Equation de Diffusion 2D

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Master : Energétique

Matire : Méthodes Numériques Appliquées II

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2014/2015

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EXAMEN

Détermination de la distribution de temperature $T(\mathrm{x}, \mathrm{y})$ à travers une plaque composée (k1 et k2) de largeur L, de hauteur H et d'éapisseur e soumise aux (C.L.) suivantes.

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x}, \mathrm{y})\right) + \frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x}, \mathrm{y})\right) = 0$$

Conditions aux limites (C.L):

$$T(L, y) = T_0 = 40,$$

$$q(0, y) = 0,$$
$$q(x, 0) = 0,$$
$$q(x, H) = \frac{100\,kW}{m^2},$$

Solution

```
> Restart : Digits := 4 :
```

```
> L := 1.6; H := 1.0; e := 0.02; k1 := 500; k2 := 200; δx := 0.4; δy := 0.2;
```

$$L := 1.6$$
$$H := 1.0$$
$$e := 0.02$$
$$k_1 := 500$$
$$k_2 := 200$$
$$\delta x := 0.4$$
$$\delta y := 0.2$$

```
> ndx := L/δx ; ndy := H/δy; Aw := δy·e; Ae := Aw; As := δx·e; An := As; km
      := (k1 + k2)/2;
```

$$ndx := 4.000$$
$$ndy := 5.000$$
$$Aw := 0.004$$
$$Ae := 0.004$$
$$As := 0.008$$
$$An := 0.008$$
$$k_m := 350$$

```
> imax := round(ndx); jmax := round(ndy);
```

$$i_{max} := 4$$
$$j_{max} := 5$$

Nombre d'équations:

```
> Ne := imax · jmax
```

$$Ne := 20$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to imax do
      x[i] := δx/2 + (i − 1)·δx;
  end do;
  x[ imax + 1 ] := L;
```

$$x_0 := 0$$
$$x_1 := 0.2000$$
$$x_2 := 0.6000$$
$$x_3 := 1.000$$
$$x_4 := 1.400$$
$$x_5 := 1.6$$

Ordonnées des noeuds:

```
> y[0] := 0;
```

for j from 1 to j_{max} do

$$y[j] := \frac{\delta y}{2} + (j-1)\cdot\delta y;$$

end do;

$$y[j_{max}+1] := H;$$

$$y_0 := 0$$
$$y_1 := 0.1000$$
$$y_2 := 0.3000$$
$$y_3 := 0.5000$$
$$y_4 := 0.7000$$
$$y_5 := 0.9000$$
$$y_6 := 1.0$$

Conditions aux Limites:

> $T0 := 40; q := 10^5; q0 := 0$

$$T0 := 40$$
$$q := 100000$$
$$q0 := 0$$

Noeuds internes gauche:

> for j from 2 to $j_{max}-1$ do

for i from 2 to $\text{trunc}\left(\frac{i_{max}}{2}\right)$ do

$$Su[i,j] := 0;$$

$$a_W[i,j] := \frac{k_1\cdot Aw}{\delta x};$$

$$a_E[i,j] := \frac{k_m\cdot Ae}{\delta x};$$

$$a_S[i,j] := \frac{k_1\cdot As}{\delta y};$$

$$a_N[i,j] := \frac{k_1\cdot An}{\delta y};$$

$$a_P[i,j] := \frac{k_1\cdot Aw}{\delta x} + \frac{k_m\cdot Ae}{\delta x} + \frac{k_1\cdot As}{\delta y} + \frac{k_1\cdot An}{\delta y};$$

$$Sp[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - a_P[i,j];$$

end do;

end do;

Noeuds internes droit:

> for j from 2 to $j_{max}-1$ do

for i from $\text{trunc}\left(\frac{i_{max}}{2}\right)+1$ to $i_{max}-1$ do

$Su[i, j] := 0;$

$a_W[i, j] := \frac{k_m \cdot Aw}{\delta x};$

$a_E[i, j] := \frac{k_2 \cdot Ae}{\delta x};$

$a_S[i, j] := \frac{k_2 \cdot As}{\delta y};$

$a_N[i, j] := \frac{k_2 \cdot An}{\delta y};$

$a_P[i, j] := \frac{k_m \cdot Aw}{\delta x} + \frac{k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y} + \frac{k_2 \cdot An}{\delta y};$

$Sp[i, j] := a_W[i, j] + a_E[i, j] + a_S[i, j] + a_N[i, j] - a_P[i, j];$

end do;

end do;

Noeuds gauche:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Su[1, j] := q0 \cdot Aw;$

$a_W[1, j] := 0;$

$a_E[1, j] := \frac{k_1 \cdot Ae}{\delta x};$

$a_S[1, j] := \frac{k_1 \cdot As}{\delta y};$

$a_N[1, j] := \frac{k_1 \cdot An}{\delta y};$

$a_P[1, j] := \frac{k_1 \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y} + \frac{k_1 \cdot An}{\delta y};$

$Sp[1, j] := a_W[1, j] + a_E[1, j] + a_S[1, j] + a_N[1, j] - a_P[1, j];$

end do;

$$Su_{1, 2} := 0.$$
$$a_{W_{1, 2}} := 0$$
$$a_{E_{1, 2}} := 5.000$$
$$a_{S_{1, 2}} := 20.00$$
$$a_{N_{1, 2}} := 20.00$$
$$a_{P_{1, 2}} := 45.00$$
$$Sp_{1, 2} := 0.$$
$$Su_{1, 3} := 0.$$
$$a_{W_{1, 3}} := 0$$

$$a_{E_{1,3}} := 5.000$$
$$a_{S_{1,3}} := 20.00$$
$$a_{N_{1,3}} := 20.00$$
$$a_{P_{1,3}} := 45.00$$
$$Sp_{1,3} := 0.$$
$$Su_{1,4} := 0.$$
$$a_{W_{1,4}} := 0$$
$$a_{E_{1,4}} := 5.000$$
$$a_{S_{1,4}} := 20.00$$
$$a_{N_{1,4}} := 20.00$$
$$a_{P_{1,4}} := 45.00$$
$$Sp_{1,4} := 0.$$

Noeuds droit:

> **for** j **from** 2 **to** $j_{max} - 1$ **do**

$$Su[i_{max}, j] := \frac{2 \cdot k_2 \cdot Ae}{\delta x} \cdot T0;$$
$$a_W[i_{max}, j] := \frac{k_2 \cdot Aw}{\delta x};$$
$$a_E[i_{max}, j] := 0;$$
$$a_S[i_{max}, j] := \frac{k_2 \cdot As}{\delta y};$$
$$a_N[i_{max}, j] := \frac{k_2 \cdot An}{\delta y};$$
$$a_P[i_{max}, j] := \frac{k_2 \cdot Aw}{\delta x} + \frac{2 \cdot k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y} + \frac{k_2 \cdot An}{\delta y};$$
$$Sp[i_{max}, j] := a_W[i_{max}, j] + a_E[i_{max}, j] + a_S[i_{max}, j] + a_N[i_{max}, j] - a_P[i_{max}, j];$$

end do;

$$Su_{4,2} := 160.0$$
$$a_{W_{4,2}} := 2.000$$
$$a_{E_{4,2}} := 0$$
$$a_{S_{4,2}} := 8.000$$
$$a_{N_{4,2}} := 8.000$$
$$a_{P_{4,2}} := 22.00$$
$$Sp_{4,2} := -4.00$$
$$Su_{4,3} := 160.0$$

$$a_{W_{4,3}} := 2.000$$
$$a_{E_{4,3}} := 0$$
$$a_{S_{4,3}} := 8.000$$
$$a_{N_{4,3}} := 8.000$$
$$a_{P_{4,3}} := 22.00$$
$$Sp_{4,3} := -4.00$$
$$Su_{4,4} := 160.0$$
$$a_{W_{4,4}} := 2.000$$
$$a_{E_{4,4}} := 0$$
$$a_{S_{4,4}} := 8.000$$
$$a_{N_{4,4}} := 8.000$$
$$a_{P_{4,4}} := 22.00$$
$$Sp_{4,4} := -4.00$$

Noeuds bas gauche:

```
> for i from 2 to trunc(imax/2) do
    Su[i, 1] := q0·As;;
    aW[i, 1] := k1·Aw/δx;
    aE[i, 1] := km·Ae/δx;
    aS[i, 1] := 0;
    aN[i, 1] := k1·An/δy;
    aP[i, 1] := k1·Aw/δx + km·Ae/δx + k1·An/δy;
    Sp[i, 1] := aW[i, 1] + aE[i, 1] + aS[i, 1] + aN[i, 1] - aP[i, 1];
  end do;
```

$$Su_{2,1} := 0.$$
$$a_{W_{2,1}} := 5.000$$
$$a_{E_{2,1}} := 3.500$$
$$a_{S_{2,1}} := 0$$
$$a_{N_{2,1}} := 20.00$$
$$a_{P_{2,1}} := 28.50$$
$$Sp_{2,1} := 0.$$

Noeuds bas droit:

```
> for i from trunc(imax/2) + 1 to imax - 1 do
    Su[i, 1] := q0·As;
    aW[i, 1] := km·Aw/δx;
    aE[i, 1] := k2·Ae/δx;
    aS[i, 1] := 0;
    aN[i, 1] := k2·An/δy;
    aP[i, 1] := km·Aw/δx + k2·Ae/δx + k2·An/δy;
    Sp[i, 1] := aW[i, 1] + aE[i, 1] + aS[i, 1] + aN[i, 1] - aP[i, 1];
  end do;
```

$$Su_{3,1} := 0.$$
$$a_{W_{3,1}} := 3.500$$
$$a_{E_{3,1}} := 2.000$$
$$a_{S_{3,1}} := 0$$
$$a_{N_{3,1}} := 8.000$$
$$a_{P_{3,1}} := 13.50$$
$$Sp_{3,1} := 0.$$

Noeuds hauts gauches:

