Equation de Laplace 2D

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LMD : Energ tique

Mati re : Outils Num riques

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D termination de la temperature $T(x, y)$ travers la surface d'une plaque rectangulaire (a x b) dont les extr mit s sont soumises des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100 * \sin\left(\frac{\pi \cdot x}{a}\right),$$
$$T(0, y) = 0,$$
$$T(a, y) = 0.$$

Solution discr tis e (formulation en 5 points):

```
> Restart : with(plots) :
> Digits := 4;
```

$$Digits := 4 \quad (1.1)$$

```
> NbIso := 15;
```

$$NbIso := 15 \quad (1.2)$$

```
> a := 0.1; b := 0.15; ndx := 10; ndy := 15
```

$$a := 0.1$$
$$b := 0.15$$
$$ndx := 10$$
$$ndy := 15 \quad \textbf{(1.3)}$$

> $\Delta x := \frac{a}{ndx}; \Delta y := \frac{b}{ndy}; \beta := \frac{\Delta x}{\Delta y};$

$$\Delta x := 0.01000$$
$$\Delta y := 0.01000$$
$$\beta := 1.000 \quad \textbf{(1.4)}$$

> $Tg := 0;$
> $Td := 0;$
> $Tb := 0;$
> $Th := 100 \cdot \sin\left(\frac{\pi \cdot (i-1) \cdot \Delta x}{a}\right); unapply(Th, i)$

$$Tg := 0$$
$$Td := 0$$
$$Tb := 0$$
$$Th := 100 \sin(0.3142\, i - 0.3142)$$
$$i \mapsto 100 \sin(0.3142\, i - 0.3142) \quad \textbf{(1.5)}$$

> $i_{max} := ndx + 1; j_{max} := ndy + 1;$

$$i_{max} := 11$$
$$j_{max} := 16 \quad \textbf{(1.6)}$$

Nombre d' quations:

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 126 \quad \textbf{(1.7)}$$

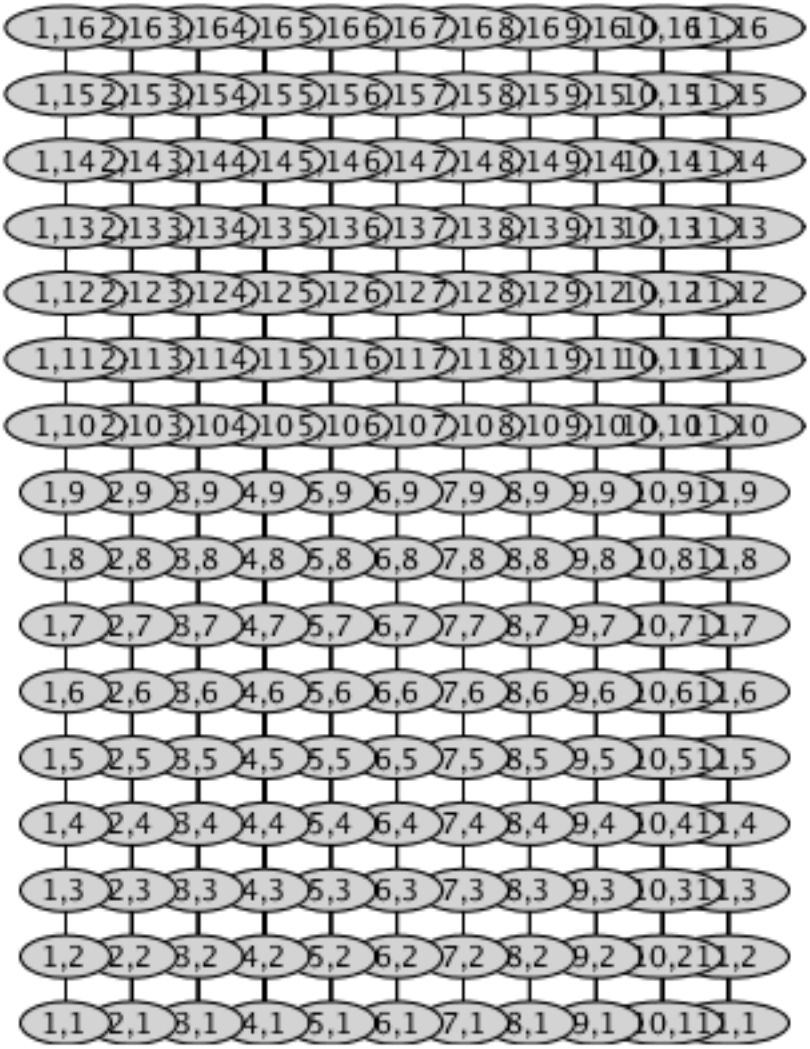
Maillage:

> $with(GraphTheory) : with(SpecialGraphs) :$
> $G := GridGraph(i_{max}, j_{max})$

$G :=$ **(1.8)**

Graph 1: an undirected unweighted graph with 176 vertices and 325 edge(s)

> $DrawGraph(G)$



Conditions aux Limites:

```
> for i from 2 to i_max − 1 do T[i, 1] := Tb end do;
```

$$T_{2,1} := 0$$
$$T_{3,1} := 0$$
$$T_{4,1} := 0$$
$$T_{5,1} := 0$$
$$T_{6,1} := 0$$
$$T_{7,1} := 0$$
$$T_{8,1} := 0$$
$$T_{9,1} := 0$$
$$T_{10,1} := 0 \qquad \textbf{(1.9)}$$

```
> for i from 2 to i_max − 1 do T[i, j_max] := evalf(Th(i)) end do;
```

$$T_{2,16} := 30.91$$
$$T_{3,16} := 58.79$$
$$T_{4,16} := 80.92$$
$$T_{5,16} := 95.12$$

$$T_{6,16} := 100.0$$
$$T_{7,16} := 95.10$$
$$T_{8,16} := 80.85$$
$$T_{9,16} := 58.72$$
$$T_{10,16} := 30.85 \quad \textbf{(1.10)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[1, j] := Tg$ **end do**;

