Equation de Laplace 2D

==========

Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

==========

LMD : Energétique

Matière : Outils Numériques

==========

2009/2010

Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100*\sin\left(\frac{\pi \cdot x}{a}\right),$$
$$T(0, y) = 0,$$
$$T(a, y) = 0.$$

Solution discrétisée (formulation en 5 points):

```
> Restart:
> a := 0.1; b := 0.15; ndx := 10; ndy := 15
```

$$a := 0.1$$
$$b := 0.15$$
$$ndx := 10$$
$$ndy := 15 \qquad \textbf{(1.1)}$$

```
> Δx := a/ndx ; Δy := b/ndy ; β := Δx/Δy ;
```

$$\Delta x := 0.01000000000$$
$$\Delta y := 0.01000000000$$
$$\beta := 1.000000000 \qquad \textbf{(1.2)}$$

> $i_{max} := ndx + 1;\ j_{max} := ndy + 1;$

$$i_{max} := 11$$
$$j_{max} := 16 \qquad \textbf{(1.3)}$$

Nombre d'équations:

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 126 \qquad \textbf{(1.4)}$$

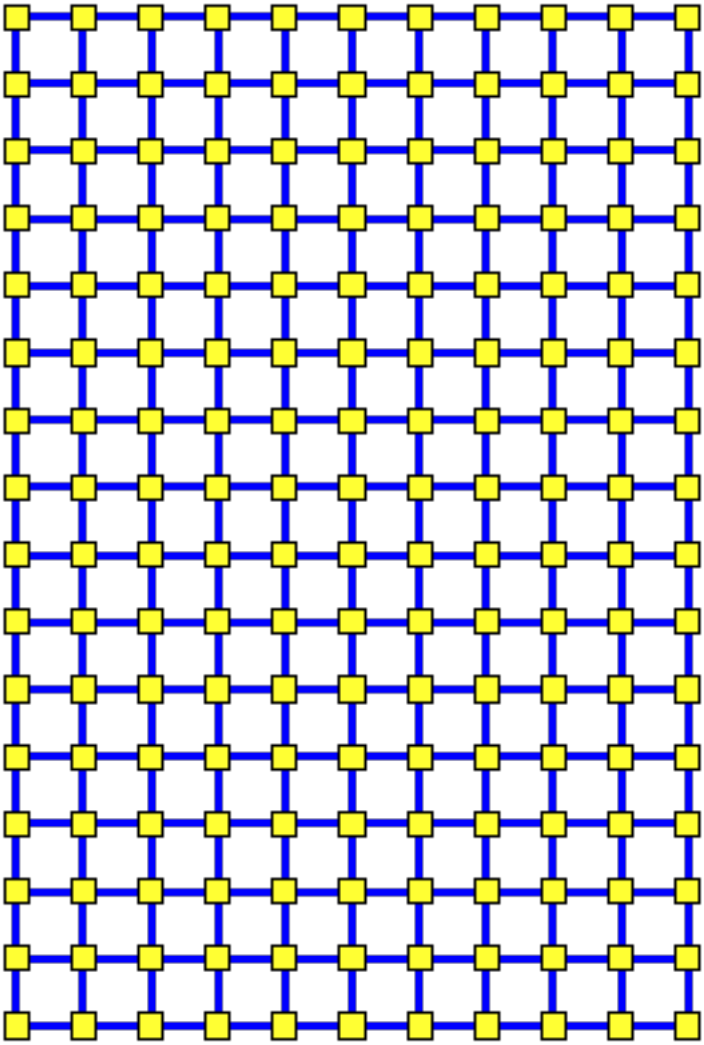
Maillage:

> $with(GraphTheory) : with(SpecialGraphs) :$

> $G := GridGraph(i_{max}, j_{max})$

$$G := Graph\ 1:\ an\ undirected\ unweighted\ graph\ with\ 176\ vertices\ and\ 325\ edge(s) \qquad \textbf{(1.5)}$$

> $DrawGraph(G)$



Conditions aux Limites:

> **for** i **from** 1 **to** i_{max} **do** $T[i, 1] := 0$ **end do**;

$$T_{1,1} := 0$$
$$T_{2,1} := 0$$
$$T_{3,1} := 0$$
$$T_{4,1} := 0$$
$$T_{5,1} := 0$$
$$T_{6,1} := 0$$
$$T_{7,1} := 0$$

$$T_{8,1} := 0$$
$$T_{9,1} := 0$$
$$T_{10,1} := 0$$
$$T_{11,1} := 0 \qquad (1.6)$$

> **for** i **from** 1 **to** $i_{\max}$ **do** $T[i, j_{\max}] := 100 \cdot evalf\left(\sin\left(\frac{\pi \cdot (i-1) \cdot \Delta x}{a}\right)\right)$ **end do**;

$$T_{1,16} := 0.$$
$$T_{2,16} := 30.90169944$$
$$T_{3,16} := 58.77852524$$
$$T_{4,16} := 80.90169944$$
$$T_{5,16} := 95.10565165$$
$$T_{6,16} := 100.$$
$$T_{7,16} := 95.10565163$$
$$T_{8,16} := 80.90169941$$
$$T_{9,16} := 58.77852522$$
$$T_{10,16} := 30.90169936$$
$$T_{11,16} := 0. \qquad (1.7)$$

> **for** j **from** 1 **to** $j_{\max}$ **do** $T[1, j] := 0$ **end do**;

$$T_{1,1} := 0$$
$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0$$
$$T_{1,5} := 0$$
$$T_{1,6} := 0$$
$$T_{1,7} := 0$$
$$T_{1,8} := 0$$
$$T_{1,9} := 0$$
$$T_{1,10} := 0$$
$$T_{1,11} := 0$$
$$T_{1,12} := 0$$
$$T_{1,13} := 0$$
$$T_{1,14} := 0$$
$$T_{1,15} := 0$$
$$T_{1,16} := 0 \qquad (1.8)$$

