Equation de diffusion (chaleur) 1D instationnaire

Dr. Lad MESSAOUDI

Département de Mécanique

Université de Batna

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LMD : Energétique

Matire : Outils Numériques

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Détermination de la temperature T(x, t) travers l'épaisseur d'une plaque dont les extrémités sont maintenues des températures constantes.

$$\frac{\partial}{\partial t} T(x, t) = \frac{\partial^2}{\partial x^2} T(x, t)$$

Conditions aux limites et initiale:

$$T(0, t) = \alpha,$$
$$T(1, t) = \beta,$$
$$T(x, 0) = \sigma$$

Forme matricielle - conditions de Dirichlet - Schéma Implicite:

```
> Restart : with(LinearAlgebra) :
>
> i_max := 11; n_max := 15; N := i_max - 2;
```

$$i_{max} := 11$$
$$n_{max} := 15$$
$$N := 9$$

```
> for i from 2 to i_max - 1 do T[i, 0] := σ end do;
```

$$T_{2,\,0} := \sigma$$
$$T_{3,\,0} := \sigma$$
$$T_{4,\,0} := \sigma$$
$$T_{5,\,0} := \sigma$$
$$T_{6,\,0} := \sigma$$
$$T_{7,\,0} := \sigma$$
$$T_{8,\,0} := \sigma$$
$$T_{9,\,0} := \sigma$$
$$T_{10,\,0} := \sigma$$

> **for** n **from** 0 **to** n_{max} **do** $T[1, n] := \alpha$ **end do**;

$$T_{1,\,0} := \alpha$$
$$T_{1,\,1} := \alpha$$
$$T_{1,\,2} := \alpha$$
$$T_{1,\,3} := \alpha$$
$$T_{1,\,4} := \alpha$$
$$T_{1,\,5} := \alpha$$
$$T_{1,\,6} := \alpha$$
$$T_{1,\,7} := \alpha$$
$$T_{1,\,8} := \alpha$$
$$T_{1,\,9} := \alpha$$
$$T_{1,\,10} := \alpha$$
$$T_{1,\,11} := \alpha$$
$$T_{1,\,12} := \alpha$$
$$T_{1,\,13} := \alpha$$
$$T_{1,\,14} := \alpha$$
$$T_{1,\,15} := \alpha$$

> **for** n **from** 0 **to** n_{max} **do** $T[i_{max}, n] := \beta$ **end do**;

$$T_{11,\,0} := \beta$$
$$T_{11,\,1} := \beta$$
$$T_{11,\,2} := \beta$$
$$T_{11,\,3} := \beta$$
$$T_{11,\,4} := \beta$$
$$T_{11,\,5} := \beta$$
$$T_{11,\,6} := \beta$$
$$T_{11,\,7} := \beta$$
$$T_{11,\,8} := \beta$$
$$T_{11,\,9} := \beta$$

$$T_{11,10} := \beta$$
$$T_{11,11} := \beta$$
$$T_{11,12} := \beta$$
$$T_{11,13} := \beta$$
$$T_{11,14} := \beta$$
$$T_{11,15} := \beta$$

Boucle principale

```
> n := n_max - 1 :  k := 1 :
> for i from 2 to i_max - 1 do
        Eq[k] := -λ·T[i - 1, n + 1] + (1 + 2·λ)·T[i, n + 1]-λ·T[i + 1, n
    + 1]   = T[i, n];
        k := k + 1;
  end do:

> Eqs := [seq(Eq[k], k = 1 .. N)] :
> Tmps := [seq(T[i, n + 1], i = 2 .. i_max - 1)] :
> A, b := GenerateMatrix( Eqs, Tmps);
```

$$A, b := \left[\begin{array}{ccccccccc} 1+2\lambda & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda & 1+2\lambda & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 1+2\lambda & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 1+2\lambda & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda & 1+2\lambda & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda & 1+2\lambda & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda & 1+2\lambda & -\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & 1+2\lambda & -\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda & 1+2\lambda \end{array}\right],$$

$$\begin{bmatrix} \alpha\,\lambda + T_{2,\,14} \\ T_{3,\,14} \\ T_{4,\,14} \\ T_{5,\,14} \\ T_{6,\,14} \\ T_{7,\,14} \\ T_{8,\,14} \\ T_{9,\,14} \\ \beta\,\lambda + T_{10,\,14} \end{bmatrix}$$