$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0$$
$$T_{1,5} := 0$$
$$T_{1,6} := 0$$
$$T_{1,7} := 0$$
$$T_{1,8} := 0$$
$$T_{1,9} := 0$$
$$T_{1,10} := 0$$
$$T_{1,11} := 0$$
$$T_{1,12} := 0$$
$$T_{1,13} := 0$$
$$T_{1,14} := 0$$
$$T_{1,15} := 0 \quad \textbf{(1.11)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[i_{max}, j] := Td$ **end do**;

$$T_{11,2} := 0$$
$$T_{11,3} := 0$$
$$T_{11,4} := 0$$
$$T_{11,5} := 0$$
$$T_{11,6} := 0$$
$$T_{11,7} := 0$$
$$T_{11,8} := 0$$
$$T_{11,9} := 0$$
$$T_{11,10} := 0$$
$$T_{11,11} := 0$$
$$T_{11,12} := 0$$
$$T_{11,13} := 0$$
$$T_{11,14} := 0$$
$$T_{11,15} := 0 \quad \textbf{(1.12)}$$

Les valeurs ci-dessous sont calcul es uniquement pour le trac graphique.

> $T[1, 1] := \frac{Tg + Tb}{2}$

$$T_{1,1} := 0 \quad \textbf{(1.13)}$$

```
> T[i_max, j_max] := (Th(i_max) + Td)/2
```

