Equation de Diffusion 2D

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EXAMEN

Détermination de la distribution de temperature $T(\mathrm{x})$ à travers une plaque de largeur L, de hauteur H et d'éapisseur e et de conductivité thermique k soumise aux (C.L.) suivantes.

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x},\mathrm{y})\right)+\frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x},\mathrm{y})\right)=0$$

Conditions aux limites (C.L):

$$\begin{aligned}T(0,y)&=T_0=50,\\ T(\mathrm{L},\mathrm{y})&=T_0=50,\\ -kA\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x},0)&=q=\frac{300\ kW}{m^2},\\ -kA\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x},\mathrm{H})&=q=\frac{300\ kW}{m^2},\end{aligned}$$

Solution

```
> Restart : Digits := 4 :
> L := 2.0; H := 0.8; e := 0.001; λ := 1000; δx := 0.4; δy := 0.2;
```

$$\begin{matrix}2.0\\0.8\\0.001\end{matrix}$$ **(1.1)**

$$1000$$
$$0.4$$
$$0.2$$

$$> ndx := \frac{L}{\delta x}\ ;\ ndy := \frac{H}{\delta y}; Se := \delta y \cdot e; Ss := \delta x \cdot e;$$

$$5.000 \qquad \textbf{(1.2)}$$
$$4.000$$
$$0.0002$$
$$0.0004$$

$$> i_{max} := \mathrm{round}(ndx);\ j_{max} := \mathrm{round}(ndy);$$

$$5 \qquad \textbf{(1.3)}$$
$$4$$

Nombre d'équations:

$$> Ne := i_{max} \cdot j_{max}$$

$$20 \qquad \textbf{(1.4)}$$

Abscisses des noeuds:

$$> x[0] := 0;$$

for i **from** 1 **to** i_{max} **do**

$$x[i] := \frac{\delta x}{2} + (i-1) \cdot \delta x\ ;$$

end do;

$$x[\, i_{max} + 1\,] := L;$$

$$0 \qquad \textbf{(1.5)}$$
$$0.2000$$
$$0.6000$$
$$1.000$$
$$1.400$$
$$1.800$$
$$2.0$$

Ordonnées des noeuds:

$$> y[0] := 0;$$

for j **from** 1 **to** j_{max} **do**

$$y[j] := \frac{\delta y}{2} + (j-1) \cdot \delta y\ ;$$

end do;

$$y[\, j_{max} + 1\,] := H;$$

$$0 \qquad \textbf{(1.6)}$$
$$0.1000$$
$$0.3000$$
$$0.5000$$
$$0.7000$$
$$0.8$$

Conditions aux Limites:

$$> T0 := 50;\ q := 300000;$$

$$50 \qquad \textbf{(1.7)}$$
$$300000$$

Noeuds internes:

$$>$$ **for** j **from** 2 **to** $j_{max} - 1$ **do**

for i **from** 2 **to** i_{max}-1 **do**

$Sp[i,j] := 0;$
$Su[i,j] := 0;$
$a_W[i,j] := \frac{\lambda \cdot Se}{\delta x};$
$a_E[i,j] := a_W[i,j];$
$a_S[i,j] := \frac{\lambda \cdot Ss}{\delta y};$
$a_N[i,j] := a_S[i,j];$
$a_P[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - Sp[i,j];$
end do;
end do;

Noeuds gauches:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Sp[1,j] := -\frac{2 \cdot \lambda \cdot Se}{\delta x};$
$Su[1,j] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0;$
$a_W[1,j] := 0;$
$a_E[1,j] := \frac{\lambda \cdot Se}{\delta x};$
$a_S[1,j] := \frac{\lambda \cdot Ss}{\delta y};$
$a_N[1,j] := a_S[1,j];$
$a_P[1,j] := a_W[1,j] + a_E[1,j] + a_S[1,j] + a_N[1,j] - Sp[1,j];$
end do;

$$\begin{matrix} -1.000 \\ 50.00 \\ 0 \\ 0.5000 \\ 2.000 \\ 2.000 \\ 5.500 \\ -1.000 \\ 50.00 \\ 0 \\ 0.5000 \\ 2.000 \\ 2.000 \\ 5.500 \end{matrix} \qquad \textbf{(1.8)}$$

Noeuds droits:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Sp[i_{\max}, j] := -\frac{2 \cdot \lambda \cdot Se}{\delta x};$
$Su[i_{\max}, j] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0;$
$a_W[i_{\max}, j] := \frac{\lambda \cdot Se}{\delta x};$