> **for** j **from** 1 **to** $j_{\max}$ **do** $T[i_{\max}, j] := 0$ **end do**;

$$T_{11,1} := 0$$
$$T_{11,2} := 0$$
$$T_{11,3} := 0$$
$$T_{11,4} := 0$$
$$T_{11,5} := 0$$
$$T_{11,6} := 0$$
$$T_{11,7} := 0$$

$$T_{11,8} := 0$$
$$T_{11,9} := 0$$
$$T_{11,10} := 0$$
$$T_{11,11} := 0$$
$$T_{11,12} := 0$$
$$T_{11,13} := 0$$
$$T_{11,14} := 0$$
$$T_{11,15} := 0$$
$$T_{11,16} := 0 \qquad \textbf{(1.9)}$$

```
> k := 1
```

$$k := 1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 2 to i_max - 1 do
    for j from 2 to j_max - 1 do
      Eq[k] := T[i+1,j] + T[i-1,j] + β^2·(T[i,j+1] + T[i,j-1]) - 2·(1
    + β^2)·T[i,j] = 0;
      Temps[k] := T[i,j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,2} + 1.000000000\,T_{2,3} - 4.000000000\,T_{2,2} = 0$$
$$T_{3,3} + 1.000000000\,T_{2,4} + 1.000000000\,T_{2,2} - 4.000000000\,T_{2,3} = 0$$
$$T_{3,4} + 1.000000000\,T_{2,5} + 1.000000000\,T_{2,3} - 4.000000000\,T_{2,4} = 0$$
$$T_{3,5} + 1.000000000\,T_{2,6} + 1.000000000\,T_{2,4} - 4.000000000\,T_{2,5} = 0$$
$$T_{3,6} + 1.000000000\,T_{2,7} + 1.000000000\,T_{2,5} - 4.000000000\,T_{2,6} = 0$$
$$T_{3,7} + 1.000000000\,T_{2,8} + 1.000000000\,T_{2,6} - 4.000000000\,T_{2,7} = 0$$
$$T_{3,8} + 1.000000000\,T_{2,9} + 1.000000000\,T_{2,7} - 4.000000000\,T_{2,8} = 0$$
$$T_{3,9} + 1.000000000\,T_{2,10} + 1.000000000\,T_{2,8} - 4.000000000\,T_{2,9} = 0$$
$$T_{3,10} + 1.000000000\,T_{2,11} + 1.000000000\,T_{2,9} - 4.000000000\,T_{2,10} = 0$$
$$T_{3,11} + 1.000000000\,T_{2,12} + 1.000000000\,T_{2,10} - 4.000000000\,T_{2,11} = 0$$
$$T_{3,12} + 1.000000000\,T_{2,13} + 1.000000000\,T_{2,11} - 4.000000000\,T_{2,12} = 0$$
$$T_{3,13} + 1.000000000\,T_{2,14} + 1.000000000\,T_{2,12} - 4.000000000\,T_{2,13} = 0$$
$$T_{3,14} + 1.000000000\,T_{2,15} + 1.000000000\,T_{2,13} - 4.000000000\,T_{2,14} = 0$$
$$T_{3,15} + 30.90169944 + 1.000000000\,T_{2,14} - 4.000000000\,T_{2,15} = 0$$
$$T_{4,2} + T_{2,2} + 1.000000000\,T_{3,3} - 4.000000000\,T_{3,2} = 0$$
$$T_{4,3} + T_{2,3} + 1.000000000\,T_{3,4} + 1.000000000\,T_{3,2} - 4.000000000\,T_{3,3} = 0$$
$$T_{4,4} + T_{2,4} + 1.000000000\,T_{3,5} + 1.000000000\,T_{3,3} - 4.000000000\,T_{3,4} = 0$$
$$T_{4,5} + T_{2,5} + 1.000000000\,T_{3,6} + 1.000000000\,T_{3,4} - 4.000000000\,T_{3,5} = 0$$
$$T_{4,6} + T_{2,6} + 1.000000000\,T_{3,7} + 1.000000000\,T_{3,5} - 4.000000000\,T_{3,6} = 0$$
$$T_{4,7} + T_{2,7} + 1.000000000\,T_{3,8} + 1.000000000\,T_{3,6} - 4.000000000\,T_{3,7} = 0$$