```
> for i from 2 to trunc(imax/2) do
    Su[i, jmax] := q·An;
    aW[i, jmax] := k1·Aw/δx;
    aE[i, jmax] := km·Ae/δx;
    aS[i, jmax] := k1·As/δy;
    aN[i, jmax] := 0;
    aP[i, jmax] := k1·Aw/δx + km·Ae/δx + k1·As/δy;
    Sp[i, jmax] := aW[i, jmax] + aE[i, jmax] + aS[i, jmax] + aN[i, jmax] - aP[i, jmax];
  end do;
```

$$Su_{2,5} := 800.0$$
$$a_{W_{2,5}} := 5.000$$

$$a_{E_{2,5}} := 3.500$$
$$a_{S_{2,5}} := 20.00$$
$$a_{N_{2,5}} := 0$$
$$a_{P_{2,5}} := 28.50$$
$$Sp_{2,5} := 0.$$

Noeuds hauts droit:

> **for** i **from** $\mathrm{trunc}\left(\frac{i_{\max}}{2}\right)+1$ **to** $i_{\max}-1$ **do**

$Su[i, j_{\max}] := q \cdot An;$

$a_W[i, j_{\max}] := \frac{k_m \cdot Aw}{\delta x};$

$a_E[i, j_{\max}] := \frac{k_2 \cdot Ae}{\delta x};$

$a_S[i, j_{\max}] := \frac{k_2 \cdot As}{\delta y};$

$a_N[i, j_{\max}] := 0;$

$a_P[i, j_{\max}] := \frac{k_m \cdot Aw}{\delta x} + \frac{k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y};$

$Sp[i, j_{\max}] := a_W[i, j_{\max}] + a_E[i, j_{\max}] + a_S[i, j_{\max}] + a_N[i, j_{\max}] - a_P[i, j_{\max}];$

end do;

$$Su_{3,5} := 800.0$$
$$a_{W_{3,5}} := 3.500$$
$$a_{E_{3,5}} := 2.000$$
$$a_{S_{3,5}} := 8.000$$
$$a_{N_{3,5}} := 0$$
$$a_{P_{3,5}} := 13.50$$
$$Sp_{3,5} := 0.$$

Noeud (1,1):

> $Su[1, 1] := q0 \cdot Aw + q0 \cdot As;$

$a_W[1, 1] := 0;$

$a_E[1, 1] := \frac{k_1 \cdot Ae}{\delta x};$

$a_S[1, 1] := 0;$

$a_N[1, 1] := \frac{k_1 \cdot An}{\delta y};$

$a_P[1, 1] := \frac{k_1 \cdot Ae}{\delta x} + \frac{k_1 \cdot An}{\delta y};$

$$Sp[1,1] := a_W[1,1] + a_E[1,1] + a_S[1,1] + a_N[1,1] - a_P[1,1];$$

$$Su_{1,1} := 0$$
$$a_{W_{1,1}} := 0$$
$$a_{E_{1,1}} := 5.000$$
$$a_{S_{1,1}} := 0$$
$$a_{N_{1,1}} := 20.00$$
$$a_{P_{1,1}} := 25.00$$
$$Sp_{1,1} := 0.$$

Noeud (imax,1):

> $$Su[i_{max},1] := q0 \cdot As + \frac{2 \cdot k_2 \cdot Ae}{\delta x} \cdot T0;$$

$$a_W[i_{max},1] := \frac{k_2 \cdot Aw}{\delta x};$$
$$a_E[i_{max},1] := 0;$$
$$a_S[i_{max},1] := 0;$$
$$a_N[i_{max},1] := \frac{k_2 \cdot An}{\delta y};$$
$$a_P[i_{max},1] := \frac{k_2 \cdot Aw}{\delta x} + \frac{2 \cdot k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot An}{\delta y};$$
$$Sp[i_{max},1] := a_W[i_{max},1] + a_E[i_{max},1] + a_S[i_{max},1] + a_N[i_{max},1] - a_P[i_{max},1];$$

$$Su_{4,1} := 160.0$$
$$a_{W_{4,1}} := 2.000$$
$$a_{E_{4,1}} := 0$$
$$a_{S_{4,1}} := 0$$
$$a_{N_{4,1}} := 8.000$$
$$a_{P_{4,1}} := 14.00$$
$$Sp_{4,1} := -4.00$$

Noeud (1,jmax):

> $$Su[1,j_{max}] := q0 \cdot Aw + q \cdot An;$$

$$a_W[1,j_{max}] := 0;$$
$$a_E[1,j_{max}] := \frac{k_1 \cdot Ae}{\delta x};$$
$$a_S[1,j_{max}] := \frac{k_1 \cdot As}{\delta y};$$
$$a_N[1,j_{max}] := 0;$$
$$a_P[1,j_{max}] := \frac{k_1 \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y};$$

$$Sp[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - a_P[1, j_{max}];$$

$$Su_{1,5} := 800.0$$
$$a_{W_{1,5}} := 0$$
$$a_{E_{1,5}} := 5.000$$
$$a_{S_{1,5}} := 20.00$$
$$a_{N_{1,5}} := 0$$
$$a_{P_{1,5}} := 25.00$$
$$Sp_{1,5} := 0.$$

Noeud (imax,jmax):

> $$Su[i_{max}, j_{max}] := q \cdot An + \frac{2 \cdot k_2 \cdot Ae}{\delta x} \cdot T0;$$

$$a_W[i_{max}, j_{max}] := \frac{k_2 \cdot Aw}{\delta x};$$
$$a_E[i_{max}, j_{max}] := 0;$$
$$a_S[i_{max}, j_{max}] := \frac{k_2 \cdot As}{\delta y};$$
$$a_N[i_{max}, j_{max}] := 0;$$
$$a_P[i_{max}, j_{max}] := \frac{k_2 \cdot Aw}{\delta x} + \frac{2 \cdot k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y};$$
$$Sp[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}] - a_P[i_{max}, j_{max}];$$

$$Su_{4,5} := 960.0$$
$$a_{W_{4,5}} := 2.000$$
$$a_{E_{4,5}} := 0$$
$$a_{S_{4,5}} := 8.000$$
$$a_{N_{4,5}} := 0$$
$$a_{P_{4,5}} := 14.00$$
$$Sp_{4,5} := -4.00$$

Equations:

> $k := 1$:

Résolution pour les noeuds internes:

> **for** j **from** 1 **to** j_{max} **do**
for i **from** 1 **to** i_{max} **do**

$$Eq[k] := a_P[i, j] \cdot T[i, j] = a_W[i, j] \cdot T[i-1, j] + a_E[i, j] \cdot T[i+1, j] + a_S[i, j] \cdot T[i, j-1] + a_N[i, j] \cdot T[i, j+1] + Su[i, j];$$
$$Var[k] := T[i, j];$$
$$k := k + 1;$$

