Equation de Diffusion 2D

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Master : Energétique

Matire : Méthodes Numériques Appliquées II

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2014/2015

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EXAMEN

Détermination de la distribution de temperature $T(\mathrm{x}, \mathrm{y})$ à travers une plaque composée (k1 et k2) de largeur L, de hauteur H et d'éapisseur e soumise aux (C.L.) suivantes.

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x}, \mathrm{y})\right) + \frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x}, \mathrm{y})\right) = 0$$

Conditions aux limites (C.L):

$$T(L, y) = T_0 = 40,$$

$$q(0, y) = 0,$$
$$q(x, 0) = 0,$$
$$q(x, H) = \frac{100\,kW}{m^2},$$

Solution

```
> Restart : Digits := 4 :
```

```
> L := 1.6; H := 1.0; e := 0.02; k_1 := 500; k_2 := 500; δx := 0.4; δy := 0.2;
```

$$L := 1.6$$
$$H := 1.0$$
$$e := 0.02$$
$$k_1 := 500$$
$$k_2 := 500$$
$$\delta x := 0.4$$
$$\delta y := 0.2 \qquad \textbf{(1.1)}$$

```
> ndx := L/δx ; ndy := H/δy; Aw := δy·e; Ae := Aw; As := δx·e; An := As; k_m :=
  (k_1 + k_2)/2;
```

$$ndx := 4.000$$
$$ndy := 5.000$$
$$Aw := 0.004$$
$$Ae := 0.004$$
$$As := 0.008$$
$$An := 0.008$$
$$k_m := 500 \qquad \textbf{(1.2)}$$

```
> i_max := round(ndx); j_max := round(ndy);
```

$$i_{max} := 4$$
$$j_{max} := 5 \qquad \textbf{(1.3)}$$

Nombre d'équations:

```
> Ne := i_max · j_max
```

$$Ne := 20 \qquad \textbf{(1.4)}$$

Conditions aux Limites:

```
> T0 := 40; q := 10^5; q0 := 0;
```

$$T0 := 40$$
$$q := 100000$$
$$q0 := 0 \qquad \textbf{(1.5)}$$

Noeuds internes gauche:

```
> for j from 2 to j_max − 1 do
    for i from 2 to trunc(i_max/2) do
      Su[i, j] := 0;
      a_W[i, j] := k_1·Aw/δx;
```

$a_E[i,j] := \frac{k_m \cdot Ae}{\delta x};$

$a_S[i,j] := \frac{k_1 \cdot As}{\delta y};$

$a_N[i,j] := \frac{k_1 \cdot An}{\delta y};$

$a_P[i,j] := \frac{k_1 \cdot Aw}{\delta x} + \frac{k_m \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y} + \frac{k_1 \cdot An}{\delta y};$

$Sp[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - a_P[i,j];$

end do;

end do;

Noeuds internes droit:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

for i **from** $\mathrm{trunc}\left(\frac{i_{\max}}{2}\right) + 1$ **to** $i_{\max} - 1$ **do**

$Su[i,j] := 0;$

$a_W[i,j] := \frac{k_m \cdot Aw}{\delta x};$

$a_E[i,j] := \frac{k_2 \cdot Ae}{\delta x};$

$a_S[i,j] := \frac{k_2 \cdot As}{\delta y};$

$a_N[i,j] := \frac{k_2 \cdot An}{\delta y};$

$a_P[i,j] := \frac{k_m \cdot Aw}{\delta x} + \frac{k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y} + \frac{k_2 \cdot An}{\delta y};$

$Sp[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - a_P[i,j];$

end do;

end do;

Noeuds gauche:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$Su[1, j] := q0 \cdot Aw;$