$a_E[i_{max}, j] := 0;$

$a_S[i_{max}, j] := \frac{\lambda \cdot Ss}{\delta y};$

$a_N[i_{max}, j] := a_S[i_{max}, j];$

$a_P[i_{max}, j] := a_W[i_{max}, j] + a_E[i_{max}, j] + a_S[i_{max}, j] + a_N[i_{max}, j] - Sp[i_{max}, j];$

end do;

$$\begin{matrix} -1.000 \\ 50.00 \\ 0.5000 \\ 0 \\ 2.000 \\ 2.000 \\ 5.500 \\ -1.000 \\ 50.00 \\ 0.5000 \\ 0 \\ 2.000 \\ 2.000 \\ 5.500 \end{matrix} \quad \textbf{(1.9)}$$

Noeuds bas:

> **for** i **from** 2 **to** $i_{max} - 1$ **do**

$Sp[i, 1] := 0;$

$Su[i, 1] := q \cdot Ss;$

$a_W[i, 1] := \frac{\lambda \cdot Se}{\delta x};$

$a_E[i, 1] := a_W[i, 1];$

$a_S[i, 1] := 0;$

$a_N[i, 1] := \frac{\lambda \cdot Ss}{\delta y};$

$a_P[i, 1] := a_W[i, 1] + a_E[i, 1] + a_S[i, 1] + a_N[i, 1] - Sp[i, 1];$

end do;

$$\begin{matrix} 0 \\ 120.0 \\ 0.5000 \\ 0.5000 \\ 0 \\ 2.000 \\ 3.000 \\ 0 \\ 120.0 \\ 0.5000 \\ 0.5000 \\ 0 \\ 2.000 \\ 3.000 \\ 0 \\ 120.0 \\ 0.5000 \\ 0.5000 \\ 0 \end{matrix} \quad \textbf{(1.10)}$$

2.000
3.000

Noeuds hauts:

```
> for i from 2 to i_max - 1 do
  Sp[i, j_max] := 0;
  Su[i, j_max] := q·Ss;
  a_W[i, j_max] := λ·Se / δx;
  a_E[i, j_max] := a_W[i, j_max];
  a_S[i, j_max] := λ·Ss / δy;
  a_N[i, j_max] := 0;
  a_P[i, j_max] := a_W[i, j_max] + a_E[i, j_max] + a_S[i, j_max] + a_N[i, j_max] - Sp[i, j_max];
  end do;
```

0
120.0
0.5000
0.5000
2.000
0
3.000
0
120.0
0.5000
0.5000
2.000
0
3.000
0
120.0
0.5000
0.5000
2.000
0
3.000

(1.11)

Noeud (1,1):

```
> Sp[1, 1] := - 2·λ·Se / δx;
  Su[1, 1] := 2·λ·Se / δx · T0 + q·Ss;
  a_W[1, 1] := 0;
  a_E[1, 1] := λ·Se / δx;
  a_S[1, 1] := 0;
  a_N[1, 1] := λ·Ss / δy;
  a_P[1, 1] := a_W[1, 1] + a_E[1, 1] + a_S[1, 1] + a_N[1, 1] - Sp[1, 1];
```

-1.000
170.0

(1.12)

0
0.5000
0
2.000
3.500

Noeud (imax,1):

$$> Sp[\, i_{max}, 1\,] := -\frac{2\cdot\lambda\cdot Se}{\delta x};$$

$$Su[\, i_{max}, 1\,] := \frac{2\cdot\lambda\cdot Se}{\delta x}\cdot T0 + q\cdot Ss;$$

$$a_W[\, i_{max}, 1\,] := \frac{\lambda\cdot Se}{\delta x};$$

$$a_E[\, i_{max}, 1\,] := 0;$$

$$a_S[\, i_{max}, 1\,] := 0;$$

$$a_N[\, i_{max}, 1\,] := \frac{\lambda\cdot Ss}{\delta y};$$

$$a_P[\, i_{max}, 1\,] := a_W[\, i_{max}, 1\,] + a_E[\, i_{max}, 1\,] + a_S[\, i_{max}, 1\,] + a_N[\, i_{max}, 1\,] - Sp[\, i_{max}, 1\,];$$

-1.000
170.0
0.5000
0
0
2.000
3.500

(1.13)

Noeud (1,jmax):

$$> Sp[1, j_{max}] := -\frac{2\cdot\lambda\cdot Se}{\delta x};$$

$$Su[1\,, j_{max}] := \frac{2\cdot\lambda\cdot Se}{\delta x}\cdot T0 + q\cdot Ss;$$

$$a_W[1, j_{max}] := 0;$$

$$a_E[1, j_{max}] := \frac{\lambda\cdot Se}{\delta x};$$

$$a_S[1, j_{max}] := \frac{\lambda\cdot Ss}{\delta y};$$

$$a_N[1, j_{max}] := 0;$$

$$a_P[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - Sp[1, j_{max}];$$

-1.000
170.0
0
0.5000
2.000
0
3.500

(1.14)