$$T_{4,8} + T_{2,8} + 1.000000000\, T_{3,9} + 1.000000000\, T_{3,7} - 4.000000000\, T_{3,8} = 0$$
$$T_{4,9} + T_{2,9} + 1.000000000\, T_{3,10} + 1.000000000\, T_{3,8} - 4.000000000\, T_{3,9} = 0$$
$$T_{4,10} + T_{2,10} + 1.000000000\, T_{3,11} + 1.000000000\, T_{3,9} - 4.000000000\, T_{3,10} = 0$$
$$T_{4,11} + T_{2,11} + 1.000000000\, T_{3,12} + 1.000000000\, T_{3,10} - 4.000000000\, T_{3,11} = 0$$
$$T_{4,12} + T_{2,12} + 1.000000000\, T_{3,13} + 1.000000000\, T_{3,11} - 4.000000000\, T_{3,12} = 0$$
$$T_{4,13} + T_{2,13} + 1.000000000\, T_{3,14} + 1.000000000\, T_{3,12} - 4.000000000\, T_{3,13} = 0$$
$$T_{4,14} + T_{2,14} + 1.000000000\, T_{3,15} + 1.000000000\, T_{3,13} - 4.000000000\, T_{3,14} = 0$$
$$T_{4,15} + T_{2,15} + 58.77852524 + 1.000000000\, T_{3,14} - 4.000000000\, T_{3,15} = 0$$
$$T_{5,2} + T_{3,2} + 1.000000000\, T_{4,3} - 4.000000000\, T_{4,2} = 0$$
$$T_{5,3} + T_{3,3} + 1.000000000\, T_{4,4} + 1.000000000\, T_{4,2} - 4.000000000\, T_{4,3} = 0$$
$$T_{5,4} + T_{3,4} + 1.000000000\, T_{4,5} + 1.000000000\, T_{4,3} - 4.000000000\, T_{4,4} = 0$$
$$T_{5,5} + T_{3,5} + 1.000000000\, T_{4,6} + 1.000000000\, T_{4,4} - 4.000000000\, T_{4,5} = 0$$
$$T_{5,6} + T_{3,6} + 1.000000000\, T_{4,7} + 1.000000000\, T_{4,5} - 4.000000000\, T_{4,6} = 0$$
$$T_{5,7} + T_{3,7} + 1.000000000\, T_{4,8} + 1.000000000\, T_{4,6} - 4.000000000\, T_{4,7} = 0$$
$$T_{5,8} + T_{3,8} + 1.000000000\, T_{4,9} + 1.000000000\, T_{4,7} - 4.000000000\, T_{4,8} = 0$$
$$T_{5,9} + T_{3,9} + 1.000000000\, T_{4,10} + 1.000000000\, T_{4,8} - 4.000000000\, T_{4,9} = 0$$
$$T_{5,10} + T_{3,10} + 1.000000000\, T_{4,11} + 1.000000000\, T_{4,9} - 4.000000000\, T_{4,10} = 0$$
$$T_{5,11} + T_{3,11} + 1.000000000\, T_{4,12} + 1.000000000\, T_{4,10} - 4.000000000\, T_{4,11} = 0$$
$$T_{5,12} + T_{3,12} + 1.000000000\, T_{4,13} + 1.000000000\, T_{4,11} - 4.000000000\, T_{4,12} = 0$$
$$T_{5,13} + T_{3,13} + 1.000000000\, T_{4,14} + 1.000000000\, T_{4,12} - 4.000000000\, T_{4,13} = 0$$
$$T_{5,14} + T_{3,14} + 1.000000000\, T_{4,15} + 1.000000000\, T_{4,13} - 4.000000000\, T_{4,14} = 0$$
$$T_{5,15} + T_{3,15} + 80.90169944 + 1.000000000\, T_{4,14} - 4.000000000\, T_{4,15} = 0$$
$$T_{6,2} + T_{4,2} + 1.000000000\, T_{5,3} - 4.000000000\, T_{5,2} = 0$$
$$T_{6,3} + T_{4,3} + 1.000000000\, T_{5,4} + 1.000000000\, T_{5,2} - 4.000000000\, T_{5,3} = 0$$
$$T_{6,4} + T_{4,4} + 1.000000000\, T_{5,5} + 1.000000000\, T_{5,3} - 4.000000000\, T_{5,4} = 0$$
$$T_{6,5} + T_{4,5} + 1.000000000\, T_{5,6} + 1.000000000\, T_{5,4} - 4.000000000\, T_{5,5} = 0$$
$$T_{6,6} + T_{4,6} + 1.000000000\, T_{5,7} + 1.000000000\, T_{5,5} - 4.000000000\, T_{5,6} = 0$$
$$T_{6,7} + T_{4,7} + 1.000000000\, T_{5,8} + 1.000000000\, T_{5,6} - 4.000000000\, T_{5,7} = 0$$
$$T_{6,8} + T_{4,8} + 1.000000000\, T_{5,9} + 1.000000000\, T_{5,7} - 4.000000000\, T_{5,8} = 0$$
$$T_{6,9} + T_{4,9} + 1.000000000\, T_{5,10} + 1.000000000\, T_{5,8} - 4.000000000\, T_{5,9} = 0$$
$$T_{6,10} + T_{4,10} + 1.000000000\, T_{5,11} + 1.000000000\, T_{5,9} - 4.000000000\, T_{5,10} = 0$$
$$T_{6,11} + T_{4,11} + 1.000000000\, T_{5,12} + 1.000000000\, T_{5,10} - 4.000000000\, T_{5,11} = 0$$
$$T_{6,12} + T_{4,12} + 1.000000000\, T_{5,13} + 1.000000000\, T_{5,11} - 4.000000000\, T_{5,12} = 0$$
$$T_{6,13} + T_{4,13} + 1.000000000\, T_{5,14} + 1.000000000\, T_{5,12} - 4.000000000\, T_{5,13} = 0$$
$$T_{6,14} + T_{4,14} + 1.000000000\, T_{5,15} + 1.000000000\, T_{5,13} - 4.000000000\, T_{5,14} = 0$$
$$T_{6,15} + T_{4,15} + 95.10565165 + 1.000000000\, T_{5,14} - 4.000000000\, T_{5,15} = 0$$
$$T_{7,2} + T_{5,2} + 1.000000000\, T_{6,3} - 4.000000000\, T_{6,2} = 0$$
$$T_{7,3} + T_{5,3} + 1.000000000\, T_{6,4} + 1.000000000\, T_{6,2} - 4.000000000\, T_{6,3} = 0$$
$$T_{7,4} + T_{5,4} + 1.000000000\, T_{6,5} + 1.000000000\, T_{6,3} - 4.000000000\, T_{6,4} = 0$$
$$T_{7,5} + T_{5,5} + 1.000000000\, T_{6,6} + 1.000000000\, T_{6,4} - 4.000000000\, T_{6,5} = 0$$
$$T_{7,6} + T_{5,6} + 1.000000000\, T_{6,7} + 1.000000000\, T_{6,5} - 4.000000000\, T_{6,6} = 0$$
$$T_{7,7} + T_{5,7} + 1.000000000\, T_{6,8} + 1.000000000\, T_{6,6} - 4.000000000\, T_{6,7} = 0$$
$$T_{7,8} + T_{5,8} + 1.000000000\, T_{6,9} + 1.000000000\, T_{6,7} - 4.000000000\, T_{6,8} = 0$$

