Equation d'onde 1D

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LMD : Energétique

Matire : Outils Numériques

2009/2010

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Détermination du déplacement u(x, t) d'une corde dont les extrémités sont maintenues.

$$\frac{\partial^2}{\partial t^2} u(x, t) = c^2 \cdot \frac{\partial^2}{\partial x^2} u(x, t)$$

Conditions aux limites et initiale:

$$u(0, t) = 0,$$
$$u(L, t) = 0,$$
$$u(x, 0) = 2 \cdot \sin\left(\frac{\pi}{L} x\right),$$
$$\frac{\partial}{\partial t}(u(x, 0)) = -\sin\left(\frac{2 \cdot \pi}{L} x\right),$$

Solution analytique:

Solution discrétisée:

> $Restart:$

> $\Delta x := 0.1;\ \Delta t := 0.05;\ L := 1;$

$$\Delta x := 0.1$$
$$\Delta t := 0.05$$
$$L := 1$$

> $\lambda := \left(\frac{\Delta t}{\Delta x}\right)^2;$

$$\lambda := 0.2500000000$$

> $i_{max} := 21;$

$$i_{max} := 21$$

> $n_{max} := 20;$

$$n_{max} := 20$$

> $\alpha := 0;\ \beta := 0;$

$$\alpha := 0$$
$$\beta := 0$$

> $f(x) := 2\cdot \sin\left(\frac{\pi\cdot x}{L}\right)$

$$f := x \rightarrow 2\sin\left(\frac{\pi x}{L}\right)$$

> $g(x) := -\sin\left(\frac{2\cdot\pi\cdot x}{L}\right)$

$$g := x \rightarrow -\sin\left(\frac{2\pi x}{L}\right)$$

> **for** i **from** 1 **to** i_{max} **do** $u[\mathrm{i}, 0] := evalf(f((i-1)\cdot\Delta x))$ **end do**;

$$u_{1,0} := 0.$$
$$u_{2,0} := 0.6180339888$$
$$u_{3,0} := 1.175570505$$
$$u_{4,0} := 1.618033989$$
$$u_{5,0} := 1.902113033$$
$$u_{6,0} := 2.$$
$$u_{7,0} := 1.902113033$$
$$u_{8,0} := 1.618033988$$
$$u_{9,0} := 1.175570504$$
$$u_{10,0} := 0.6180339872$$
$$u_{11,0} := 0.$$
$$u_{12,0} := -0.6180339888$$
$$u_{13,0} := -1.175570506$$
$$u_{14,0} := -1.618033989$$
$$u_{15,0} := -1.902113033$$
$$u_{16,0} := -2.$$

$$u_{17,\,0} := -1.902113032$$
$$u_{18,\,0} := -1.618033988$$
$$u_{19,\,0} := -1.175570504$$
$$u_{20,\,0} := -0.6180339866$$
$$u_{21,\,0} := 0.$$

```
> for n from 1 to n_max do u[1, n] := α end do;
```

$$u_{1,\,1} := 0$$
$$u_{1,\,2} := 0$$
$$u_{1,\,3} := 0$$
$$u_{1,\,4} := 0$$
$$u_{1,\,5} := 0$$
$$u_{1,\,6} := 0$$
$$u_{1,\,7} := 0$$
$$u_{1,\,8} := 0$$
$$u_{1,\,9} := 0$$
$$u_{1,\,10} := 0$$
$$u_{1,\,11} := 0$$
$$u_{1,\,12} := 0$$
$$u_{1,\,13} := 0$$
$$u_{1,\,14} := 0$$
$$u_{1,\,15} := 0$$
$$u_{1,\,16} := 0$$
$$u_{1,\,17} := 0$$
$$u_{1,\,18} := 0$$
$$u_{1,\,19} := 0$$
$$u_{1,\,20} := 0$$

```
> for n from 1 to n_max do u[i_max, n] := β end do;
```