Noeud (imax,jmax):

$$> Sp[i_{max}, j_{max}] := -\frac{2\cdot\lambda\cdot Se}{\delta x};$$

$$Su[i_{max}, j_{max}] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0 + q \cdot Ss;$$

$$a_W[i_{max}, j_{max}] := \frac{\lambda \cdot Se}{\delta x};$$

$$a_E[i_{max}, j_{max}] := 0;$$

$$a_S[i_{max}, j_{max}] := \frac{\lambda \cdot Ss}{\delta y};$$

$$a_N[i_{max}, j_{max}] := 0;$$

$$a_P[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}] - Sp[i_{max}, j_{max}];$$

$$\begin{matrix} -1.000 \\ 170.0 \\ 0.5000 \\ 0 \\ 2.000 \\ 0 \\ 3.500 \end{matrix} \qquad \textbf{(1.15)}$$

Equations:

$> k := 1$

$$1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for j from 1 to j_max do
  for i from 1 to i_max do
        Eq[k] := a_P[i,j]·T[i,j] = a_W[i,j]·T[i-1,j] + a_E[i,j]·T[i+1,j] + a_S[i,j]
    ·T[i,j-1] + a_N[i,j]·T[i,j+1] + Su[i,j];
      k := k + 1;
  end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$\begin{aligned}
3.500\, T_{1,1} &= 170.0 + 0.5000\, T_{2,1} + 2.000\, T_{1,2} \qquad \textbf{(1.1.2)}\\
3.000\, T_{2,1} &= 0.5000\, T_{1,1} + 0.5000\, T_{3,1} + 120.0 + 2.000\, T_{2,2}\\
3.000\, T_{3,1} &= 0.5000\, T_{2,1} + 0.5000\, T_{4,1} + 120.0 + 2.000\, T_{3,2}\\
3.000\, T_{4,1} &= 0.5000\, T_{3,1} + 0.5000\, T_{5,1} + 120.0 + 2.000\, T_{4,2}\\
3.500\, T_{5,1} &= 0.5000\, T_{4,1} + 170.0 + 2.000\, T_{5,2}\\
5.500\, T_{1,2} &= 50.00 + 0.5000\, T_{2,2} + 2.000\, T_{1,1} + 2.000\, T_{1,3}\\
5.000\, T_{2,2} &= 0.5000\, T_{1,2} + 0.5000\, T_{3,2} + 2.000\, T_{2,1} + 2.000\, T_{2,3}\\
5.000\, T_{3,2} &= 0.5000\, T_{2,2} + 0.5000\, T_{4,2} + 2.000\, T_{3,1} + 2.000\, T_{3,3}\\
5.000\, T_{4,2} &= 0.5000\, T_{3,2} + 0.5000\, T_{5,2} + 2.000\, T_{4,1} + 2.000\, T_{4,3}\\
5.500\, T_{5,2} &= 0.5000\, T_{4,2} + 50.00 + 2.000\, T_{5,1} + 2.000\, T_{5,3}\\
5.500\, T_{1,3} &= 50.00 + 0.5000\, T_{2,3} + 2.000\, T_{1,2} + 2.000\, T_{1,4}\\
5.000\, T_{2,3} &= 0.5000\, T_{1,3} + 0.5000\, T_{3,3} + 2.000\, T_{2,2} + 2.000\, T_{2,4}\\
5.000\, T_{3,3} &= 0.5000\, T_{2,3} + 0.5000\, T_{4,3} + 2.000\, T_{3,2} + 2.000\, T_{3,4}\\
5.000\, T_{4,3} &= 0.5000\, T_{3,3} + 0.5000\, T_{5,3} + 2.000\, T_{4,2} + 2.000\, T_{4,4}
\end{aligned}$$

$$5.500\,T_{5,3} = 0.5000\,T_{4,3} + 50.00 + 2.000\,T_{5,2} + 2.000\,T_{5,4}$$
$$3.500\,T_{1,4} = 170.0 + 0.5000\,T_{2,4} + 2.000\,T_{1,3}$$
$$3.000\,T_{2,4} = 0.5000\,T_{1,4} + 0.5000\,T_{3,4} + 2.000\,T_{2,3} + 120.0$$
$$3.000\,T_{3,4} = 0.5000\,T_{2,4} + 0.5000\,T_{4,4} + 2.000\,T_{3,3} + 120.0$$
$$3.000\,T_{4,4} = 0.5000\,T_{3,4} + 0.5000\,T_{5,4} + 2.000\,T_{4,3} + 120.0$$
$$3.500\,T_{5,4} = 0.5000\,T_{4,4} + 170.0 + 2.000\,T_{5,3}$$

```
> Eqs := {seq(Eq[k], k = 1 ..Ne)};
```