$$T_{7,9} + T_{5,9} + 1.000000000\, T_{6,10} + 1.000000000\, T_{6,8} - 4.000000000\, T_{6,9} = 0$$
$$T_{7,10} + T_{5,10} + 1.000000000\, T_{6,11} + 1.000000000\, T_{6,9} - 4.000000000\, T_{6,10} = 0$$
$$T_{7,11} + T_{5,11} + 1.000000000\, T_{6,12} + 1.000000000\, T_{6,10} - 4.000000000\, T_{6,11} = 0$$
$$T_{7,12} + T_{5,12} + 1.000000000\, T_{6,13} + 1.000000000\, T_{6,11} - 4.000000000\, T_{6,12} = 0$$
$$T_{7,13} + T_{5,13} + 1.000000000\, T_{6,14} + 1.000000000\, T_{6,12} - 4.000000000\, T_{6,13} = 0$$
$$T_{7,14} + T_{5,14} + 1.000000000\, T_{6,15} + 1.000000000\, T_{6,13} - 4.000000000\, T_{6,14} = 0$$
$$T_{7,15} + T_{5,15} + 100.0000000 + 1.000000000\, T_{6,14} - 4.000000000\, T_{6,15} = 0$$
$$T_{8,2} + T_{6,2} + 1.000000000\, T_{7,3} - 4.000000000\, T_{7,2} = 0$$
$$T_{8,3} + T_{6,3} + 1.000000000\, T_{7,4} + 1.000000000\, T_{7,2} - 4.000000000\, T_{7,3} = 0$$
$$T_{8,4} + T_{6,4} + 1.000000000\, T_{7,5} + 1.000000000\, T_{7,3} - 4.000000000\, T_{7,4} = 0$$
$$T_{8,5} + T_{6,5} + 1.000000000\, T_{7,6} + 1.000000000\, T_{7,4} - 4.000000000\, T_{7,5} = 0$$
$$T_{8,6} + T_{6,6} + 1.000000000\, T_{7,7} + 1.000000000\, T_{7,5} - 4.000000000\, T_{7,6} = 0$$
$$T_{8,7} + T_{6,7} + 1.000000000\, T_{7,8} + 1.000000000\, T_{7,6} - 4.000000000\, T_{7,7} = 0$$
$$T_{8,8} + T_{6,8} + 1.000000000\, T_{7,9} + 1.000000000\, T_{7,7} - 4.000000000\, T_{7,8} = 0$$
$$T_{8,9} + T_{6,9} + 1.000000000\, T_{7,10} + 1.000000000\, T_{7,8} - 4.000000000\, T_{7,9} = 0$$
$$T_{8,10} + T_{6,10} + 1.000000000\, T_{7,11} + 1.000000000\, T_{7,9} - 4.000000000\, T_{7,10} = 0$$
$$T_{8,11} + T_{6,11} + 1.000000000\, T_{7,12} + 1.000000000\, T_{7,10} - 4.000000000\, T_{7,11} = 0$$
$$T_{8,12} + T_{6,12} + 1.000000000\, T_{7,13} + 1.000000000\, T_{7,11} - 4.000000000\, T_{7,12} = 0$$
$$T_{8,13} + T_{6,13} + 1.000000000\, T_{7,14} + 1.000000000\, T_{7,12} - 4.000000000\, T_{7,13} = 0$$
$$T_{8,14} + T_{6,14} + 1.000000000\, T_{7,15} + 1.000000000\, T_{7,13} - 4.000000000\, T_{7,14} = 0$$
$$T_{8,15} + T_{6,15} + 95.10565163 + 1.000000000\, T_{7,14} - 4.000000000\, T_{7,15} = 0$$
$$T_{9,2} + T_{7,2} + 1.000000000\, T_{8,3} - 4.000000000\, T_{8,2} = 0$$
$$T_{9,3} + T_{7,3} + 1.000000000\, T_{8,4} + 1.000000000\, T_{8,2} - 4.000000000\, T_{8,3} = 0$$
$$T_{9,4} + T_{7,4} + 1.000000000\, T_{8,5} + 1.000000000\, T_{8,3} - 4.000000000\, T_{8,4} = 0$$
$$T_{9,5} + T_{7,5} + 1.000000000\, T_{8,6} + 1.000000000\, T_{8,4} - 4.000000000\, T_{8,5} = 0$$
$$T_{9,6} + T_{7,6} + 1.000000000\, T_{8,7} + 1.000000000\, T_{8,5} - 4.000000000\, T_{8,6} = 0$$
$$T_{9,7} + T_{7,7} + 1.000000000\, T_{8,8} + 1.000000000\, T_{8,6} - 4.000000000\, T_{8,7} = 0$$
$$T_{9,8} + T_{7,8} + 1.000000000\, T_{8,9} + 1.000000000\, T_{8,7} - 4.000000000\, T_{8,8} = 0$$
$$T_{9,9} + T_{7,9} + 1.000000000\, T_{8,10} + 1.000000000\, T_{8,8} - 4.000000000\, T_{8,9} = 0$$
$$T_{9,10} + T_{7,10} + 1.000000000\, T_{8,11} + 1.000000000\, T_{8,9} - 4.000000000\, T_{8,10} = 0$$
$$T_{9,11} + T_{7,11} + 1.000000000\, T_{8,12} + 1.000000000\, T_{8,10} - 4.000000000\, T_{8,11} = 0$$
$$T_{9,12} + T_{7,12} + 1.000000000\, T_{8,13} + 1.000000000\, T_{8,11} - 4.000000000\, T_{8,12} = 0$$
$$T_{9,13} + T_{7,13} + 1.000000000\, T_{8,14} + 1.000000000\, T_{8,12} - 4.000000000\, T_{8,13} = 0$$
$$T_{9,14} + T_{7,14} + 1.000000000\, T_{8,15} + 1.000000000\, T_{8,13} - 4.000000000\, T_{8,14} = 0$$
$$T_{9,15} + T_{7,15} + 80.90169941 + 1.000000000\, T_{8,14} - 4.000000000\, T_{8,15} = 0$$
$$T_{10,2} + T_{8,2} + 1.000000000\, T_{9,3} - 4.000000000\, T_{9,2} = 0$$
$$T_{10,3} + T_{8,3} + 1.000000000\, T_{9,4} + 1.000000000\, T_{9,2} - 4.000000000\, T_{9,3} = 0$$
$$T_{10,4} + T_{8,4} + 1.000000000\, T_{9,5} + 1.000000000\, T_{9,3} - 4.000000000\, T_{9,4} = 0$$
$$T_{10,5} + T_{8,5} + 1.000000000\, T_{9,6} + 1.000000000\, T_{9,4} - 4.000000000\, T_{9,5} = 0$$
$$T_{10,6} + T_{8,6} + 1.000000000\, T_{9,7} + 1.000000000\, T_{9,5} - 4.000000000\, T_{9,6} = 0$$
$$T_{10,7} + T_{8,7} + 1.000000000\, T_{9,8} + 1.000000000\, T_{9,6} - 4.000000000\, T_{9,7} = 0$$
$$T_{10,8} + T_{8,8} + 1.000000000\, T_{9,9} + 1.000000000\, T_{9,7} - 4.000000000\, T_{9,8} = 0$$
$$T_{10,9} + T_{8,9} + 1.000000000\, T_{9,10} + 1.000000000\, T_{9,8} - 4.000000000\, T_{9,9} = 0$$

