Equation de Laplace 2D

==========

Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

Départment de Mécanique

==========

Master : Energétique

Matière : Méthodes Numériques Appliquées I

==========

2010/2011

==========

Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100,$$
$$T(0, y) = 60,$$
$$T(a, y) = 20.$$

Maillage et conditions aux limites:

```
> Restart :
> a := 5; b := 4; ndx := 5; ndy := 4
```

$$a := 5$$
$$b := 4$$
$$ndx := 5$$
$$ndy := 4 \qquad \textbf{(1.1)}$$

```
> Δx := a/ndx ; Δy := b/ndy ; β := Δx/Δy ;
```

$$\Delta x := 1$$
$$\Delta y := 1$$
$$\beta := 1 \quad \textbf{(1.2)}$$

> $i_{\max} := ndx + 1;\ j_{\max} := ndy + 1;$

$$i_{max} := 6$$
$$j_{max} := 5 \quad \textbf{(1.3)}$$

Nombre d'équations:

> $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$

$$N := 12 \quad \textbf{(1.4)}$$

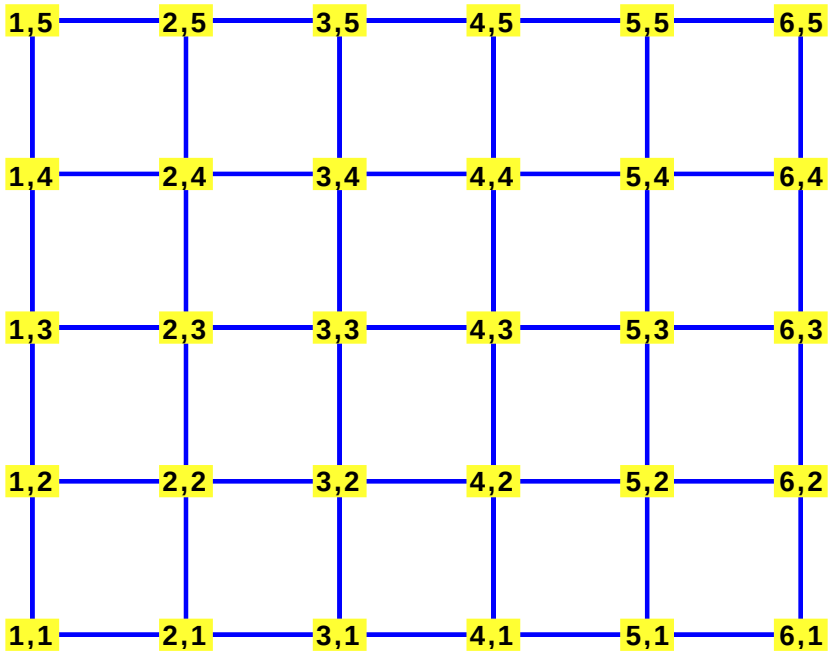
Maillage:

> $with(GraphTheory) : with(SpecialGraphs) :$

> $G := GridGraph(i_{\max}, j_{\max})$

$$G := Graph\ 1:\ an\ undirected\ unweighted\ graph\ with\ 30\ vertices\ and\ 49\ edge(s) \quad \textbf{(1.5)}$$

> $DrawGraph(G)$



Conditions aux Limites:

> $Tb := 100.;\ Tg := 20;\ Td := 40;\ Th := 10.;$

$$Tb := 100.$$
$$Tg := 20$$
$$Td := 40$$
$$Th := 10. \quad \textbf{(1.6)}$$

> **for** i **from** 2 **to** $i_{\max} - 1$ **do** $T[i, 1] := Tb$ **end do**;

$$T_{2,1} := 100.$$
$$T_{3,1} := 100.$$
$$T_{4,1} := 100.$$
$$T_{5,1} := 100. \quad \textbf{(1.7)}$$

> **for** i **from** 2 **to** $i_{max} - 1$ **do** $T[i, j_{max}] := Th$ **end do**;

$$T_{2,5} := 10.$$
$$T_{3,5} := 10.$$
$$T_{4,5} := 10.$$
$$T_{5,5} := 10. \quad \textbf{(1.8)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[1, j] := Tg$ **end do**;

$$T_{1,2} := 20$$
$$T_{1,3} := 20$$
$$T_{1,4} := 20 \quad \textbf{(1.9)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[i_{max}, j] := Td$ **end do**;

$$T_{6,2} := 40$$
$$T_{6,3} := 40$$
$$T_{6,4} := 40 \quad \textbf{(1.10)}$$

> $k := 1$:

Solution discrétisée par la formulation à 5 points:

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
        Eq[k] := T[i+1,j] + T[i-1,j] + β^2·(T[i,j+1] + T[i,j-1]) - 2·(1+β^2)
      ·T[i,j] = 0;
        Temps[k] := T[i,j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