$$\{3.500\,T_{1,1} = 170.0 + 0.5000\,T_{2,1} + 2.000\,T_{1,2},\ 5.500\,T_{1,2} = 50.00 + 0.5000\,T_{2,2} + 2.000\,T_{1,1} + 2.000\,T_{1,3},\ 5.500\,T_{1,3} = 50.00 + 0.5000\,T_{2,3} + 2.000\,T_{1,2} + 2.000\,T_{1,4},\ 3.500\,T_{1,4} = 170.0 + 0.5000\,T_{2,4} + 2.000\,T_{1,3},\ 3.000\,T_{2,1} = 0.5000\,T_{1,1} + 0.5000\,T_{3,1} + 120.0 + 2.000\,T_{2,2},\ 5.000\,T_{2,2} = 0.5000\,T_{1,2} + 0.5000\,T_{3,2} + 2.000\,T_{2,1} + 2.000\,T_{2,3},\ 5.000\,T_{2,3} = 0.5000\,T_{1,3} + 0.5000\,T_{3,3} + 2.000\,T_{2,2} + 2.000\,T_{2,4},\ 3.000\,T_{2,4} = 0.5000\,T_{1,4} + 0.5000\,T_{3,4} + 2.000\,T_{2,3} + 120.0,\ 3.000\,T_{3,1} = 0.5000\,T_{2,1} + 0.5000\,T_{4,1} + 120.0 + 2.000\,T_{3,2},\ 5.000\,T_{3,2} = 0.5000\,T_{2,2} + 0.5000\,T_{4,2} + 2.000\,T_{3,1} + 2.000\,T_{3,3},\ 5.000\,T_{3,3} = 0.5000\,T_{2,3} + 0.5000\,T_{4,3} + 2.000\,T_{3,2} + 2.000\,T_{3,4},\ 3.000\,T_{3,4} = 0.5000\,T_{2,4} + 0.5000\,T_{4,4} + 2.000\,T_{3,3} + 120.0,\ 3.000\,T_{4,1} = 0.5000\,T_{3,1} + 0.5000\,T_{5,1} + 120.0 + 2.000\,T_{4,2},\ 5.000\,T_{4,2} = 0.5000\,T_{3,2} + 0.5000\,T_{5,2} + 2.000\,T_{4,1} + 2.000\,T_{4,3},\ 5.000\,T_{4,3} = 0.5000\,T_{3,3} + 0.5000\,T_{5,3} + 2.000\,T_{4,2} + 2.000\,T_{4,4},\ 3.000\,T_{4,4} = 0.5000\,T_{3,4} + 0.5000\,T_{5,4} + 2.000\,T_{4,3} + 120.0,\ 3.500\,T_{5,1} = 0.5000\,T_{4,1} + 170.0 + 2.000\,T_{5,2},\ 5.500\,T_{5,2} = 0.5000\,T_{4,2} + 50.00 + 2.000\,T_{5,1} + 2.000\,T_{5,3},\ 5.500\,T_{5,3} = 0.5000\,T_{4,3} + 50.00 + 2.000\,T_{5,2} + 2.000\,T_{5,4},\ 3.500\,T_{5,4} = 0.5000\,T_{4,4} + 170.0 + 2.000\,T_{5,3}\} \quad \textbf{(1.1.3)}$$

```
> SolT := solve(Eqs);
```

$$\{T_{1,1} = 212.2,\ T_{1,2} = 187.8,\ T_{1,3} = 187.8,\ T_{1,4} = 212.2,\ T_{2,1} = 394.7,\ T_{2,2} = 365.3,\ T_{2,3} = 365.3,\ T_{2,4} = 394.7,\ T_{3,1} = 454.9,\ T_{3,2} = 425.1,\ T_{3,3} = 425.1,\ T_{3,4} = 454.9,\ T_{4,1} = 394.7,\ T_{4,2} = 365.3,\ T_{4,3} = 365.3,\ T_{4,4} = 394.7,\ T_{5,1} = 212.2,\ T_{5,2} = 187.8,\ T_{5,3} = 187.8,\ T_{5,4} = 212.2\} \quad \textbf{(1.1.4)}$$

```
> Tmps := [seq(lhs(SolT_i), i = 1 ..Ne)];
```

$$[T_{1,1}, T_{1,2}, T_{1,3}, T_{1,4}, T_{2,1}, T_{2,2}, T_{2,3}, T_{2,4}, T_{3,1}, T_{3,2}, T_{3,3}, T_{3,4}, T_{4,1}, T_{4,2}, T_{4,3}, T_{4,4}, T_{5,1}, T_{5,2}, T_{5,3}, T_{5,4}] \quad \textbf{(1.1.5)}$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Tmps)
```

$$Matrix(\%id = 4687190530),\ Vector_{column}(\%id = 4687190338) \quad \textbf{(1.1.6)}$$

```
> b[8]
```

$$120.0 \quad \textbf{(1.1.7)}$$

```
> ?
```

`?` **(1.1.8)**