$$T_{10,10} + T_{8,10} + 1.000000000\,T_{9,11} + 1.000000000\,T_{9,9} - 4.000000000\,T_{9,10} = 0$$
$$T_{10,11} + T_{8,11} + 1.000000000\,T_{9,12} + 1.000000000\,T_{9,10} - 4.000000000\,T_{9,11} = 0$$
$$T_{10,12} + T_{8,12} + 1.000000000\,T_{9,13} + 1.000000000\,T_{9,11} - 4.000000000\,T_{9,12} = 0$$
$$T_{10,13} + T_{8,13} + 1.000000000\,T_{9,14} + 1.000000000\,T_{9,12} - 4.000000000\,T_{9,13} = 0$$
$$T_{10,14} + T_{8,14} + 1.000000000\,T_{9,15} + 1.000000000\,T_{9,13} - 4.000000000\,T_{9,14} = 0$$
$$T_{10,15} + T_{8,15} + 58.77852522 + 1.000000000\,T_{9,14} - 4.000000000\,T_{9,15} = 0$$
$$T_{9,2} + 1.000000000\,T_{10,3} - 4.000000000\,T_{10,2} = 0$$
$$T_{9,3} + 1.000000000\,T_{10,4} + 1.000000000\,T_{10,2} - 4.000000000\,T_{10,3} = 0$$
$$T_{9,4} + 1.000000000\,T_{10,5} + 1.000000000\,T_{10,3} - 4.000000000\,T_{10,4} = 0$$
$$T_{9,5} + 1.000000000\,T_{10,6} + 1.000000000\,T_{10,4} - 4.000000000\,T_{10,5} = 0$$
$$T_{9,6} + 1.000000000\,T_{10,7} + 1.000000000\,T_{10,5} - 4.000000000\,T_{10,6} = 0$$
$$T_{9,7} + 1.000000000\,T_{10,8} + 1.000000000\,T_{10,6} - 4.000000000\,T_{10,7} = 0$$
$$T_{9,8} + 1.000000000\,T_{10,9} + 1.000000000\,T_{10,7} - 4.000000000\,T_{10,8} = 0$$
$$T_{9,9} + 1.000000000\,T_{10,10} + 1.000000000\,T_{10,8} - 4.000000000\,T_{10,9} = 0$$
$$T_{9,10} + 1.000000000\,T_{10,11} + 1.000000000\,T_{10,9} - 4.000000000\,T_{10,10} = 0$$
$$T_{9,11} + 1.000000000\,T_{10,12} + 1.000000000\,T_{10,10} - 4.000000000\,T_{10,11} = 0$$
$$T_{9,12} + 1.000000000\,T_{10,13} + 1.000000000\,T_{10,11} - 4.000000000\,T_{10,12} = 0$$
$$T_{9,13} + 1.000000000\,T_{10,14} + 1.000000000\,T_{10,12} - 4.000000000\,T_{10,13} = 0$$
$$T_{9,14} + 1.000000000\,T_{10,15} + 1.000000000\,T_{10,13} - 4.000000000\,T_{10,14} = 0$$
$$30.90169936 + T_{9,15} + 1.000000000\,T_{10,14} - 4.000000000\,T_{10,15} = 0 \qquad \textbf{(1.1.2)}$$