$a_W[1,j] := 0;$

$a_E[1,j] := \frac{k_1 \cdot Ae}{\delta x};$

$a_S[1,j] := \frac{k_1 \cdot As}{\delta y};$

$a_N[1,j] := \frac{k_1 \cdot An}{\delta y};$

$$a_P[1, j] := \frac{k_1 \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y} + \frac{k_1 \cdot An}{\delta y};$$

$$Sp[1, j] := a_W[1, j] + a_E[1, j] + a_S[1, j] + a_N[1, j] - a_P[1, j];$$

end do;

$$Su_{1,2} := 0.$$
$$(a_W)_{1,2} := 0$$
$$(a_E)_{1,2} := 5.000$$
$$(a_S)_{1,2} := 20.00$$
$$(a_N)_{1,2} := 20.00$$
$$(a_P)_{1,2} := 45.00$$
$$Sp_{1,2} := 0.$$
$$Su_{1,3} := 0.$$
$$(a_W)_{1,3} := 0$$
$$(a_E)_{1,3} := 5.000$$
$$(a_S)_{1,3} := 20.00$$
$$(a_N)_{1,3} := 20.00$$
$$(a_P)_{1,3} := 45.00$$
$$Sp_{1,3} := 0.$$
$$Su_{1,4} := 0.$$
$$(a_W)_{1,4} := 0$$
$$(a_E)_{1,4} := 5.000$$
$$(a_S)_{1,4} := 20.00$$
$$(a_N)_{1,4} := 20.00$$
$$(a_P)_{1,4} := 45.00$$
$$Sp_{1,4} := 0. \qquad \textbf{(1.6)}$$

Noeuds droit:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$$Su[i_{\max}, j] := \frac{2 \cdot k_2 \cdot Ae}{\delta x} \cdot T0;$$

$$a_W[i_{\max}, j] := \frac{k_2 \cdot Aw}{\delta x};$$

$$a_E[i_{\max}, j] := 0;$$

$$a_S[i_{\max}, j] := \frac{k_2 \cdot As}{\delta y};$$

$$a_N[i_{max}, j] := \frac{k_2 \cdot An}{\delta y};$$

$$a_P[i_{max}, j] := \frac{k_2 \cdot Aw}{\delta x} + \frac{2 \cdot k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y} + \frac{k_2 \cdot An}{\delta y};$$

$$Sp[i_{max}, j] := a_W[i_{max}, j] + a_E[i_{max}, j] + a_S[i_{max}, j] + a_N[i_{max}, j] - a_P[i_{max}, j];$$

end do;

$$Su_{4,2} := 400.0$$
$$(a_W)_{4,2} := 5.000$$
$$(a_E)_{4,2} := 0$$
$$(a_S)_{4,2} := 20.00$$
$$(a_N)_{4,2} := 20.00$$
$$(a_P)_{4,2} := 55.00$$
$$Sp_{4,2} := -10.00$$
$$Su_{4,3} := 400.0$$
$$(a_W)_{4,3} := 5.000$$
$$(a_E)_{4,3} := 0$$
$$(a_S)_{4,3} := 20.00$$
$$(a_N)_{4,3} := 20.00$$
$$(a_P)_{4,3} := 55.00$$
$$Sp_{4,3} := -10.00$$
$$Su_{4,4} := 400.0$$
$$(a_W)_{4,4} := 5.000$$
$$(a_E)_{4,4} := 0$$
$$(a_S)_{4,4} := 20.00$$
$$(a_N)_{4,4} := 20.00$$
$$(a_P)_{4,4} := 55.00$$
$$Sp_{4,4} := -10.00 \qquad \textbf{(1.7)}$$

Noeuds bas gauche:

> **for** i **from** 2 **to** $\mathrm{trunc}\left(\frac{i_{max}}{2}\right)$ **do**

$$Su[i, 1] := q0 \cdot As;;$$

$$a_W[i, 1] := \frac{k_1 \cdot Aw}{\delta x};$$

$$a_E[i, 1] := \frac{k_m \cdot Ae}{\delta x};$$

$$a_S[i, 1] := 0;$$

$a_N[i,1] := \frac{k_1 \cdot An}{\delta y};$

$a_P[i,1] := \frac{k_1 \cdot Aw}{\delta x} + \frac{k_m \cdot Ae}{\delta x} + \frac{k_1 \cdot An}{\delta y};$