> **for** k **from** 1 **to** N **do** $Eq[k]$ **end do**;

$$T_{3,2} + 120. + T_{2,3} - 4\,T_{2,2} = 0$$
$$T_{4,2} + T_{2,2} + T_{3,3} + 100. - 4\,T_{3,2} = 0$$
$$T_{5,2} + T_{3,2} + T_{4,3} + 100. - 4\,T_{4,2} = 0$$
$$140. + T_{4,2} + T_{5,3} - 4\,T_{5,2} = 0$$
$$T_{3,3} + 20 + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0$$
$$T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0$$
$$T_{5,3} + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0$$
$$40 + T_{4,3} + T_{5,4} + T_{5,2} - 4\,T_{5,3} = 0$$
$$T_{3,4} + 30. + T_{2,3} - 4\,T_{2,4} = 0$$
$$T_{4,4} + T_{2,4} + 10. + T_{3,3} - 4\,T_{3,4} = 0$$
$$T_{5,4} + T_{3,4} + 10. + T_{4,3} - 4\,T_{4,4} = 0$$

(2.1)

$$50. + T_{4,4} + T_{5,3} - 4\,T_{5,4} = 0 \tag{2.1}$$

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)];
```

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{5,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{5,3}, T_{2,4}, T_{3,4}, T_{4,4}, T_{5,4}] \tag{2.2}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 55.76542343, T_{3,2} = 67.33702312, T_{4,2} = 69.48756075, T_{5,2} = 63.72241268, T_{2,3} = 35.72467060, T_{3,3} = 44.09510828, T_{4,3} = 46.89080721, T_{5,3} = 45.40208996, T_{2,4} = 23.03815070, T_{3,4} = 26.42793221, T_{4,4} = 28.57846984, T_{5,4} = 30.99513995]] \tag{2.3}$$

```
> Solution := evalf(SolT);
```

$$Solution := [[T_{2,2} = 55.76542343, T_{3,2} = 67.33702312, T_{4,2} = 69.48756075, T_{5,2} = 63.72241268, T_{2,3} = 35.72467060, T_{3,3} = 44.09510828, T_{4,3} = 46.89080721, T_{5,3} = 45.40208996, T_{2,4} = 23.03815070, T_{3,4} = 26.42793221, T_{4,4} = 28.57846984, T_{5,4} = 30.99513995]] \tag{2.4}$$

```
> Sys := [seq(Eq[i], i = 1 ..N)];
```

$$Sys := [T_{3,2} + 120. + T_{2,3} - 4\,T_{2,2} = 0, T_{4,2} + T_{2,2} + T_{3,3} + 100. - 4\,T_{3,2} = 0, T_{5,2} + T_{3,2} + T_{4,3} + 100. - 4\,T_{4,2} = 0, 140. + T_{4,2} + T_{5,3} - 4\,T_{5,2} = 0, T_{3,3} + 20 + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0, T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0, T_{5,3} + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0, 40 + T_{4,3} + T_{5,4} + T_{5,2} - 4\,T_{5,3} = 0, T_{3,4} + 30. + T_{2,3} - 4\,T_{2,4} = 0, T_{4,4} + T_{2,4} + 10. + T_{3,3} - 4\,T_{3,4} = 0, T_{5,4} + T_{3,4} + 10. + T_{4,3} - 4\,T_{4,4} = 0, 50. + T_{4,4} + T_{5,3} - 4\,T_{5,4} = 0] \tag{2.5}$$

```
> Var := [seq(Temps[i], i = 1 ..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{4,2}, T_{5,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{5,3}, T_{2,4}, T_{3,4}, T_{4,4}, T_{5,4}] \tag{2.6}$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := \begin{bmatrix} 12\ x\ 12\ Matrix \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix}, \begin{bmatrix} 1\ ..\ 12\ Vector_{column} \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix} \tag{2.7}$$

```
> [seq(b[k], k = 1 ..N)]
```

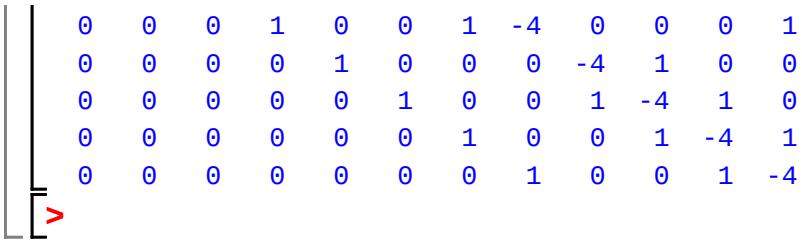
$$[-120., -100., -100., -140., -20, 0, 0, -40, -30., -10., -10., -50.] \tag{2.8}$$

```
> seq([seq(A[i, j], i = 1 ..N)], j = 1 ..N)
```

$$[-4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0], [1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0], [0, 1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 1, -4, 0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, -4, 1, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0, 1, -4, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0, -4, 1, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0], [0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1], [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4] \tag{2.9}$$

```
> printf("%3d\n", A);
```

```
 -4   1   0   0   1   0   0   0   0   0   0   0
  1  -4   1   0   0   1   0   0   0   0   0   0
  0   1  -4   1   0   0   1   0   0   0   0   0
  0   0   1  -4   0   0   0   1   0   0   0   0
  1   0   0   0  -4   1   0   0   1   0   0   0
  0   1   0   0   1  -4   1   0   0   1   0   0
  0   0   1   0   0   1  -4   1   0   0   1   0
```


0 0 0 1 0 0 1 -4 0 0 0 1
0 0 0 0 1 0 0 0 -4 1 0 0
0 0 0 0 0 1 0 0 1 -4 1 0
0 0 0 0 0 0 1 0 0 1 -4 1
0 0 0 0 0 0 0 1 0 0 1 -4
v