```
    end do;
    end do;
```

Ecriture du systme d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$25.00\,T_{1,1} = 20.00\,T_{1,2} + 5.000\,T_{2,1}$$
$$28.50\,T_{2,1} = 20.00\,T_{2,2} + 5.000\,T_{1,1} + 3.500\,T_{3,1}$$
$$13.50\,T_{3,1} = 8.000\,T_{3,2} + 3.500\,T_{2,1} + 2.000\,T_{4,1}$$
$$14.00\,T_{4,1} = 8.000\,T_{4,2} + 2.000\,T_{3,1} + 160.0$$
$$45.00\,T_{1,2} = 20.00\,T_{1,1} + 20.00\,T_{1,3} + 5.000\,T_{2,2}$$
$$48.50\,T_{2,2} = 20.00\,T_{2,1} + 20.00\,T_{2,3} + 5.000\,T_{1,2} + 3.500\,T_{3,2}$$
$$21.50\,T_{3,2} = 8.000\,T_{3,1} + 8.000\,T_{3,3} + 3.500\,T_{2,2} + 2.000\,T_{4,2}$$
$$22.00\,T_{4,2} = 8.000\,T_{4,1} + 8.000\,T_{4,3} + 2.000\,T_{3,2} + 160.0$$
$$45.00\,T_{1,3} = 20.00\,T_{1,2} + 20.00\,T_{1,4} + 5.000\,T_{2,3}$$
$$48.50\,T_{2,3} = 20.00\,T_{2,2} + 20.00\,T_{2,4} + 5.000\,T_{1,3} + 3.500\,T_{3,3}$$
$$21.50\,T_{3,3} = 8.000\,T_{3,2} + 8.000\,T_{3,4} + 3.500\,T_{2,3} + 2.000\,T_{4,3}$$
$$22.00\,T_{4,3} = 8.000\,T_{4,2} + 8.000\,T_{4,4} + 2.000\,T_{3,3} + 160.0$$
$$45.00\,T_{1,4} = 20.00\,T_{1,3} + 20.00\,T_{1,5} + 5.000\,T_{2,4}$$
$$48.50\,T_{2,4} = 20.00\,T_{2,3} + 20.00\,T_{2,5} + 5.000\,T_{1,4} + 3.500\,T_{3,4}$$
$$21.50\,T_{3,4} = 8.000\,T_{3,3} + 8.000\,T_{3,5} + 3.500\,T_{2,4} + 2.000\,T_{4,4}$$
$$22.00\,T_{4,4} = 8.000\,T_{4,3} + 8.000\,T_{4,5} + 2.000\,T_{3,4} + 160.0$$
$$25.00\,T_{1,5} = 20.00\,T_{1,4} + 5.000\,T_{2,5} + 800.0$$
$$28.50\,T_{2,5} = 20.00\,T_{2,4} + 5.000\,T_{1,5} + 3.500\,T_{3,5} + 800.0$$
$$13.50\,T_{3,5} = 8.000\,T_{3,4} + 3.500\,T_{2,5} + 2.000\,T_{4,5} + 800.0$$
$$14.00\,T_{4,5} = 8.000\,T_{4,4} + 2.000\,T_{3,5} + 960.0$$