$Sp[i,1] := a_W[i,1] + a_E[i,1] + a_S[i,1] + a_N[i,1] - a_P[i,1];$

end do;

$$Su_{2,1} := 0.$$
$$(a_W)_{2,1} := 5.000$$
$$(a_E)_{2,1} := 5.000$$
$$(a_S)_{2,1} := 0$$
$$(a_N)_{2,1} := 20.00$$
$$(a_P)_{2,1} := 30.00$$
$$Sp_{2,1} := 0. \qquad \textbf{(1.8)}$$

Noeuds bas droit:

> **for** i **from** $\mathrm{trunc}\left(\frac{i_{\max}}{2}\right) + 1$ **to** $i_{\max} - 1$ **do**

$Su[i,1] := q0 \cdot As;$

$a_W[i,1] := \frac{k_m \cdot Aw}{\delta x};$

$a_E[i,1] := \frac{k_2 \cdot Ae}{\delta x};$

$a_S[i,1] := 0;$

$a_N[i,1] := \frac{k_2 \cdot An}{\delta y};$

$a_P[i,1] := \frac{k_m \cdot Aw}{\delta x} + \frac{k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot An}{\delta y};$

$Sp[i,1] := a_W[i,1] + a_E[i,1] + a_S[i,1] + a_N[i,1] - a_P[i,1];$

end do;

$$Su_{3,1} := 0.$$
$$(a_W)_{3,1} := 5.000$$
$$(a_E)_{3,1} := 5.000$$
$$(a_S)_{3,1} := 0$$
$$(a_N)_{3,1} := 20.00$$
$$(a_P)_{3,1} := 30.00$$
$$Sp_{3,1} := 0. \qquad \textbf{(1.9)}$$

Noeuds hauts gauches:

> **for** i **from** 2 **to** $\mathrm{trunc}\left(\frac{i_{\max}}{2}\right)$ **do**

$Su[i, j_{max}] := q \cdot An;$

$a_W[i, j_{max}] := \frac{k_1 \cdot Aw}{\delta x};$

$a_E[i, j_{max}] := \frac{k_m \cdot Ae}{\delta x};$

$a_S[i, j_{max}] := \frac{k_1 \cdot As}{\delta y};$

$a_N[i, j_{max}] := 0;$

$a_P[i, j_{max}] := \frac{k_1 \cdot Aw}{\delta x} + \frac{k_m \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y};$

$Sp[i, j_{max}] := a_W[i, j_{max}] + a_E[i, j_{max}] + a_S[i, j_{max}] + a_N[i, j_{max}] - a_P[i, j_{max}];$

end do;

$$Su_{2,5} := 800.0$$
$$(a_W)_{2,5} := 5.000$$
$$(a_E)_{2,5} := 5.000$$
$$(a_S)_{2,5} := 20.00$$
$$(a_N)_{2,5} := 0$$
$$(a_P)_{2,5} := 30.00$$
$$Sp_{2,5} := 0. \qquad \textbf{(1.10)}$$

Noeuds hauts droit:

> **for** i **from** $\text{trunc}\left(\frac{i_{max}}{2}\right) + 1$ **to** $i_{max} - 1$ **do**

$Su[i, j_{max}] := q \cdot An;$

$a_W[i, j_{max}] := \frac{k_m \cdot Aw}{\delta x};$

$a_E[i, j_{max}] := \frac{k_2 \cdot Ae}{\delta x};$

$a_S[i, j_{max}] := \frac{k_2 \cdot As}{\delta y};$

$a_N[i, j_{max}] := 0;$

$a_P[i, j_{max}] := \frac{k_m \cdot Aw}{\delta x} + \frac{k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y};$

$Sp[i, j_{max}] := a_W[i, j_{max}] + a_E[i, j_{max}] + a_S[i, j_{max}] + a_N[i, j_{max}] - a_P[i, j_{max}];$

end do;

$$Su_{3,5} := 800.0$$
$$(a_W)_{3,5} := 5.000$$
$$(a_E)_{3,5} := 5.000$$
$$(a_S)_{3,5} := 20.00$$

$$(a_N)_{3,5} := 0$$
$$(a_P)_{3,5} := 30.00$$
$$Sp_{3,5} := 0. \qquad \textbf{(1.11)}$$

