $\left(1 - \frac{\Delta x}{2}\right) \cdot T[i-2, j] + 2 \cdot (\Delta x - 1) \cdot T[i-1, j] + \beta^2 \cdot (T[i, j-1]$ 
     $+ T[i, j+1]) + \left(1 - 2 \cdot \beta^2 - \frac{3 \cdot \Delta x}{2}\right) \cdot T[i, j] = 0;$ 
    Temps[k] := T[i, j];
    k := k + 1
  end do;
end do;
```

Ecriture du syst me d' equations:

```
> for k from 1 to N do Eq[k] end do;
360 + 4 T2,3 - 10 T2,2 = 0
160 + 2 T2,2 + 4 T3,3 - 10 T3,2 = 0
200 + 4 T2,2 + 4 T2,4 - 10 T2,3 = 0
2 T2,3 + 4 T3,2 + 4 T3,4 - 10 T3,3 = 0
336. + 4 T2,3 - 10 T2,4 = 0
152. + 2 T2,4 + 4 T3,3 - 10 T3,4 = 0 (1.1.2)
```

```
> Eqs := {seq(Eq[i], i = 1..N)};
> Tmps := [seq(Temps[i], i = 1..N)];
Tmps := [T2,2, T3,2, T2,3, T3,3, T2,4, T3,4] (1.1.3)
```

```
> SolT := solve(Eqs, Tmps);
SolT := [[T2,2 = 64.14117647, T3,2 = 50.37010381, T2,3 = 70.35294118, (1.1.4)
T3,3 = 53.85467128, T2,4 = 61.74117647, T3,4 = 49.09010381]]
```

```
> Solution := evalf(SolT);
Solution := [[T2,2 = 64.14117647, T3,2 = 50.37010381, T2,3 (1.1.5)
= 70.35294118, T3,3 = 53.85467128, T2,4 = 61.74117647, T3,4
= 49.09010381]]
```

```
> Sys := [seq(Eq[i], i = 1..N)];
Sys := [360 + 4 T2,3 - 10 T2,2 = 0, 160 + 2 T2,2 + 4 T3,3 - 10 T3,2 = 0, 200 (1.1.6)
+ 4 T2,2 + 4 T2,4 - 10 T2,3 = 0, 2 T2,3 + 4 T3,2 + 4 T3,4 - 10 T3,3 = 0,
336. + 4 T2,3 - 10 T2,4 = 0, 152. + 2 T2,4 + 4 T3,3 - 10 T3,4 = 0]
```

```
> Var := [seq(Temps[i], i = 1..N)];
Var := [T2,2, T3,2, T2,3, T3,3, T2,4, T3,4] (1.1.7)
```

```
> with(LinearAlgebra):
```

```
> A, b := GenerateMatrix(Sys, Var);
```

$$\left[\begin{array}{c} \left[\begin{array}{c} \left[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right] \\ \vdots \\ \vdots \end{array} \right] \\ \vdots \\ \vdots \end{array} \right] > ?$$

$$A, b := \begin{bmatrix} -10 & 0 & 4 & 0 & 0 & 0 \\ 2 & -10 & 0 & 4 & 0 & 0 \\ 4 & 0 & -10 & 0 & 4 & 0 \\ 0 & 4 & 2 & -10 & 0 & 4 \\ 0 & 0 & 4 & 0 & -10 & 0 \\ 0 & 0 & 0 & 4 & 2 & -10 \end{bmatrix}, \begin{bmatrix} -360 \\ -160 \\ -200 \\ 0 \\ -336. \\ -152. \end{bmatrix} \tag{1.1.8}$$

$$`?` \tag{1.1.9}$$