Equation de diffusion (chaleur) 1D instationnaire

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Détermination de la temperature $T(x, t)$ à travers l'épaisseur d'une plaque dont les extrémités sont maintenues à des températures constantes.

$$\frac{\partial}{\partial t} T(x, t) = \frac{\partial^2}{\partial x^2} T(x, t)$$

Conditions aux limites et initiale:

$$T(0, t) = \alpha,$$
$$T(1, t) = \beta,$$
$$T(x, 0) = \sigma$$

Forme matricielle - conditions de Dirichlet - Schéma Implicite:

```
> Restart : with(LinearAlgebra) :
>
> i_max := 11; n_max := 15; N := i_max - 2;
```

$$i_{max} := 11$$
$$n_{max} := 15$$
$$N := 9 \quad (1.1)$$

```
> for i from 2 to i_max - 1 do T[i, 0] := σ end do;
```

$$T_{2,0} := \sigma$$
$$T_{3,0} := \sigma$$
$$T_{4,0} := \sigma$$
$$T_{5,0} := \sigma$$

$$T_{6,\,0} := \sigma$$
$$T_{7,\,0} := \sigma$$
$$T_{8,\,0} := \sigma$$
$$T_{9,\,0} := \sigma$$
$$T_{10,\,0} := \sigma \tag{1.2}$$

```
> for n from 0 to n_max do T[1, n] := α  end do;
```

$$T_{1,\,0} := \alpha$$
$$T_{1,\,1} := \alpha$$
$$T_{1,\,2} := \alpha$$
$$T_{1,\,3} := \alpha$$
$$T_{1,\,4} := \alpha$$
$$T_{1,\,5} := \alpha$$
$$T_{1,\,6} := \alpha$$
$$T_{1,\,7} := \alpha$$
$$T_{1,\,8} := \alpha$$
$$T_{1,\,9} := \alpha$$
$$T_{1,\,10} := \alpha$$
$$T_{1,\,11} := \alpha$$
$$T_{1,\,12} := \alpha$$
$$T_{1,\,13} := \alpha$$
$$T_{1,\,14} := \alpha$$
$$T_{1,\,15} := \alpha \tag{1.3}$$

```
> for n from 0 to n_max do T[i_max, n] := β end do;
```

$$T_{11,\,0} := \beta$$
$$T_{11,\,1} := \beta$$
$$T_{11,\,2} := \beta$$
$$T_{11,\,3} := \beta$$
$$T_{11,\,4} := \beta$$
$$T_{11,\,5} := \beta$$
$$T_{11,\,6} := \beta$$
$$T_{11,\,7} := \beta$$
$$T_{11,\,8} := \beta$$
$$T_{11,\,9} := \beta$$
$$T_{11,\,10} := \beta$$
$$T_{11,\,11} := \beta$$
$$T_{11,\,12} := \beta$$
$$T_{11,\,13} := \beta$$
$$T_{11,\,14} := \beta$$

$$T_{11,\,15} := \beta \tag{1.4}$$

Boucle principale

```
> n := n_max - 1 :  k := 1 :
> for i from 2 to i_max - 1 do
        Eq[k] := -λ·T[i - 1, n + 1] + (1 + 2·λ)·T[i, n + 1]-λ·T[i + 1, n
    + 1]   = T[i, n];
        k := k + 1;
  end do:
> Eqs := [seq(Eq[k], k = 1 ..N)] :
> Tmps := [seq(T[i, n + 1], i = 2 ..i_max - 1)] :
> A, b := GenerateMatrix( Eqs, Tmps);
```

$$A, b := [[1+2\lambda, -\lambda, 0, 0, 0, 0, 0, 0, 0], [-\lambda, 1+2\lambda, -\lambda, 0, 0, 0, 0, 0, 0], [0, -\lambda, 1+2\lambda, -\lambda, 0, 0, 0, 0, 0], [0, 0, -\lambda, 1+2\lambda, -\lambda, 0, 0, 0, 0], [0, 0, 0, -\lambda, 1+2\lambda, -\lambda, 0, 0, 0], [0, 0, 0, 0, -\lambda, 1+2\lambda, -\lambda, 0, 0], [0, 0, 0, 0, 0, -\lambda, 1+2\lambda, -\lambda, 0], [0, 0, 0, 0, 0, 0, -\lambda, 1+2\lambda, -\lambda], [0, 0, 0, 0, 0, 0, 0, -\lambda, 1+2\lambda]], \begin{bmatrix} \lambda\alpha + T_{2,\,14} \\ T_{3,\,14} \\ T_{4,\,14} \\ T_{5,\,14} \\ T_{6,\,14} \\ T_{7,\,14} \\ T_{8,\,14} \\ T_{9,\,14} \\ \lambda\beta + T_{10,\,14} \end{bmatrix} \tag{1.1.1}$$