$$T_{11,\,16} := -0.02036 \tag{1.14}$$

```
> T[i_max, 1] := (Tb + Td)/2
```

$$T_{11,\,1} := 0 \tag{1.15}$$

```
> T[1, j_max] := (Tg + Th(1))/2
```

$$T_{1,\,16} := -0.02036 \tag{1.16}$$

```
> `?`
```

`?` (1.17)

```
> k := 1 :
```

R solution pour les noeuds internes:

```
> for i from 2 to i_max - 1 do
    for j from 2 to j_max - 1 do
      Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2·(T[i, j + 1] + T[i, j - 1]) - 2·(1
      + β^2)·T[i, j] = 0;
      Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Ecriture du syst me d' quations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,\,2} + 1.000\,T_{2,\,3} - 4.000\,T_{2,\,2} = 0$$
$$T_{3,\,3} + 1.000\,T_{2,\,4} + 1.000\,T_{2,\,2} - 4.000\,T_{2,\,3} = 0$$
$$T_{3,\,4} + 1.000\,T_{2,\,5} + 1.000\,T_{2,\,3} - 4.000\,T_{2,\,4} = 0$$
$$T_{3,\,5} + 1.000\,T_{2,\,6} + 1.000\,T_{2,\,4} - 4.000\,T_{2,\,5} = 0$$
$$T_{3,\,6} + 1.000\,T_{2,\,7} + 1.000\,T_{2,\,5} - 4.000\,T_{2,\,6} = 0$$
$$T_{3,\,7} + 1.000\,T_{2,\,8} + 1.000\,T_{2,\,6} - 4.000\,T_{2,\,7} = 0$$
$$T_{3,\,8} + 1.000\,T_{2,\,9} + 1.000\,T_{2,\,7} - 4.000\,T_{2,\,8} = 0$$
$$T_{3,\,9} + 1.000\,T_{2,\,10} + 1.000\,T_{2,\,8} - 4.000\,T_{2,\,9} = 0$$
$$T_{3,\,10} + 1.000\,T_{2,\,11} + 1.000\,T_{2,\,9} - 4.000\,T_{2,\,10} = 0$$
$$T_{3,\,11} + 1.000\,T_{2,\,12} + 1.000\,T_{2,\,10} - 4.000\,T_{2,\,11} = 0$$
$$T_{3,\,12} + 1.000\,T_{2,\,13} + 1.000\,T_{2,\,11} - 4.000\,T_{2,\,12} = 0$$
$$T_{3,\,13} + 1.000\,T_{2,\,14} + 1.000\,T_{2,\,12} - 4.000\,T_{2,\,13} = 0$$
$$T_{3,\,14} + 1.000\,T_{2,\,15} + 1.000\,T_{2,\,13} - 4.000\,T_{2,\,14} = 0$$
$$T_{3,\,15} + 30.91 + 1.000\,T_{2,\,14} - 4.000\,T_{2,\,15} = 0$$
$$T_{4,\,2} + T_{2,\,2} + 1.000\,T_{3,\,3} - 4.000\,T_{3,\,2} = 0$$
$$T_{4,\,3} + T_{2,\,3} + 1.000\,T_{3,\,4} + 1.000\,T_{3,\,2} - 4.000\,T_{3,\,3} = 0$$

$$
\begin{aligned}
T_{4,4} + T_{2,4} + 1.000\,T_{3,5} + 1.000\,T_{3,3} - 4.000\,T_{3,4} &= 0\\
T_{4,5} + T_{2,5} + 1.000\,T_{3,6} + 1.000\,T_{3,4} - 4.000\,T_{3,5} &= 0\\
T_{4,6} + T_{2,6} + 1.000\,T_{3,7} + 1.000\,T_{3,5} - 4.000\,T_{3,6} &= 0\\
T_{4,7} + T_{2,7} + 1.000\,T_{3,8} + 1.000\,T_{3,6} - 4.000\,T_{3,7} &= 0\\
T_{4,8} + T_{2,8} + 1.000\,T_{3,9} + 1.000\,T_{3,7} - 4.000\,T_{3,8} &= 0\\
T_{4,9} + T_{2,9} + 1.000\,T_{3,10} + 1.000\,T_{3,8} - 4.000\,T_{3,9} &= 0\\
T_{4,10} + T_{2,10} + 1.000\,T_{3,11} + 1.000\,T_{3,9} - 4.000\,T_{3,10} &= 0\\
T_{4,11} + T_{2,11} + 1.000\,T_{3,12} + 1.000\,T_{3,10} - 4.000\,T_{3,11} &= 0\\
T_{4,12} + T_{2,12} + 1.000\,T_{3,13} + 1.000\,T_{3,11} - 4.000\,T_{3,12} &= 0\\
T_{4,13} + T_{2,13} + 1.000\,T_{3,14} + 1.000\,T_{3,12} - 4.000\,T_{3,13} &= 0\\
T_{4,14} + T_{2,14} + 1.000\,T_{3,15} + 1.000\,T_{3,13} - 4.000\,T_{3,14} &= 0\\
T_{4,15} + T_{2,15} + 58.79 + 1.000\,T_{3,14} - 4.000\,T_{3,15} &= 0\\
T_{5,2} + T_{3,2} + 1.000\,T_{4,3} - 4.000\,T_{4,2} &= 0\\
T_{5,3} + T_{3,3} + 1.000\,T_{4,4} + 1.000\,T_{4,2} - 4.000\,T_{4,3} &= 0\\
T_{5,4} + T_{3,4} + 1.000\,T_{4,5} + 1.000\,T_{4,3} - 4.000\,T_{4,4} &= 0\\
T_{5,5} + T_{3,5} + 1.000\,T_{4,6} + 1.000\,T_{4,4} - 4.000\,T_{4,5} &= 0\\
T_{5,6} + T_{3,6} + 1.000\,T_{4,7} + 1.000\,T_{4,5} - 4.000\,T_{4,6} &= 0\\
T_{5,7} + T_{3,7} + 1.000\,T_{4,8} + 1.000\,T_{4,6} - 4.000\,T_{4,7} &= 0\\
T_{5,8} + T_{3,8} + 1.000\,T_{4,9} + 1.000\,T_{4,7} - 4.000\,T_{4,8} &= 0\\
T_{5,9} + T_{3,9} + 1.000\,T_{4,10} + 1.000\,T_{4,8} - 4.000\,T_{4,9} &= 0\\
T_{5,10} + T_{3,10} + 1.000\,T_{4,11} + 1.000\,T_{4,9} - 4.000\,T_{4,10} &= 0\\