```
> Eqs := [seq(Eq[k], k = 1 ..Ne)]:
> Vars := [seq(Var[k], k = 1 ..Ne)]
```

$$Vars := [T_{1,1}, T_{2,1}, T_{3,1}, T_{4,1}, T_{1,2}, T_{2,2}, T_{3,2}, T_{4,2}, T_{1,3}, T_{2,3}, T_{3,3}, T_{4,3}, T_{1,4}, T_{2,4}, T_{3,4}, T_{4,4}, T_{1,5}, T_{2,5}, T_{3,5}, T_{4,5}]$$

```
> SolT := solve(Eqs, Vars);
```

$$SolT := [[T_{1,1} = 529.1, T_{2,1} = 494.1, T_{3,1} = 389.8, T_{4,1} = 164.9, T_{1,2} = 537.9, T_{2,2} = 503.5, T_{3,2} = 400.4, T_{4,2} = 171.1, T_{1,3} = 555.3, T_{2,3} = 522.5, T_{3,3} = 423.2, T_{4,3} = 185.5, T_{1,4} = 580.8, T_{2,4} = 550.5, T_{3,4} = 462.1, T_{4,4} = 213.2, T_{1,5} = 614.0, T_{2,5} = 586.5, T_{3,5} = 524.5, T_{4,5} = 265.3]]$$

```
> with(LinearAlgebra):
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Vars)
```

$$A,\ b := \left[\begin{array}{c} 20\ x\ 20\ Matrix \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right], \left[\begin{array}{c} 1\ ..\ 20\ Vector_{column} \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right]$$

```
> seq(b[i], i = 1 ..Ne)
```

0, 0, 0, 160.0, 0, 0, 0, 160.0, 0, 0, 0, 160.0, 0, 0, 0, 160.0, 800.0, 800.0, 800.0, 960.0

```
> seq(A[i, i], i = 1 ..Ne)
```

25.00, 28.50, 13.50, 14.00, 45.00, 48.50, 21.50, 22.00, 45.00, 48.50, 21.50, 22.00, 45.00, 48.50, 21.50, 22.00, 25.00, 28.50, 13.50, 14.00

Récapitulation:

```
> seq( seq( a_W[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000

```
> seq( seq( a_E[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0, 5.000, 3.500, 2.000, 0

```
> seq( seq( a_S[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

0, 0, 0, 0, 20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000

```
> seq( seq( a_N[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000, 20.00, 20.00, 8.000, 8.000, 0, 0, 0, 0

```
> seq( seq( Su[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

0, 0., 0., 160.0, 0., 0, 0, 160.0, 0., 0, 0, 160.0, 0., 0, 0, 160.0, 800.0, 800.0, 800.0, 960.0

```
> seq( seq( a_P[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

25.00, 28.50, 13.50, 14.00, 45.00, 48.50, 21.50, 22.00, 45.00, 48.50, 21.50, 22.00, 45.00, 48.50, 21.50, 22.00, 25.00, 28.50, 13.50, 14.00

```
> seq( seq( Sp[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

0., 0., 0., -4.00, 0., 0., 0., -4.00, 0., 0., 0., -4.00, 0., 0., 0., -4.00, 0., 0., 0., -4.00

```
>
```