$$u_{21,\,1} := 0$$
$$u_{21,\,2} := 0$$
$$u_{21,\,3} := 0$$
$$u_{21,\,4} := 0$$
$$u_{21,\,5} := 0$$
$$u_{21,\,6} := 0$$
$$u_{21,\,7} := 0$$
$$u_{21,\,8} := 0$$
$$u_{21,\,9} := 0$$
$$u_{21,\,10} := 0$$
$$u_{21,\,11} := 0$$
$$u_{21,\,12} := 0$$

$$u_{21,13} := 0$$
$$u_{21,14} := 0$$
$$u_{21,15} := 0$$
$$u_{21,16} := 0$$
$$u_{21,17} := 0$$
$$u_{21,18} := 0$$
$$u_{21,19} := 0$$
$$u_{21,20} := 0$$

Schéma explicite:

```
> for i from 1 to i_max do
    u[i, 1] := (1 - λ)·evalf(f((i-1)·Δx)) + λ/2·(evalf(f(i·Δx)) + evalf(f((i
        - 2)·Δx))) + Δt·evalf(g((i-1)·Δx));
  end do;
```

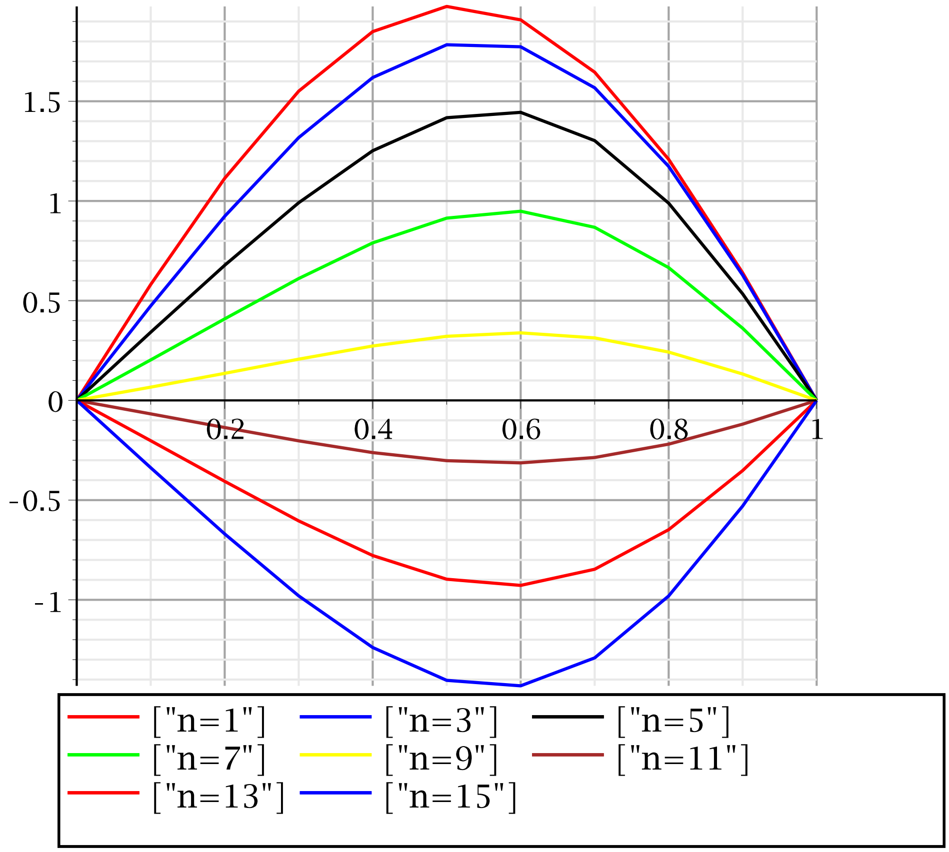
$$u_{1,1} := 0.$$
$$u_{2,1} := 0.5810825421$$
$$u_{3,1} := 1.113633550$$
$$u_{4,1} := 1.550683108$$
$$u_{5,1} := 1.849449761$$
$$u_{6,1} := 1.975528258$$
$$u_{7,1} := 1.908228287$$
$$u_{8,1} := 1.645788759$$
$$u_{9,1} := 1.208739201$$
$$u_{10,1} := 0.6398610660$$
$$u_{11,1} := -2.000000000\ 10^{-10}$$
$$u_{12,1} := -0.6398610675$$
$$u_{13,1} := -1.208739203$$
$$u_{14,1} := -1.645788760$$
$$u_{15,1} := -1.908228287$$
$$u_{16,1} := -1.975528258$$
$$u_{17,1} := -1.849449759$$
$$u_{18,1} := -1.550683107$$
$$u_{19,1} := -1.113633549$$
$$u_{20,1} := -0.5810825406$$
$$u_{21,1} := 3.750000000\ 10^{-10}$$

```
> for n from 1 to n_max do
    for i from 2 to i_max - 1 do
        u[i, n + 1] := 2·(1 - λ)·u[i, n] + λ·(u[i + 1, n] + u[i - 1, n]) - u[i, n-1]
    end do;