T_{5,11} + T_{3,11} + 1.000\,T_{4,12} + 1.000\,T_{4,10} - 4.000\,T_{4,11} &= 0\\
T_{5,12} + T_{3,12} + 1.000\,T_{4,13} + 1.000\,T_{4,11} - 4.000\,T_{4,12} &= 0\\
T_{5,13} + T_{3,13} + 1.000\,T_{4,14} + 1.000\,T_{4,12} - 4.000\,T_{4,13} &= 0\\
T_{5,14} + T_{3,14} + 1.000\,T_{4,15} + 1.000\,T_{4,13} - 4.000\,T_{4,14} &= 0\\
T_{5,15} + T_{3,15} + 80.92 + 1.000\,T_{4,14} - 4.000\,T_{4,15} &= 0\\
T_{6,2} + T_{4,2} + 1.000\,T_{5,3} - 4.000\,T_{5,2} &= 0\\
T_{6,3} + T_{4,3} + 1.000\,T_{5,4} + 1.000\,T_{5,2} - 4.000\,T_{5,3} &= 0\\
T_{6,4} + T_{4,4} + 1.000\,T_{5,5} + 1.000\,T_{5,3} - 4.000\,T_{5,4} &= 0\\
T_{6,5} + T_{4,5} + 1.000\,T_{5,6} + 1.000\,T_{5,4} - 4.000\,T_{5,5} &= 0\\
T_{6,6} + T_{4,6} + 1.000\,T_{5,7} + 1.000\,T_{5,5} - 4.000\,T_{5,6} &= 0\\
T_{6,7} + T_{4,7} + 1.000\,T_{5,8} + 1.000\,T_{5,6} - 4.000\,T_{5,7} &= 0\\
T_{6,8} + T_{4,8} + 1.000\,T_{5,9} + 1.000\,T_{5,7} - 4.000\,T_{5,8} &= 0\\
T_{6,9} + T_{4,9} + 1.000\,T_{5,10} + 1.000\,T_{5,8} - 4.000\,T_{5,9} &= 0\\
T_{6,10} + T_{4,10} + 1.000\,T_{5,11} + 1.000\,T_{5,9} - 4.000\,T_{5,10} &= 0\\
T_{6,11} + T_{4,11} + 1.000\,T_{5,12} + 1.000\,T_{5,10} - 4.000\,T_{5,11} &= 0\\
T_{6,12} + T_{4,12} + 1.000\,T_{5,13} + 1.000\,T_{5,11} - 4.000\,T_{5,12} &= 0\\
T_{6,13} + T_{4,13} + 1.000\,T_{5,14} + 1.000\,T_{5,12} - 4.000\,T_{5,13} &= 0\\
T_{6,14} + T_{4,14} + 1.000\,T_{5,15} + 1.000\,T_{5,13} - 4.000\,T_{5,14} &= 0\\
T_{6,15} + T_{4,15} + 95.12 + 1.000\,T_{5,14} - 4.000\,T_{5,15} &= 0
\end{aligned}
$$

$$T_{7,2} + T_{5,2} + 1.000\ T_{6,3} - 4.000\ T_{6,2} = 0$$
$$T_{7,3} + T_{5,3} + 1.000\ T_{6,4} + 1.000\ T_{6,2} - 4.000\ T_{6,3} = 0$$
$$T_{7,4} + T_{5,4} + 1.000\ T_{6,5} + 1.000\ T_{6,3} - 4.000\ T_{6,4} = 0$$
$$T_{7,5} + T_{5,5} + 1.000\ T_{6,6} + 1.000\ T_{6,4} - 4.000\ T_{6,5} = 0$$
$$T_{7,6} + T_{5,6} + 1.000\ T_{6,7} + 1.000\ T_{6,5} - 4.000\ T_{6,6} = 0$$
$$T_{7,7} + T_{5,7} + 1.000\ T_{6,8} + 1.000\ T_{6,6} - 4.000\ T_{6,7} = 0$$
$$T_{7,8} + T_{5,8} + 1.000\ T_{6,9} + 1.000\ T_{6,7} - 4.000\ T_{6,8} = 0$$
$$T_{7,9} + T_{5,9} + 1.000\ T_{6,10} + 1.000\ T_{6,8} - 4.000\ T_{6,9} = 0$$
$$T_{7,10} + T_{5,10} + 1.000\ T_{6,11} + 1.000\ T_{6,9} - 4.000\ T_{6,10} = 0$$
$$T_{7,11} + T_{5,11} + 1.000\ T_{6,12} + 1.000\ T_{6,10} - 4.000\ T_{6,11} = 0$$
$$T_{7,12} + T_{5,12} + 1.000\ T_{6,13} + 1.000\ T_{6,11} - 4.000\ T_{6,12} = 0$$
$$T_{7,13} + T_{5,13} + 1.000\ T_{6,14} + 1.000\ T_{6,12} - 4.000\ T_{6,13} = 0$$
$$T_{7,14} + T_{5,14} + 1.000\ T_{6,15} + 1.000\ T_{6,13} - 4.000\ T_{6,14} = 0$$
$$T_{7,15} + T_{5,15} + 100.0 + 1.000\ T_{6,14} - 4.000\ T_{6,15} = 0$$
$$T_{8,2} + T_{6,2} + 1.000\ T_{7,3} - 4.000\ T_{7,2} = 0$$
$$T_{8,3} + T_{6,3} + 1.000\ T_{7,4} + 1.000\ T_{7,2} - 4.000\ T_{7,3} = 0$$
$$T_{8,4} + T_{6,4} + 1.000\ T_{7,5} + 1.000\ T_{7,3} - 4.000\ T_{7,4} = 0$$
$$T_{8,5} + T_{6,5} + 1.000\ T_{7,6} + 1.000\ T_{7,4} - 4.000\ T_{7,5} = 0$$
$$T_{8,6} + T_{6,6} + 1.000\ T_{7,7} + 1.000\ T_{7,5} - 4.000\ T_{7,6} = 0$$
$$T_{8,7} + T_{6,7} + 1.000\ T_{7,8} + 1.000\ T_{7,6} - 4.000\ T_{7,7} = 0$$
$$T_{8,8} + T_{6,8} + 1.000\ T_{7,9} + 1.000\ T_{7,7} - 4.000\ T_{7,8} = 0$$
$$T_{8,9} + T_{6,9} + 1.000\ T_{7,10} + 1.000\ T_{7,8} - 4.000\ T_{7,9} = 0$$
$$T_{8,10} + T_{6,10} + 1.000\ T_{7,11} + 1.000\ T_{7,9} - 4.000\ T_{7,10} = 0$$
$$T_{8,11} + T_{6,11} + 1.000\ T_{7,12} + 1.000\ T_{7,10} - 4.000\ T_{7,11} = 0$$
$$T_{8,12} + T_{6,12} + 1.000\ T_{7,13} + 1.000\ T_{7,11} - 4.000\ T_{7,12} = 0$$
$$T_{8,13} + T_{6,13} + 1.000\ T_{7,14} + 1.000\ T_{7,12} - 4.000\ T_{7,13} = 0$$
$$T_{8,14} + T_{6,14} + 1.000\ T_{7,15} + 1.000\ T_{7,13} - 4.000\ T_{7,14} = 0$$
$$T_{8,15} + T_{6,15} + 95.10 + 1.000\ T_{7,14} - 4.000\ T_{7,15} = 0$$
$$T_{9,2} + T_{7,2} + 1.000\ T_{8,3} - 4.000\ T_{8,2} = 0$$
$$T_{9,3} + T_{7,3} + 1.000\ T_{8,4} + 1.000\ T_{8,2} - 4.000\ T_{8,3} = 0$$
$$T_{9,4} + T_{7,4} + 1.000\ T_{8,5} + 1.000\ T_{8,3} - 4.000\ T_{8,4} = 0$$
$$T_{9,5} + T_{7,5} + 1.000\ T_{8,6} + 1.000\ T_{8,4} - 4.000\ T_{8,5} = 0$$
$$T_{9,6} + T_{7,6} + 1.000\ T_{8,7} + 1.000\ T_{8,5} - 4.000\ T_{8,6} = 0$$
$$T_{9,7} + T_{7,7} + 1.000\ T_{8,8} + 1.000\ T_{8,6} - 4.000\ T_{8,7} = 0$$
$$T_{9,8} + T_{7,8} + 1.000\ T_{8,9} + 1.000\ T_{8,7} - 4.000\ T_{8,8} = 0$$
$$T_{9,9} + T_{7,9} + 1.000\ T_{8,10} + 1.000\ T_{8,8} - 4.000\ T_{8,9} = 0$$
$$T_{9,10} + T_{7,10} + 1.000\ T_{8,11} + 1.000\ T_{8,9} - 4.000\ T_{8,10} = 0$$
$$T_{9,11} + T_{7,11} + 1.000\ T_{8,12} + 1.000\ T_{8,10} - 4.000\ T_{8,11} = 0$$
$$T_{9,12} + T_{7,12} + 1.000\ T_{8,13} + 1.000\ T_{8,11} - 4.000\ T_{8,12} = 0$$
$$T_{9,13} + T_{7,13} + 1.000\ T_{8,14} + 1.000\ T_{8,12} - 4.000\ T_{8,13} = 0$$

$$T_{9,14} + T_{7,14} + 1.000\,T_{8,15} + 1.000\,T_{8,13} - 4.000\,T_{8,14} = 0$$
$$T_{9,15} + T_{7,15} + 80.85 + 1.000\,T_{8,14} - 4.000\,T_{8,15} = 0$$
$$T_{10,2} + T_{8,2} + 1.000\,T_{9,3} - 4.000\,T_{9,2} = 0$$
$$T_{10,3} + T_{8,3} + 1.000\,T_{9,4} + 1.000\,T_{9,2} - 4.000\,T_{9,3} = 0$$
$$T_{10,4} + T_{8,4} + 1.000\,T_{9,5} + 1.000\,T_{9,3} - 4.000\,T_{9,4} = 0$$
$$T_{10,5} + T_{8,5} + 1.000\,T_{9,6} + 1.000\,T_{9,4} - 4.000\,T_{9,5} = 0$$
$$T_{10,6} + T_{8,6} + 1.000\,T_{9,7} + 1.000\,T_{9,5} - 4.000\,T_{9,6} = 0$$
$$T_{10,7} + T_{8,7} + 1.000\,T_{9,8} + 1.000\,T_{9,6} - 4.000\,T_{9,7} = 0$$
$$T_{10,8} + T_{8,8} + 1.000\,T_{9,9} + 1.000\,T_{9,7} - 4.000\,T_{9,8} = 0$$
$$T_{10,9} + T_{8,9} + 1.000\,T_{9,10} + 1.000\,T_{9,8} - 4.000\,T_{9,9} = 0$$
$$T_{10,10} + T_{8,10} + 1.000\,T_{9,11} + 1.000\,T_{9,9} - 4.000\,T_{9,10} = 0$$
$$T_{10,11} + T_{8,11} + 1.000\,T_{9,12} + 1.000\,T_{9,10} - 4.000\,T_{9,11} = 0$$
$$T_{10,12} + T_{8,12} + 1.000\,T_{9,13} + 1.000\,T_{9,11} - 4.000\,T_{9,12} = 0$$
$$T_{10,13} + T_{8,13} + 1.000\,T_{9,14} + 1.000\,T_{9,12} - 4.000\,T_{9,13} = 0$$
$$T_{10,14} + T_{8,14} + 1.000\,T_{9,15} + 1.000\,T_{9,13} - 4.000\,T_{9,14} = 0$$
$$T_{10,15} + T_{8,15} + 58.72 + 1.000\,T_{9,14} - 4.000\,T_{9,15} = 0$$
$$T_{9,2} + 1.000\,T_{10,3} - 4.000\,T_{10,2} = 0$$
$$T_{9,3} + 1.000\,T_{10,4} + 1.000\,T_{10,2} - 4.000\,T_{10,3} = 0$$
$$T_{9,4} + 1.000\,T_{10,5} + 1.000\,T_{10,3} - 4.000\,T_{10,4} = 0$$
$$T_{9,5} + 1.000\,T_{10,6} + 1.000\,T_{10,4} - 4.000\,T_{10,5} = 0$$
$$T_{9,6} + 1.000\,T_{10,7} + 1.000\,T_{10,5} - 4.000\,T_{10,6} = 0$$
$$T_{9,7} + 1.000\,T_{10,8} + 1.000\,T_{10,6} - 4.000\,T_{10,7} = 0$$
$$T_{9,8} + 1.000\,T_{10,9} + 1.000\,T_{10,7} - 4.000\,T_{10,8} = 0$$
$$T_{9,9} + 1.000\,T_{10,10} + 1.000\,T_{10,8} - 4.000\,T_{10,9} = 0$$
$$T_{9,10} + 1.000\,T_{10,11} + 1.000\,T_{10,9} - 4.000\,T_{10,10} = 0$$
$$T_{9,11} + 1.000\,T_{10,12} + 1.000\,T_{10,10} - 4.000\,T_{10,11} = 0$$
$$T_{9,12} + 1.000\,T_{10,13} + 1.000\,T_{10,11} - 4.000\,T_{10,12} = 0$$
$$T_{9,13} + 1.000\,T_{10,14} + 1.000\,T_{10,12} - 4.000\,T_{10,13} = 0$$
$$T_{9,14} + 1.000\,T_{10,15} + 1.000\,T_{10,13} - 4.000\,T_{10,14} = 0$$
$$30.85 + T_{9,15} + 1.000\,T_{10,14} - 4.000\,T_{10,15} = 0 \qquad \textbf{(1.1.1)}$$

```
> Eqs := {seq(Eq[i], i = 1..N)}:
> Tmps := [seq(Temps[i], i = 1..N)];
```

$$Tmps := [T_{2,2}, T_{2,3}, T_{2,4}, T_{2,5}, T_{2,6}, T_{2,7}, T_{2,8}, T_{2,9}, T_{2,10}, T_{2,11}, T_{2,12}, T_{2,13}, T_{2,14}, T_{2,15}, T_{3,2}, T_{3,3}, T_{3,4}, T_{3,5}, T_{3,6}, T_{3,7}, T_{3,8}, T_{3,9}, T_{3,10}, T_{3,11}, T_{3,12}, T_{3,13}, T_{3,14}, T_{3,15}, T_{4,2}, T_{4,3}, T_{4,4}, T_{4,5}, T_{4,6}, T_{4,7}, T_{4,8}, T_{4,9}, T_{4,10}, T_{4,11}, T_{4,12}, T_{4,13}, T_{4,14}, T_{4,15}, T_{5,2}, T_{5,3}, T_{5,4}, T_{5,5}, T_{5,6}, T_{5,7}, T_{5,8}, T_{5,9}, T_{5,10}, T_{5,11}, T_{5,12}, T_{5,13}, T_{5,14}, T_{5,15}, T_{6,2}, T_{6,3}, T_{6,4}, T_{6,5}, T_{6,6}, T_{6,7}, T_{6,8}, T_{6,9}, T_{6,10}, T_{6,11}, T_{6,12}, T_{6,13}, T_{6,14}, T_{6,15}, T_{7,2}, T_{7,3}, T_{7,4}, T_{7,5}, T_{7,6}, T_{7,7}, T_{7,8}, T_{7,9}, T_{7,10}, T_{7,11}, T_{7,12}, T_{7,13}, T_{7,14}, T_{7,15}, T_{8,2}, T_{8,3}, T_{8,4}, T_{8,5}, T_{8,6}, T_{8,7}, T_{8,8}, T_{8,9}, T_{8,10}, T_{8,11}, T_{8,12}, T_{8,13}, T_{8,14}, T_{8,15}, T_{9,2}, T_{9,3}, T_{9,4}, T_{9,5}, T_{9,6},$$ **(1.1.2)**

$T_{9,7}, T_{9,8}, T_{9,9}, T_{9,10}, T_{9,11}, T_{9,12}, T_{9,13}, T_{9,14}, T_{9,15}, T_{10,2}, T_{10,3}, T_{10,4}, T_{10,5}, T_{10,6}, T_{10,7}, T_{10,8}, T_{10,9}, T_{10,10}, T_{10,11}, T_{10,12}, T_{10,13}, T_{10,14}, T_{10,15}]$

```
> SolT := solve(Eqs, Tmps);
```

$SolT := [[T_{2,2} = 0.1827, T_{2,3} = 0.3832, T_{2,4} = 0.6213, T_{2,5} = 0.9202, T_{2,6} = 1.309, T_{2,7} = 1.826, T_{2,8} = 2.522, T_{2,9} = 3.465, T_{2,10} = 4.747, T_{2,11} = 6.494, T_{2,12} = 8.877, T_{2,13} = 12.13, T_{2,14} = 16.57, T_{2,15} = 22.63, T_{3,2} = 0.3475, T_{3,3} = 0.7290, T_{3,4} = 1.182, T_{3,5} = 1.750, T_{3,6} = 2.490, T_{3,7} = 3.474, T_{3,8} = 4.798, T_{3,9} = 6.591, T_{3,10} = 9.030, T_{3,11} = 12.35, T_{3,12} = 16.88, T_{3,13} = 23.07, T_{3,14} = 31.51, T_{3,15} = 43.05, T_{4,2} = 0.4782, T_{4,3} = 1.003, T_{4,4} = 1.627, T_{4,5} = 2.409, T_{4,6} = 3.427, T_{4,7} = 4.781, T_{4,8} = 6.603, T_{4,9} = 9.072, T_{4,10} = 12.43, T_{4,11} = 17.00, T_{4,12} = 23.24, T_{4,13} = 31.75, T_{4,14} = 43.38, T_{4,15} = 59.25, T_{5,2} = 0.5622, T_{5,3} = 1.179, T_{5,4} = 1.912, T_{5,5} = 2.832, T_{5,6} = 4.029, T_{5,7} = 5.621, T_{5,8} = 7.762, T_{5,9} = 10.66, T_{5,10} = 14.61, T_{5,11} = 19.99, T_{5,12} = 27.32, T_{5,13} = 37.33, T_{5,14} = 50.99, T_{5,15} = 69.64, T_{6,2} = 0.5911, T_{6,3} = 1.240, T_{6,4} = 2.011, T_{6,5} = 2.978, T_{6,6} = 4.236, T_{6,7} = 5.910, T_{6,8} = 8.162, T_{6,9} = 11.21, T_{6,10} = 15.36, T_{6,11} = 21.01, T_{6,12} = 28.72, T_{6,13} = 39.24, T_{6,14} = 53.61, T_{6,15} = 73.22, T_{7,2} = 0.5622, T_{7,3} = 1.179, T_{7,4} = 1.912, T_{7,5} = 2.832, T_{7,6} = 4.029, T_{7,7} = 5.621, T_{7,8} = 7.762, T_{7,9} = 10.66, T_{7,10} = 14.61, T_{7,11} = 19.98, T_{7,12} = 27.31, T_{7,13} = 37.32, T_{7,14} = 50.98, T_{7,15} = 69.63, T_{8,2} = 0.4782, T_{8,3} = 1.003, T_{8,4} = 1.627, T_{8,5} = 2.409, T_{8,6} = 3.427, T_{8,7} = 4.781, T_{8,8} = 6.603, T_{8,9} = 9.071, T_{8,10} = 12.43, T_{8,11} = 17.00, T_{8,12} = 23.23, T_{8,13} = 31.74, T_{8,14} = 43.36, T_{8,15} = 59.21, T_{9,2} = 0.3475, T_{9,3} = 0.7289, T_{9,4} = 1.182, T_{9,5} = 1.750, T_{9,6} = 2.490, T_{9,7} = 3.474, T_{9,8} = 4.797, T_{9,9} = 6.590, T_{9,10} = 9.028, T_{9,11} = 12.35, T_{9,12} = 16.88, T_{9,13} = 23.06, T_{9,14} = 31.49, T_{9,15} = 43.01, T_{10,2} = 0.1827, T_{10,3} = 0.3832, T_{10,4} = 0.6213, T_{10,5} = 0.9202, T_{10,6} = 1.309, T_{10,7} = 1.826, T_{10,8} = 2.522, T_{10,9} = 3.465, T_{10,10} = 4.746, T_{10,11} = 6.492, T_{10,12} = 8.873, T_{10,13} = 12.12, T_{10,14} = 16.55, T_{10,15} = 22.60]]$ **(1.1.3)**

Extraction des valeurs des temp ratures:

```
> k := 1 :
  for i from 2 to i_max - 1 do
   for j from 2 to j_max - 1 do
       T[i, j] := rhs(SolT_{1,k});
       k := k + 1
   end do;
  end do
```

```
> `?`
```

`?` **(1.1.4)**

Calcul du nombre de listes pour v rification:

> $NL := \frac{N}{j_{max} - 2}$;

$$NL := 9 \quad \textbf{(1.1.5)}$$

Cr ation des listes pour le trac :

> $GTemps := [seq([seq(T[i, j], j = 1..j_{max})], i = 1..i_{max})]$

$GTemps := [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.02036], [0, 0.1827, 0.3832, 0.6213, 0.9202, 1.309, 1.826, 2.522, 3.465, 4.747, 6.494, 8.877, 12.13, 16.57, 22.63, 30.91], [0, 0.3475, 0.7290, 1.182, 1.750, 2.490, 3.474, 4.798, 6.591, 9.030, 12.35, 16.88, 23.07, 31.51, 43.05, 58.79], [0, 0.4782, 1.003, 1.627, 2.409, 3.427, 4.781, 6.603, 9.072, 12.43, 17.00, 23.24, 31.75, 43.38, 59.25, 80.92], [0, 0.5622, 1.179, 1.912, 2.832, 4.029, 5.621, 7.762, 10.66, 14.61, 19.99, 27.32, 37.33, 50.99, 69.64, 95.12], [0, 0.5911, 1.240, 2.011, 2.978, 4.236, 5.910, 8.162, 11.21, 15.36, 21.01, 28.72, 39.24, 53.61, 73.22, 100.0], [0, 0.5622, 1.179, 1.912, 2.832, 4.029, 5.621, 7.762, 10.66, 14.61, 19.98, 27.31, 37.32, 50.98, 69.63, 95.10], [0, 0.4782, 1.003, 1.627, 2.409, 3.427, 4.781, 6.603, 9.071, 12.43, 17.00, 23.23, 31.74, 43.36, 59.21, 80.85], [0, 0.3475, 0.7289, 1.182, 1.750, 2.490, 3.474, 4.797, 6.590, 9.028, 12.35, 16.88, 23.06, 31.49, 43.01, 58.72], [0, 0.1827, 0.3832, 0.6213, 0.9202, 1.309, 1.826, 2.522, 3.465, 4.746, 6.492, 8.873, 12.12, 16.55, 22.60, 30.85], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.02036]] **(1.1.6)**

Calcul des isothermes tracer:

> $TMax := \max(seq(seq(T[i, j], j = 1..j_{max}), i = 1..i_{max}))$;

$TMin := \min(seq(seq(T[i, j], j = 1..j_{max}), i = 1..i_{max}))$;

$DeltaT := evalf\left(\frac{TMax - TMin}{NbIso}\right)$;

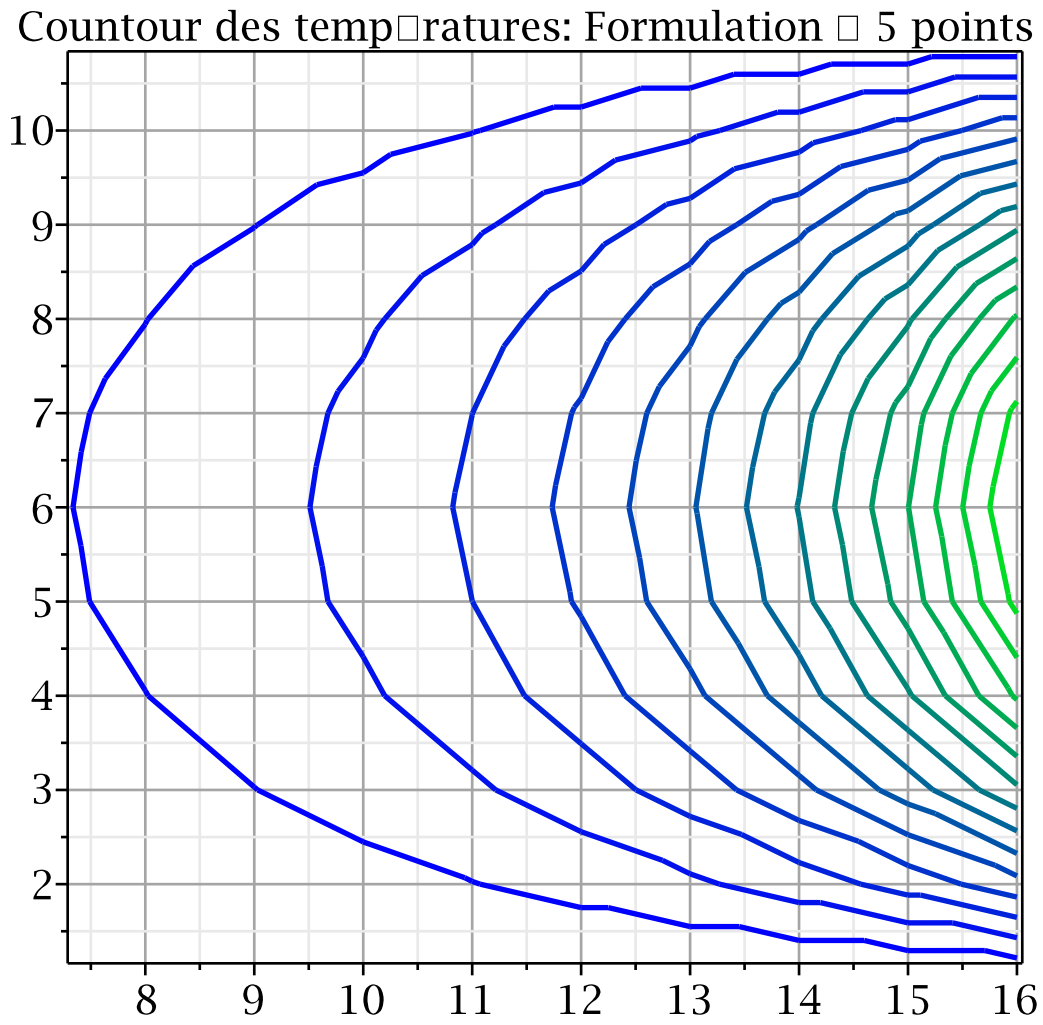
for k **from** 1 **to** $NbIso$ **do** $Iso[k] := k \cdot DeltaT$ **end do**

$$TMax := 100.0$$
$$TMin := -0.02036$$
$$DeltaT := 6.667$$
$$Iso_1 := 6.667$$
$$Iso_2 := 13.33$$
$$Iso_3 := 20.00$$
$$Iso_4 := 26.67$$
$$Iso_5 := 33.34$$
$$Iso_6 := 40.00$$
$$Iso_7 := 46.67$$
$$Iso_8 := 53.34$$
$$Iso_9 := 60.00$$
$$Iso_{10} := 66.67$$
$$Iso_{11} := 73.34$$
$$Iso_{12} := 80.00$$
$$Iso_{13} := 86.67$$
$$Iso_{14} := 93.34$$

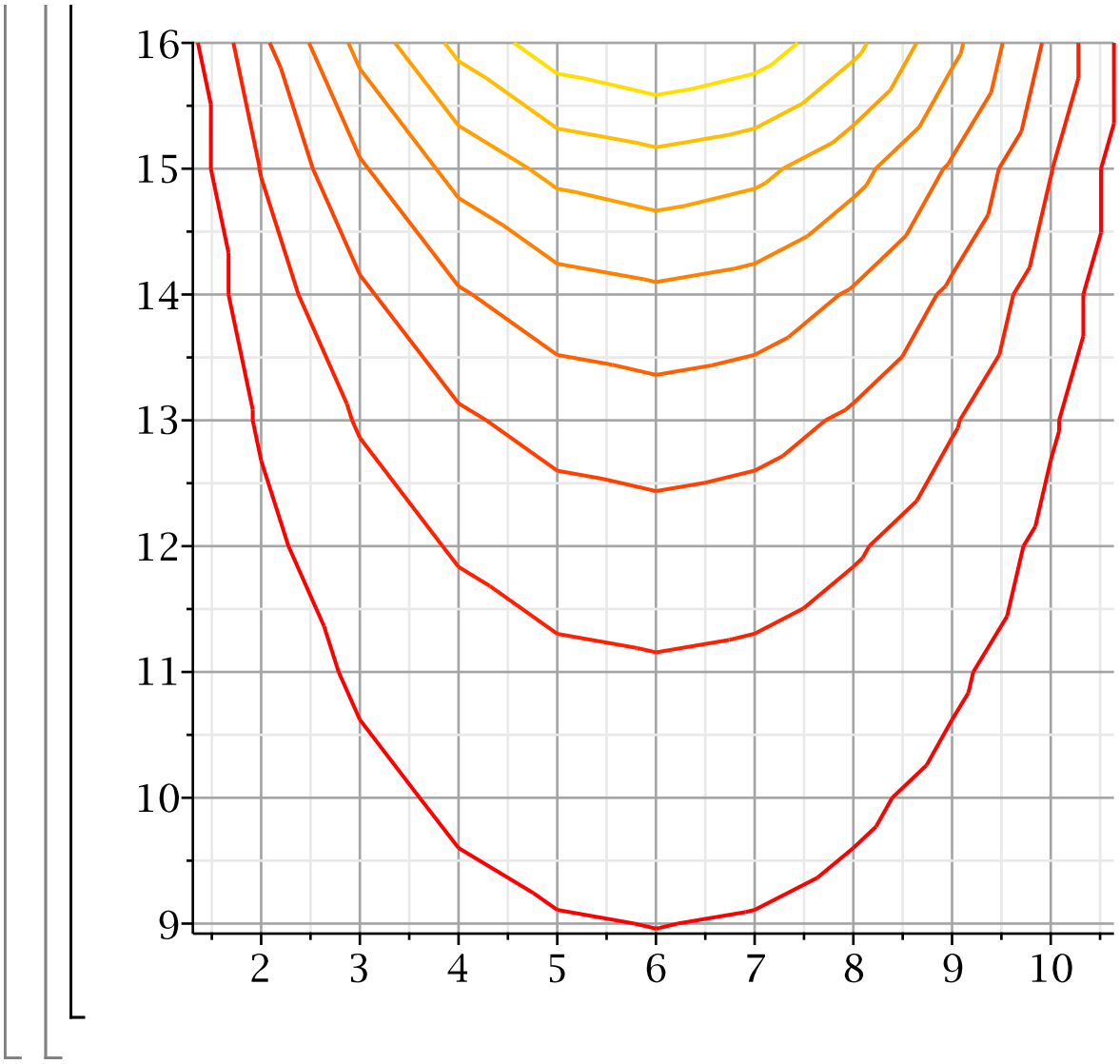
$$Iso_{15} := 100.0 \qquad \textbf{(1.1.7)}$$

Trac des isothermes:

```
> listcontplot(GTemps, title
      = "Countour des temp□ratures: Formulation □ 5 points", axes = boxed,
      gridlines = true, thickness = 2, coloring = [blue, green], contours
      = [seq(Iso[k], k = 1..NbIso)])
```




```
> listcontplot([seq([seq(T[i, j], i = 1..imax)], j = 1..jmax)])
```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