Conditions Limites haut et droite de Neumann discrétisée par des schémas décentrés d'ordre 1

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> restart : with(LinearAlgebra) :
> Digits := 4 :
> L := 24;  H := 18;  ndx := 4;  ndy := 3;
```

$$L := 24$$
$$H := 18$$
$$ndx := 4$$
$$ndy := 3 \quad (1.1)$$

> $\Delta x := \frac{L}{ndx}$; $\Delta y := \frac{H}{ndy}$; $\beta := \frac{\Delta x}{\Delta y}$

$$\Delta x := 6$$
$$\Delta y := 6$$
$$\beta := 1 \quad (1.2)$$

> $i_{max} := ndx + 1$; $j_{max} := ndy + 1$;

$$i_{max} := 5$$
$$j_{max} := 4 \quad (1.3)$$

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$;

$$N := 6 \quad (1.4)$$

> $Tb := 2 \cdot (i - 1) \cdot \Delta x + 10$;

$$Tb := 12\, i - 2 \quad (1.5)$$

> $Tg := 3 \cdot (j - 1) \cdot \Delta y + 20$;

$$Tg := 18\, j + 2 \quad (1.6)$$

> $\alpha[h] := 5$; $\alpha[d] := 10$

$$\alpha_h := 5$$
$$\alpha_d := 10 \quad (1.7)$$

> **for** i **from** 2 **to** i_{max} **do** $T[i, 1] := Tb$ **end do**;

$$T_{2,\,1} := 22$$
$$T_{3,\,1} := 34$$
$$T_{4,\,1} := 46$$
$$T_{5,\,1} := 58 \quad (1.8)$$

> **for** j ***from*** 2 **to** j_{max} **do** $T[1, j] := Tg$ **end do**;

$$T_{1,2} := 38$$
$$T_{1,3} := 56$$
$$T_{1,4} := 74 \quad \textbf{(1.9)}$$

> $i := 1 : j := 1 :$

> $T[1, 1] := \frac{(Tb + Tg)}{2}$

$$T_{1,1} := 15 \quad \textbf{(1.10)}$$

$k := 1 :$

for j **from** 2 **to** $j_{max} - 1$ **do**
$T[i_{max}, j] := T[i_{max} - 1, j] + \alpha[d] \cdot \Delta x :$

for i **from** 2 **to** $i_{max} - 1$ **do**
$T[i, j_{max}] := T[i, j_{max}\text{-}1] + \alpha[h] \cdot \Delta y :$
$T[i_{max}, j_{max}] := \frac{(\ T[i_{max} - 1, j_{max}] + T[i_{max}, j_{max} - 1])}{2} :$

$Eq[k] := T[i + 1, j + 1] + T[i + 1, j - 1] + T[i - 1, j + 1] + T[i - 1, j - 1] + 2 \cdot \frac{5 - \beta^2}{1 + \beta^2} \cdot (T[i + 1, j] + T[i - 1, j]) + 2 \cdot \frac{5 \cdot \ \beta^2 - 1}{1 + \beta^2} \cdot (T[i, j + 1] + T[i, j - 1]) - 20 \cdot T[i, j] = 0 :$
$Temps[k] := T[i, j] :$
$k \ := \ k + 1 :$
end do
end do:

> **for** k **from** 1 **to** N **do** $Eq[k]$ **end do**;

$$T_{3,3} + 345 + 4\ T_{3,2} + 4\ T_{2,3} - 20\ T_{2,2} = 0$$
$$T_{4,3} + 204 + T_{2,3} + 4\ T_{4,2} + 4\ T_{2,2} + 4\ T_{3,3} - 20\ T_{3,2} = 0$$
$$5\ T_{4,3} + 576 + T_{3,3} - 16\ T_{4,2} + 4\ T_{3,2} = 0$$
$$5\ T_{3,3} + 486 + T_{3,2} - 16\ T_{2,3} + 4\ T_{2,2} = 0$$
$$5\ T_{4,3} + 180 + T_{4,2} + 5\ T_{2,3} + T_{2,2} - 16\ T_{3,3} + 4\ T_{3,2} = 0$$
$$-11\ T_{4,3} + 495 + 5\ T_{4,2} + 5\ T_{3,3} + T_{3,2} = 0 \quad \textbf{(1.11)}$$

> $N := k - 1;$

$$N := 6 \quad \textbf{(1.12)}$$

> $Eqs := \{seq(Eq[k], k = 1..N)\};$

$$Eqs := \{T_{3,3} + 345 + 4\ T_{3,2} + 4\ T_{2,3} - 20\ T_{2,2} = 0,\ 5\ T_{3,3} + 486 + T_{3,2} - 16\ T_{2,3} \quad \textbf{(1.13)}$$

$+ 4\,T_{2,2} = 0,\ -11\,T_{4,3} + 495 + 5\,T_{4,2} + 5\,T_{3,3} + T_{3,2} = 0,\ 5\,T_{4,3} + 576 + T_{3,3} - 16\,T_{4,2} + 4\,T_{3,2} = 0,\ T_{4,3} + 204 + T_{2,3} + 4\,T_{4,2} + 4\,T_{2,2} + 4\,T_{3,3} - 20\,T_{3,2} = 0,\ 5\,T_{4,3} + 180 + T_{4,2} + 5\,T_{2,3} + T_{2,2} - 16\,T_{3,3} + 4\,T_{3,2} = 0\}$

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> Tmps := [seq(Temps[k], k=1..N)];
```

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}] \quad \textbf{(1.14)}$$

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> SolT := evalf(solve(Eqs, Tmps));
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$$SolT := [[T_{2,2} = 56.26,\ T_{3,2} = 79.53,\ T_{4,2} = 112.2,\ T_{2,3} = 86.14,\ T_{3,3} = 117.5,\ T_{4,3} = 156.6]] \quad \textbf{(1.15)}$$

```
> Eqs := [seq(Eq[k], k=1..N)];
```

$$Eqs := [T_{3,3} + 345 + 4\,T_{3,2} + 4\,T_{2,3} - 20\,T_{2,2} = 0,\ T_{4,3} + 204 + T_{2,3} + 4\,T_{4,2} + 4\,T_{2,2} + 4\,T_{3,3} - 20\,T_{3,2} = 0,\ 5\,T_{4,3} + 576 + T_{3,3} - 16\,T_{4,2} + 4\,T_{3,2} = 0,\ 5\,T_{3,3} + 486 + T_{3,2} - 16\,T_{2,3} + 4\,T_{2,2} = 0,\ 5\,T_{4,3} + 180 + T_{4,2} + 5\,T_{2,3} + T_{2,2} - 16\,T_{3,3} + 4\,T_{3,2} = 0,\ -11\,T_{4,3} + 495 + 5\,T_{4,2} + 5\,T_{3,3} + T_{3,2} = 0] \quad \textbf{(1.16)}$$

```
> M, R := GenerateMatrix(Eqs, Tmps)
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$$M, R := \begin{bmatrix} -20 & 4 & 0 & 4 & 1 & 0 \\ 4 & -20 & 4 & 1 & 4 & 1 \\ 0 & 4 & -16 & 0 & 1 & 5 \\ 4 & 1 & 0 & -16 & 5 & 0 \\ 1 & 4 & 1 & 5 & -16 & 5 \\ 0 & 1 & 5 & 0 & 5 & -11 \end{bmatrix}, \begin{bmatrix} -345 \\ -204 \\ -576 \\ -486 \\ -180 \\ -495 \end{bmatrix} \quad \textbf{(1.17)}$$