

Equation de diffusion (chaleur) 1D instationnaire

Dr. Lad MESSAOUDI

Département de Mécanique

Université de Batna

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Matire : Outils Numériques

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Détermination de la temperature $T(x, t)$ travers l'épaisseur d'une plaque dont les extrémités sont maintenues des températures constantes.

$$\frac{\partial}{\partial t} T(x, t) = \frac{\partial^2}{\partial x^2} T(x, t)$$

Conditions aux limites et initiale:

$$\begin{aligned} T(0, t) &= \alpha, \\ T(1, t) &= \beta, \\ T(x, 0) &= \sigma \end{aligned}$$

Forme matricielle - conditions de Dirichlet - Schéma explicite:

```
[> Restart : with(LinearAlgebra) :  
[>  
[> i_max := 9; n_max := 15; N := i_max - 2;  
i_max := 9  
n_max := 15  
N := 7  
[> for i from 2 to i_max - 1 do T[i, 0] := sigma end do;  
T_{2,0} := sigma  
T_{3,0} := sigma
```

$T_{4,0} := \sigma$

$T_{5,0} := \sigma$

$T_{6,0} := \sigma$

$T_{7,0} := \sigma$

$T_{8,0} := \sigma$

> **for** n **from** 0 **to** n_{\max} **do** $T[1, n] := \alpha$ **end do**;

$T_{1,0} := \alpha$

$T_{1,1} := \alpha$

$T_{1,2} := \alpha$

$T_{1,3} := \alpha$

$T_{1,4} := \alpha$

$T_{1,5} := \alpha$

$T_{1,6} := \alpha$

$T_{1,7} := \alpha$

$T_{1,8} := \alpha$

$T_{1,9} := \alpha$

$T_{1,10} := \alpha$

$T_{1,11} := \alpha$

$T_{1,12} := \alpha$

$T_{1,13} := \alpha$

$T_{1,14} := \alpha$

$T_{1,15} := \alpha$

> **for** n **from** 0 **to** n_{\max} **do** $T[i_{\max}, n] := \beta$ **end do**;

$T_{9,0} := \beta$

$T_{9,1} := \beta$

$T_{9,2} := \beta$

$T_{9,3} := \beta$

$T_{9,4} := \beta$

$T_{9,5} := \beta$

$T_{9,6} := \beta$

$T_{9,7} := \beta$

$T_{9,8} := \beta$

$T_{9,9} := \beta$

$T_{9,10} := \beta$

$T_{9,11} := \beta$

$T_{9,12} := \beta$

$T_{9,13} := \beta$

$$T_{9,14} := \beta$$

$$T_{9,15} := \beta$$

Boucle principale

```
> n := nmax - 1 : k := 1 :  
> for i from 2 to imax - 1 do  
    Eq[k] := λ · T[i - 1, n] + (1 - 2 · λ) · T[i, n] + λ · T[i + 1, n]  
    = T[i, n + 1];  
    k := k + 1;  
end do;
```

```
> Eqs := [seq(Eq[k], k = 1 .. N)]:  
> Tmps := [seq(T[i, n], i = 2 .. imax - 1)]:  
> A, b := GenerateMatrix( Eqs, Tmps);
```

$$A, b := \begin{bmatrix} 1 - 2\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \lambda & 1 - 2\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \lambda & 1 - 2\lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 - 2\lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 - 2\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 - 2\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \lambda & 1 - 2\lambda \end{bmatrix},$$

$$\begin{bmatrix} -\alpha\lambda + T_{2,15} \\ T_{3,15} \\ T_{4,15} \\ T_{5,15} \\ T_{6,15} \\ T_{7,15} \\ -\beta\lambda + T_{8,15} \end{bmatrix}$$