Noeud (1,1):

```
> Su[1 , 1] := q0·Aw + q0·As;
  aW[1, 1] := 0;
  aE[1, 1] := k1·Ae / δx;
  aS[1, 1] := 0;
  aN[1, 1] := k1·An / δy;
  aP[1, 1] := k1·Ae / δx + k1·An / δy ;
  Sp[1, 1] := aW[1, 1] + aE[1, 1] + aS[1, 1] + aN[1, 1] − aP[1, 1];
```

$$Su_{1,1} := 0$$
$$(a_W)_{1,1} := 0$$
$$(a_E)_{1,1} := 5.000$$
$$(a_S)_{1,1} := 0$$
$$(a_N)_{1,1} := 20.00$$
$$(a_P)_{1,1} := 25.00$$
$$Sp_{1,1} := 0. \qquad \textbf{(1.12)}$$

Noeud (imax,1):

```
> Su[ imax , 1] := q0·As + 2· k2·Ae / δx · T0;
  aW[ imax, 1] := k2·Aw / δx;
  aE[ imax, 1] := 0;
  aS[ imax, 1] := 0;
  aN[ imax, 1] := k2·An / δy;
  aP[ imax, 1] := k2·Aw / δx + 2· k2·Ae / δx + k2·An / δy;
  Sp[ imax, 1] := aW[ imax, 1] + aE[ imax, 1] + aS[ imax, 1] + aN[ imax, 1] − aP[ imax,
      1];
```

$$Su_{4,1} := 400.0$$
$$(a_W)_{4,1} := 5.000$$
$$(a_E)_{4,1} := 0$$
$$(a_S)_{4,1} := 0$$

$$(a_N)_{4,1} := 20.00$$
$$(a_P)_{4,1} := 35.00$$
$$Sp_{4,1} := -10.00 \quad \textbf{(1.13)}$$

Noeud (1,jmax):

> $Su[1, j_{max}] := q0 \cdot Aw + q \cdot An;$
> $a_W[1, j_{max}] := 0;$
> $a_E[1, j_{max}] := \frac{k_1 \cdot Ae}{\delta x};$
> $a_S[1, j_{max}] := \frac{k_1 \cdot As}{\delta y};$
> $a_N[1, j_{max}] := 0;$
> $a_P[1, j_{max}] := \frac{k_1 \cdot Ae}{\delta x} + \frac{k_1 \cdot As}{\delta y};$
> $Sp[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - a_P[1, j_{max}];$

$$Su_{1,5} := 800.0$$
$$(a_W)_{1,5} := 0$$
$$(a_E)_{1,5} := 5.000$$
$$(a_S)_{1,5} := 20.00$$
$$(a_N)_{1,5} := 0$$
$$(a_P)_{1,5} := 25.00$$
$$Sp_{1,5} := 0. \quad \textbf{(1.14)}$$

Noeud (imax,jmax):

> $Su[i_{max}, j_{max}] := q \cdot An + \frac{2 \cdot k_2 \cdot Ae}{\delta x} \cdot T0;$
> $a_W[i_{max}, j_{max}] := \frac{k_2 \cdot Aw}{\delta x};$
> $a_E[i_{max}, j_{max}] := 0;$
> $a_S[i_{max}, j_{max}] := \frac{k_2 \cdot As}{\delta y};$
> $a_N[i_{max}, j_{max}] := 0;$
> $a_P[i_{max}, j_{max}] := \frac{k_2 \cdot Aw}{\delta x} + \frac{2 \cdot k_2 \cdot Ae}{\delta x} + \frac{k_2 \cdot As}{\delta y};$
> $Sp[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}] - a_P[i_{max}, j_{max}];$

$$Su_{4,5} := 1200.$$
$$(a_W)_{4,5} := 5.000$$
$$(a_E)_{4,5} := 0$$

$$(a_S)_{4,5} := 20.00$$
$$(a_N)_{4,5} := 0$$
$$(a_P)_{4,5} := 35.00$$
$$Sp_{4,5} := -10.00 \qquad \textbf{(1.15)}$$

Equations:

```
> k := 1 :
```

Résolution pour les noeuds internes:

```
> for j from 1 to j_max do
  for i from 1 to i_max do
        Eq[k] := a_P[i,j]·T[i,j] = a_W[i,j]·T[i-1,j] + a_E[i,j]·T[i+1,j]
      + a_S[i,j]·T[i,j-1] + a_N[i,j]·T[i,j+1] + Su[i,j];
        Var[k] := T[i,j];
        k := k+1;
  end do;
  end do;
```

Ecriture du systme d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$25.00\,T_{1,1} = 20.00\,T_{1,2} + 5.000\,T_{2,1}$$
$$30.00\,T_{2,1} = 20.00\,T_{2,2} + 5.000\,T_{1,1} + 5.000\,T_{3,1}$$
$$30.00\,T_{3,1} = 20.00\,T_{3,2} + 5.000\,T_{2,1} + 5.000\,T_{4,1}$$
$$35.00\,T_{4,1} = 20.00\,T_{4,2} + 5.000\,T_{3,1} + 400.0$$
$$45.00\,T_{1,2} = 20.00\,T_{1,1} + 20.00\,T_{1,3} + 5.000\,T_{2,2}$$
$$50.00\,T_{2,2} = 20.00\,T_{2,1} + 20.00\,T_{2,3} + 5.000\,T_{1,2} + 5.000\,T_{3,2}$$
$$50.00\,T_{3,2} = 20.00\,T_{3,1} + 20.00\,T_{3,3} + 5.000\,T_{2,2} + 5.000\,T_{4,2}$$
$$55.00\,T_{4,2} = 20.00\,T_{4,1} + 20.00\,T_{4,3} + 5.000\,T_{3,2} + 400.0$$
$$45.00\,T_{1,3} = 20.00\,T_{1,2} + 20.00\,T_{1,4} + 5.000\,T_{2,3}$$
$$50.00\,T_{2,3} = 20.00\,T_{2,2} + 20.00\,T_{2,4} + 5.000\,T_{1,3} + 5.000\,T_{3,3}$$
$$50.00\,T_{3,3} = 20.00\,T_{3,2} + 20.00\,T_{3,4} + 5.000\,T_{2,3} + 5.000\,T_{4,3}$$
$$55.00\,T_{4,3} = 20.00\,T_{4,2} + 20.00\,T_{4,4} + 5.000\,T_{3,3} + 400.0$$
$$45.00\,T_{1,4} = 20.00\,T_{1,3} + 20.00\,T_{1,5} + 5.000\,T_{2,4}$$
$$50.00\,T_{2,4} = 20.00\,T_{2,3} + 20.00\,T_{2,5} + 5.000\,T_{1,4} + 5.000\,T_{3,4}$$
$$50.00\,T_{3,4} = 20.00\,T_{3,3} + 20.00\,T_{3,5} + 5.000\,T_{2,4} + 5.000\,T_{4,4}$$
$$55.00\,T_{4,4} = 20.00\,T_{4,3} + 20.00\,T_{4,5} + 5.000\,T_{3,4} + 400.0$$
$$25.00\,T_{1,5} = 20.00\,T_{1,4} + 5.000\,T_{2,5} + 800.0$$
$$30.00\,T_{2,5} = 20.00\,T_{2,4} + 5.000\,T_{1,5} + 5.000\,T_{3,5} + 800.0$$
$$30.00\,T_{3,5} = 20.00\,T_{3,4} + 5.000\,T_{2,5} + 5.000\,T_{4,5} + 800.0$$
$$35.00\,T_{4,5} = 20.00\,T_{4,4} + 5.000\,T_{3,5} + 1200. \qquad \textbf{(1.1.1)}$$

```
> Eqs := [seq(Eq[k], k = 1..Ne)] :
> Vars := [seq(Var[k], k = 1..Ne)] :
```

```
> SolT := solve(Eqs, Vars);
```