```

```
end do;

> for i from 2 to i_max - 1 do u[i, n_max] end do;
                    -0.6105053354
                    -1.163380623
                    -1.605823279
                    -1.894523924
                    -1.999905827
                    -1.909523014
                    -1.630092323
                    -1.187649672
                    -0.6255044323
                    4.757041261 10^-9
                    0.6255044419
                    1.187649673
                    1.630092321
                    1.909523018
                    1.999905826
                    1.894523919
                    1.605823277
                    1.163380630
                    0.6105053433
> with(plots) :
> for n from 1 to n_max do
    liste[n] := [[0.0, u[1, n]], [0.1, u[2, n]], [0.2, u[3, n]], [0.3, u[4, n]], [0.4,
      u[5, n]], [0.5, u[6, n]], [0.6, u[7, n]], [0.7, u[8, n]], [0.8, u[9, n]], [0.9,
      u[10, n]], [1.0, u[11, n]]]
  end do:

> multiple(listplot, [liste[1], color = red, legend = ["n=1"]], [liste[3], color
      = blue, legend = ["n=3"]], [liste[5], color = black, legend = ["n=5"]],
      [liste[7], color = green, legend = ["n=7"]], [liste[9], color = yellow, legend
      = ["n=9"]], [liste[11], color = brown, legend = ["n=11"]], [liste[13], color
      = red, legend = ["n=13"]], [liste[15], color = blue, legend = ["n=15"]],
      gridlines = true);
```