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)];
```

$$Tmps := [T_{2,2}, T_{2,3}, T_{2,4}, T_{2,5}, T_{2,6}, T_{2,7}, T_{2,8}, T_{2,9}, T_{2,10}, T_{2,11}, T_{2,12}, T_{2,13}, T_{2,14}, T_{2,15}, T_{3,2}, T_{3,3}, T_{3,4}, T_{3,5}, T_{3,6}, T_{3,7}, T_{3,8}, T_{3,9}, T_{3,10}, T_{3,11}, T_{3,12}, T_{3,13}, T_{3,14}, T_{3,15}, T_{4,2}, T_{4,3}, T_{4,4}, T_{4,5}, T_{4,6}, T_{4,7}, T_{4,8}, T_{4,9}, T_{4,10}, T_{4,11}, T_{4,12}, T_{4,13}, T_{4,14}, T_{4,15}, T_{5,2}, T_{5,3}, T_{5,4}, T_{5,5}, T_{5,6}, T_{5,7}, T_{5,8}, T_{5,9}, T_{5,10}, T_{5,11}, T_{5,12}, T_{5,13}, T_{5,14}, T_{5,15}, T_{6,2}, T_{6,3}, T_{6,4}, T_{6,5}, T_{6,6}, T_{6,7}, T_{6,8}, T_{6,9}, T_{6,10}, T_{6,11}, T_{6,12}, T_{6,13}, T_{6,14}, T_{6,15}, T_{7,2}, T_{7,3}, T_{7,4}, T_{7,5}, T_{7,6}, T_{7,7}, T_{7,8}, T_{7,9}, T_{7,10}, T_{7,11}, T_{7,12}, T_{7,13}, T_{7,14}, T_{7,15}, T_{8,2}, T_{8,3}, T_{8,4}, T_{8,5}, T_{8,6}, T_{8,7}, T_{8,8}, T_{8,9}, T_{8,10}, T_{8,11}, T_{8,12}, T_{8,13}, T_{8,14}, T_{8,15}, T_{9,2}, T_{9,3}, T_{9,4}, T_{9,5}, T_{9,6}, T_{9,7}, T_{9,8}, T_{9,9}, T_{9,10}, T_{9,11}, T_{9,12}, T_{9,13}, T_{9,14}, T_{9,15}, T_{10,2}, T_{10,3}, T_{10,4}, T_{10,5}, T_{10,6}, T_{10,7}, T_{10,8}, T_{10,9}, T_{10,10}, T_{10,11}, T_{10,12}, T_{10,13}, T_{10,14}, T_{10,15}] \qquad \textbf{(1.1.3)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 0.1826949441, T_{2,3} = 0.3832733422, T_{2,4} = 0.6213692054, T_{2,5} = 0.9202890158, T_{2,6} = 1.309293127, T_{2,7} = 1.826459972, T_{2,8} = 2.522413445, T_{2,9} = 3.465278320, T_{2,10} = 4.747348782, T_{2,11} = 6.494122819, T_{2,12} = 8.876586845, T_{2,13} = 12.12795304, T_{2,14} = 16.56648777, T_{2,15} = 22.62666576, T_{3,2} = 0.3475064341, T_{3,3} = 0.7290292192, T_{3,4} = 1.181914464, T_{3,5} = 1.750493731, T_{3,6} = 2.490423521, T_{3,7} = 3.474133316, T_{3,8} = 4.797915487, T_{3,9} = 6.591351055, T_{3,10} = 9.029993988, T_{3,11} = 12.35255565, T_{3,12} = 16.88427152, T_{3,13} = 23.06873753, T_{3,14} = 31.51133230, T_{3,15} = 43.03847583, T_{4,2} = 0.4783015731, T_{4,3} = 1.003422637, T_{4,4} = 1.626765699, T_{4,5} = 2.409347923, T_{4,6} = 3.427773908, T_{4,7} = 4.781734286, T_{4,8} \qquad \textbf{(1.1.4)}$$

$= 6.603764132,\ T_{4,\,9} = 9.072216423,\ T_{4,\,10} = 12.42872047,\ T_{4,\,11} = 17.00183427,$
$T_{4,\,12} = 23.23920606,\ T_{4,\,13} = 31.75139327,\ T_{4,\,14} = 43.37162807,\ T_{4,\,15}$
$= 59.23738001,\ T_{5,\,2} = 0.5622772217,\ T_{5,\,3} = 1.179594055,\ T_{5,\,4} = 1.912377774,$
$T_{5,\,5} = 2.832358353,\ T_{5,\,6} = 4.029589903,\ T_{5,\,7} = 5.621265787,\ T_{5,\,8}$
$= 7.763190333,\ T_{5,\,9} = 10.66503004,\ T_{5,\,10} = 14.61083719,\ T_{5,\,11} = 19.98685489,$
$T_{5,\,12} = 27.31932520,\ T_{5,\,13} = 37.32600140,\ T_{5,\,14} = 50.98640669,\ T_{5,\,15}$
$= 69.63771671,\ T_{6,\,2} = 0.5912132581,\ T_{6,\,3} = 1.240298589,\ T_{6,\,4} = 2.010792988,$
$T_{6,\,5} = 2.978117814,\ T_{6,\,6} = 4.236961562,\ T_{6,\,7} = 5.910548628,\ T_{6,\,8}$
$= 8.162701375,\ T_{6,\,9} = 11.21387620,\ T_{6,\,10} = 15.36274337,\ T_{6,\,11} = 21.01542289,$
$T_{6,\,12} = 28.72523844,\ T_{6,\,13} = 39.24688045,\ T_{6,\,14} = 53.61028058,\ T_{6,\,15}$
$= 73.22142850,\ T_{7,\,2} = 0.5622772216,\ T_{7,\,3} = 1.179594055,\ T_{7,\,4} = 1.912377774,$
$T_{7,\,5} = 2.832358353,\ T_{7,\,6} = 4.029589903,\ T_{7,\,7} = 5.621265787,\ T_{7,\,8}$
$= 7.763190333,\ T_{7,\,9} = 10.66503004,\ T_{7,\,10} = 14.61083719,\ T_{7,\,11} = 19.98685489,$
$T_{7,\,12} = 27.31932519,\ T_{7,\,13} = 37.32600140,\ T_{7,\,14} = 50.98640669,\ T_{7,\,15}$
$= 69.63771670,\ T_{8,\,2} = 0.4783015731,\ T_{8,\,3} = 1.003422637,\ T_{8,\,4} = 1.626765699,$
$T_{8,\,5} = 2.409347923,\ T_{8,\,6} = 3.427773908,\ T_{8,\,7} = 4.781734286,\ T_{8,\,8}$
$= 6.603764132,\ T_{8,\,9} = 9.072216422,\ T_{8,\,10} = 12.42872047,\ T_{8,\,11} = 17.00183426,$
$T_{8,\,12} = 23.23920606,\ T_{8,\,13} = 31.75139326,\ T_{8,\,14} = 43.37162806,\ T_{8,\,15}$
$= 59.23737999,\ T_{9,\,2} = 0.3475064341,\ T_{9,\,3} = 0.7290292191,\ T_{9,\,4} = 1.181914464,$
$T_{9,\,5} = 1.750493731,\ T_{9,\,6} = 2.490423520,\ T_{9,\,7} = 3.474133316,\ T_{9,\,8}$
$= 4.797915487,\ T_{9,\,9} = 6.591351054,\ T_{9,\,10} = 9.029993987,\ T_{9,\,11} = 12.35255565,$
$T_{9,\,12} = 16.88427152,\ T_{9,\,13} = 23.06873752,\ T_{9,\,14} = 31.51133229,\ T_{9,\,15}$
$= 43.03847581,\ T_{10,\,2} = 0.1826949441,\ T_{10,\,3} = 0.3832733421,\ T_{10,\,4}$
$= 0.6213692054,\ T_{10,\,5} = 0.9202890158,\ T_{10,\,6} = 1.309293127,\ T_{10,\,7}$
$= 1.826459972,\ T_{10,\,8} = 2.522413445,\ T_{10,\,9} = 3.465278320,\ T_{10,\,10}$
$= 4.747348781,\ T_{10,\,11} = 6.494122818,\ T_{10,\,12} = 8.876586842,\ T_{10,\,13}$
$= 12.12795303,\ T_{10,\,14} = 16.56648776,\ T_{10,\,15} = 22.62666573\ ]]$

$> LT := \left[seq\left(T_{1,\,j}, j = 1..j_{\max}\right), seq\left(rhs\left(SolT_{1,\,i}\right), i = 1..\mathrm{N}\right), seq\left(T_{i_{\max},\,j}, j = 1..j_{\max}\right) \right];$

$LT :=$ [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1826949441, 0.3832733422, **(1.1.5)**
0.6213692054, 0.9202890158, 1.309293127, 1.826459972, 2.522413445,
3.465278320, 4.747348782, 6.494122819, 8.876586845, 12.12795304,
16.56648777, 22.62666576, 0.3475064341, 0.7290292192, 1.181914464,
1.750493731, 2.490423521, 3.474133316, 4.797915487, 6.591351055,
9.029993988, 12.35255565, 16.88427152, 23.06873753, 31.51133230,
43.03847583, 0.4783015731, 1.003422637, 1.626765699, 2.409347923,
3.427773908, 4.781734286, 6.603764132, 9.072216423, 12.42872047,
17.00183427, 23.23920606, 31.75139327, 43.37162807, 59.23738001,
0.5622772217, 1.179594055, 1.912377774, 2.832358353, 4.029589903,
5.621265787, 7.763190333, 10.66503004, 14.61083719, 19.98685489,
27.31932520, 37.32600140, 50.98640669, 69.63771671, 0.5912132581,
1.240298589, 2.010792988, 2.978117814, 4.236961562, 5.910548628,
8.162701375, 11.21387620, 15.36274337, 21.01542289, 28.72523844,
39.24688045, 53.61028058, 73.22142850, 0.5622772216, 1.179594055,
1.912377774, 2.832358353, 4.029589903, 5.621265787, 7.763190333,
10.66503004, 14.61083719, 19.98685489, 27.31932519, 37.32600140,
50.98640669, 69.63771670, 0.4783015731, 1.003422637, 1.626765699,
2.409347923, 3.427773908, 4.781734286, 6.603764132, 9.072216422,

12.42872047, 17.00183426, 23.23920606, 31.75139326, 43.37162806, 59.23737999, 0.3475064341, 0.7290292191, 1.181914464, 1.750493731, 2.490423520, 3.474133316, 4.797915487, 6.591351054, 9.029993987, 12.35255565, 16.88427152, 23.06873752, 31.51133229, 43.03847581, 0.1826949441, 0.3832733421, 0.6213692054, 0.9202890158, 1.309293127, 1.826459972, 2.522413445, 3.465278320, 4.747348781, 6.494122818, 8.876586842, 12.12795303, 16.56648776, 22.62666573, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

> with(plots) :

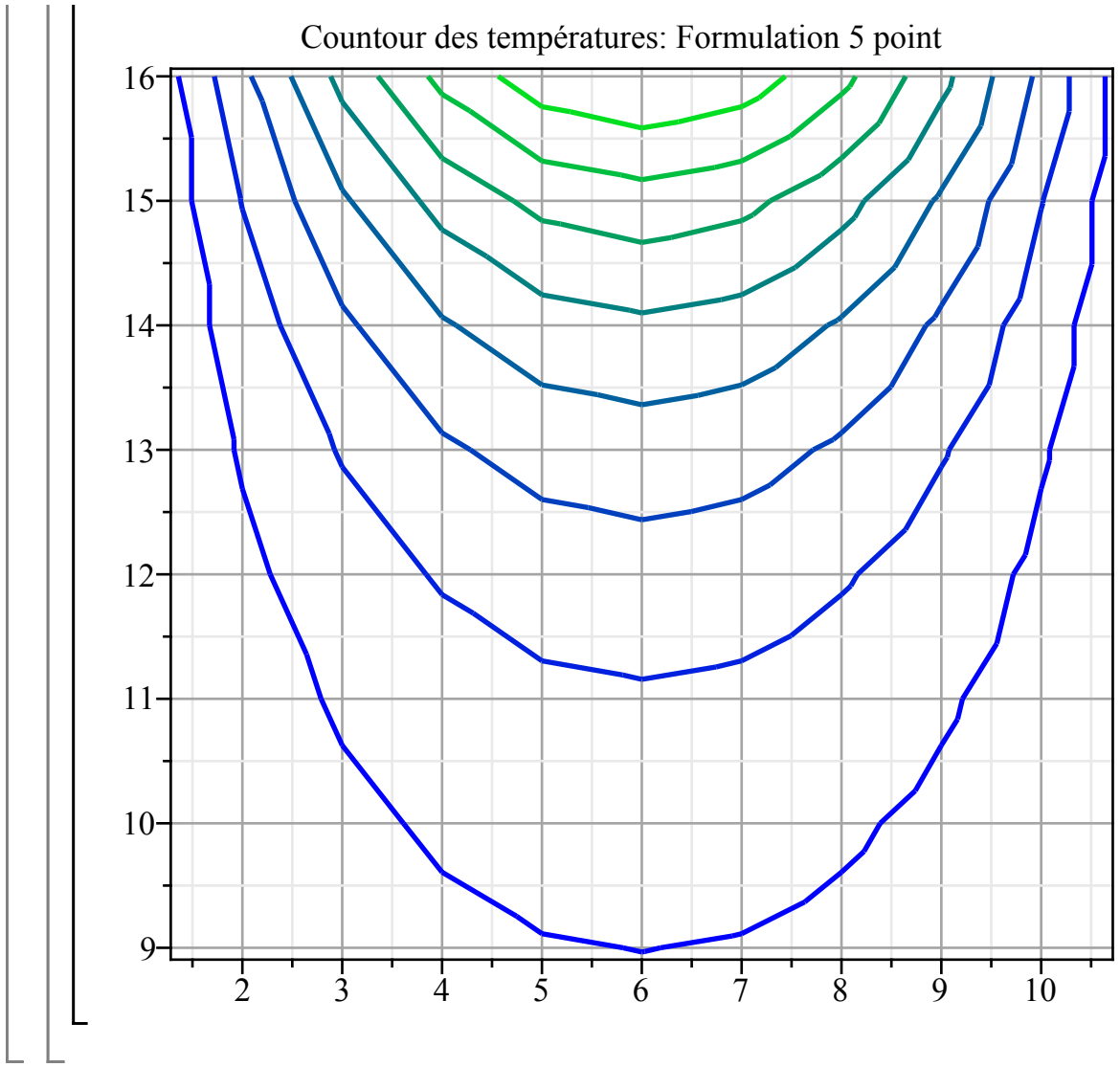
> **for** i **from** 1 **to** $i_{max} - 2$ **do** $Ns[i] := i \cdot \frac{N}{i_{max} - 2}$ **end do**;

$$Ns_1 := 14$$
$$Ns_2 := 28$$
$$Ns_3 := 42$$
$$Ns_4 := 56$$
$$Ns_5 := 70$$
$$Ns_6 := 84$$
$$Ns_7 := 98$$
$$Ns_8 := 112$$
$$Ns_9 := 126 \qquad \textbf{(1.1.6)}$$

> $GTemps := \left[\left[seq\left(T_{1,j}, j = 1 .. j_{max} \right) \right], \left[T_{2,1}, seq\left(rhs\left(SolT_{1,i} \right), i = 1 .. Ns_1 \right), T_{2,j_{max}} \right], \left[T_{3,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_1 + 1 .. Ns_2 \right), T_{3,j_{max}} \right], \left[T_{4,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_2 + 1 .. Ns_3 \right), T_{4,j_{max}} \right], \left[T_{5,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_3 + 1 .. Ns_4 \right), T_{5,j_{max}} \right], \left[T_{6,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_4 + 1 .. Ns_5 \right), T_{6,j_{max}} \right], \left[T_{7,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_5 + 1 .. Ns_6 \right), T_{7,j_{max}} \right], \left[T_{8,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_6 + 1 .. Ns_7 \right), T_{8,j_{max}} \right], \left[T_{9,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_7 + 1 .. Ns_8 \right), T_{9,j_{max}} \right], \left[T_{10,1}, seq\left(rhs\left(SolT_{1,i} \right), i = Ns_8 + 1 .. Ns_9 \right), T_{10,j_{max}} \right], \left[seq\left(T_{i_{max},j}, j = 1 .. j_{max} \right) \right] \right] :$

Tracé des isothermes:

> listcontplot(GTemps, title = "Countour des températures: Formulation 5 point", axes = boxed, gridlines = true, thickness = 2, coloring = [blue, green])


Countour des températures: Formulation 5 point
16
15
14
13
12
11
10
9
2
3
4
5
6
7
8
9
10