$$SolT := [[T_{1,1} = 264.7,\ T_{2,1} = 233.8,\ T_{3,1} = 173.4,\ T_{4,1} = 88.43,\ T_{1,2} = 272.4,\ T_{2,2} = 241.1,\ T_{3,2} = 179.5,\ T_{4,2} = 91.41,\ T_{1,3} = 288.0,\ T_{2,3} = 256.0,\ T_{3,3} = 192.3,\ T_{4,3} = 98.08,\ T_{1,4} = 311.6,\ T_{2,4} = 278.9,\ T_{3,4} = 212.7,\ T_{4,4} = 110.2,\ T_{1,5} = 343.3,\ T_{2,5} = 310.2,\ T_{3,5} = 242.1,\ T_{4,5} = 131.9]] \quad \textbf{(1.1.2)}$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Vars)
```

$$A, b := \begin{bmatrix} 25.00 & -5.000 & 0 & 0 & -20.00 & 0 & 0 & 0 & 0 \\ -5.000 & 30.00 & -5.000 & 0 & 0 & -20.00 & 0 & 0 & 0 \\ 0 & -5.000 & 30.00 & -5.000 & 0 & 0 & -20.00 & 0 & 0 \\ 0 & 0 & -5.000 & 35.00 & 0 & 0 & 0 & -20.00 & 0 \\ -20.00 & 0 & 0 & 0 & 45.00 & -5.000 & 0 & 0 & -20 \\ 0 & -20.00 & 0 & 0 & -5.000 & 50.00 & -5.000 & 0 & 0 \\ 0 & 0 & -20.00 & 0 & 0 & -5.000 & 50.00 & -5.000 & 0 \\ 0 & 0 & 0 & -20.00 & 0 & 0 & -5.000 & 55.00 & 0 \\ 0 & 0 & 0 & 0 & -20.00 & 0 & 0 & 0 & 45.0 \\ 0 & 0 & 0 & 0 & 0 & -20.00 & 0 & 0 & -5.0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 400.0 \\ 0 \\ 0 \\ 0 \\ 400.0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

20 element Vector[column]

```
> seq(b[i], i = 1 ..Ne)
```

0, 0, 0, 400.0, 0, 0, 0, 400.0, 0, 0, 0, 400.0, 0, 0, 0, 400.0, 800.0, 800.0, **(1.1.4)**

800.0, 1200.

> $seq(A[i, i], i = 1..Ne)$

25.00, 30.00, 30.00, 35.00, 45.00, 50.00, 50.00, 55.00, 45.00, 50.00, 50.00, 55.00, 45.00, 50.00, 50.00, 55.00, 25.00, 30.00, 30.00, 35.00 **(1.1.5)**

Récapitulation:

> $seq(seq(a_W[i, j], i = 1..i_{max}), j = 1..j_{max})$

0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000 **(1.1.6)**

> $seq(seq(a_E[i, j], i = 1..i_{max}), j = 1..j_{max})$

5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0, 5.000, 5.000, 5.000, 0 **(1.1.7)**

> $seq(seq(a_S[i, j], i = 1..i_{max}), j = 1..j_{max})$

0, 0, 0, 0, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00 **(1.1.8)**

> $seq(seq(a_N[i, j], i = 1..i_{max}), j = 1..j_{max})$

20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 20.00, 0, 0, 0, 0 **(1.1.9)**

> $seq(seq(Su[i, j], i = 1..i_{max}), j = 1..j_{max})$

0, 0., 0., 400.0, 0., 0, 0, 400.0, 0., 0, 0, 400.0, 0., 0, 0, 400.0, 800.0, 800.0, 800.0, 1200. **(1.1.10)**

> $seq(seq(a_P[i, j], i = 1..i_{max}), j = 1..j_{max})$

25.00, 30.00, 30.00, 35.00, 45.00, 50.00, 50.00, 55.00, 45.00, 50.00, 50.00, 55.00, 45.00, 50.00, 50.00, 55.00, 25.00, 30.00, 30.00, 35.00 **(1.1.11)**

> $seq(seq(Sp[i, j], i = 1..i_{max}), j = 1..j_{max})$

0., 0., 0., −10.00, 0., 0., 0., −10.00, 0., 0., 0., −10.00, 0., 0., 0., −10.00, 0., 0., 0., −10.00 **(1.1.12)**

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