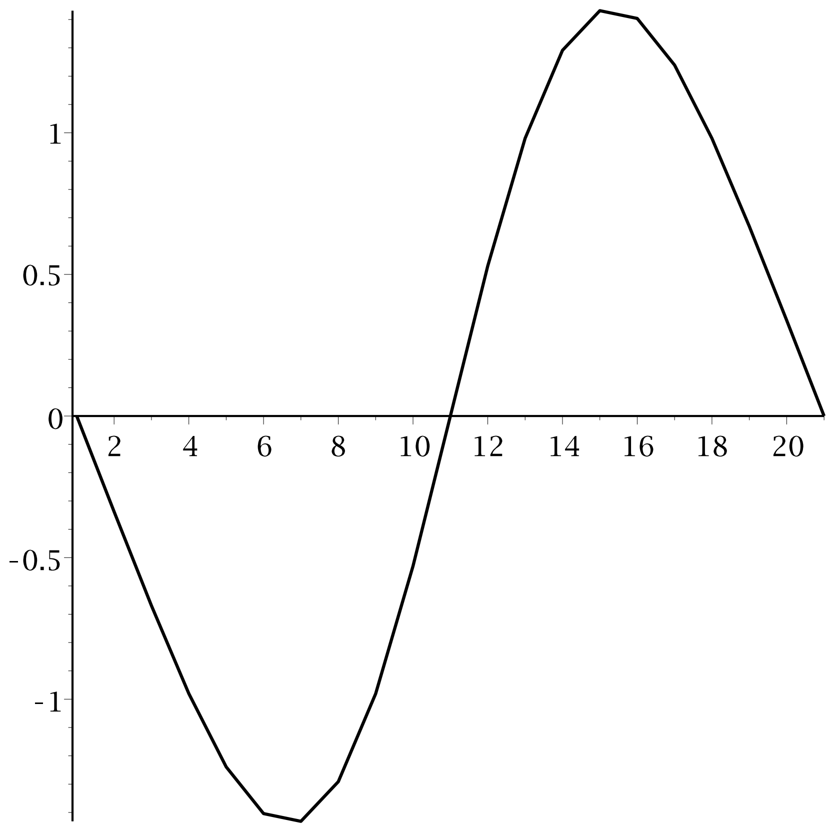


> **for** n **from** 1 **to** n_{max} **do**
    $liste[n] := [\alpha, seq(u[i, n], i = 2 .. i_{max} - 1), \beta]$
  end do:

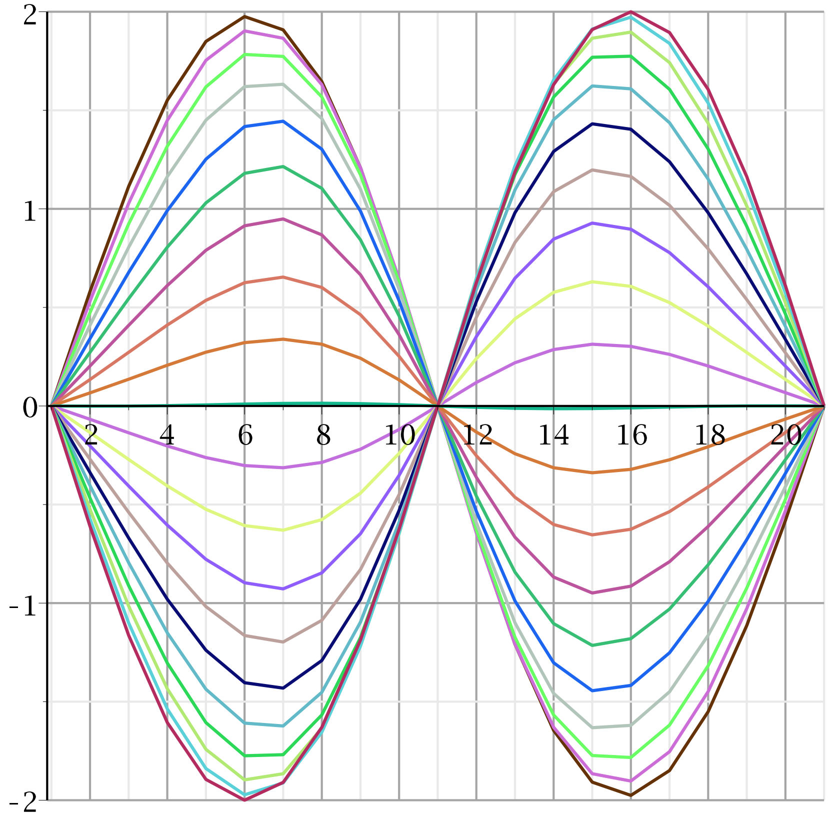
> SolExplicite := $liste[n_{max}]$

SolExplicite := [0, -0.6105053354, -1.163380623, -1.605823279, -1.894523924, -1.999905827, -1.909523014, -1.630092323, -1.187649672, -0.6255044323, $4.757041261\ 10^{-9}$, 0.6255044419, 1.187649673, 1.630092321, 1.909523018, 1.999905826, 1.894523919, 1.605823277, 1.163380630, 0.6105053433, 0]

> *listplot*(*liste*[15])



$$> \; multiple\left(listplot,\ seq\left(\left[liste[i],\ color = COLOR\left(RGB,\ \frac{rand(\,)}{10^{12}},\ \frac{rand(\,)}{10^{12}},\ \frac{rand(\,)}{10^{12}}\right)\right],\ i = 1\,..n_{\mathrm{max}}\right),\ gridlines = true\right);$$



> `?`

Schéma implicite: