

Equation de Laplace 2D

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Détermination de la température $T(x, y)$ à travers la surface d'une plaque rectangulaire ($a \times b$) dont les extrémités sont soumises à des (C.L.) de Dirichlet

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L.):

$$\begin{aligned} T(x, 0) &= 0, \\ T(x, b) &= 100 \cdot \sin\left(\frac{\pi \cdot x}{a}\right), \\ T(0, y) &= 0, \\ T(a, y) &= 0. \end{aligned}$$

Solution discrétisée (formulation en 9 points):

```
> Restart ;
> a := 0.1; b := 0.15; ndx := 10; ndy := 15
      a := 0.1
      b := 0.15
      ndx := 10
      ndy := 15
```

(1.1)

```
> Δx := a / ndx ; Δy := b / ndy ; β := Δx / Δy ;
      Δx := 0.01000000000
      Δy := 0.01000000000
      β := 1.000000000
```

(1.2)

```
>  $i_{\max} := ndx + 1; j_{\max} := ndy + 1;$ 
```

```
 $i_{\max} := 11$ 
```

```
 $j_{\max} := 16$ 
```

(1.3)

Nombre d'équations:

```
>  $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$ 
```

```
 $N := 126$ 
```

(1.4)

Maillage:

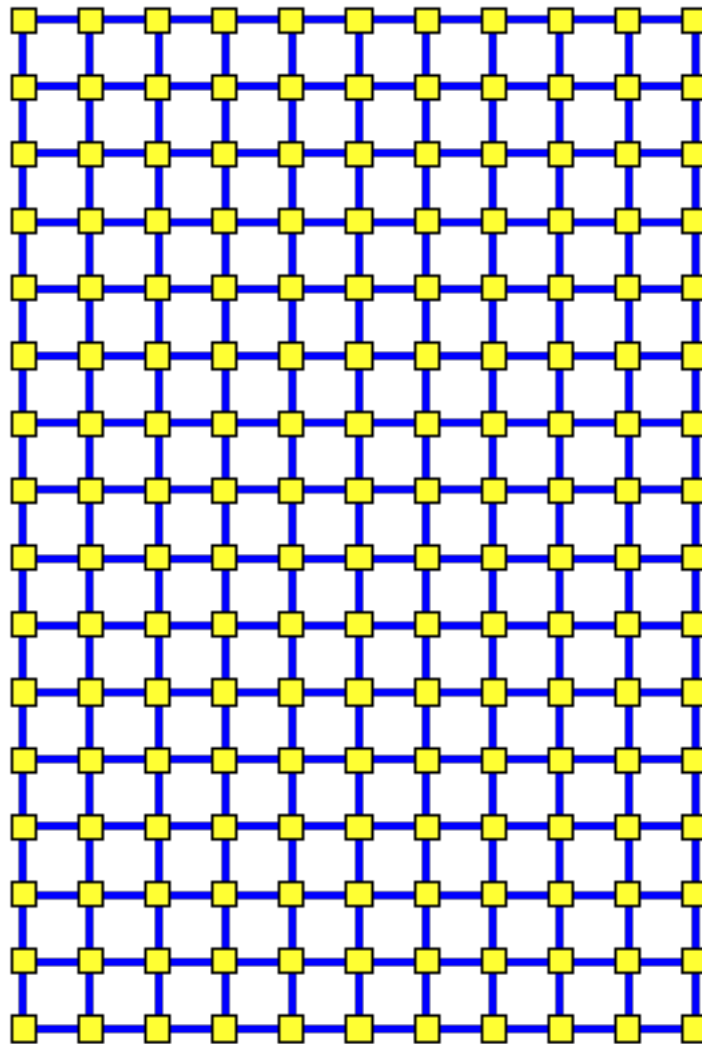
```
> with(GraphTheory) : with(SpecialGraphs) :
```

```
>  $G := \text{GridGraph}(i_{\max}, j_{\max})$ 
```

```
 $G := \text{Graph 1: an undirected unweighted graph with 176 vertices and 325 edge(s)}$ 
```

(1.5)

```
> DrawGraph(G)
```



Conditions aux Limites:

```
> for i from 1 to  $i_{\max}$  do  $T[i, 1] := 0$  end do;
```

```
 $T_{1,1} := 0$ 
```

```
 $T_{2,1} := 0$ 
```

```
 $T_{3,1} := 0$ 
```

```
 $T_{4,1} := 0$ 
```

```
 $T_{5,1} := 0$ 
```

```
 $T_{6,1} := 0$ 
```

```
 $T_{7,1} := 0$ 
```

$$\begin{aligned}
T_{8,1} &:= 0 \\
T_{9,1} &:= 0 \\
T_{10,1} &:= 0 \\
T_{11,1} &:= 0
\end{aligned}
\tag{1.6}$$

> for i from 1 to i_{\max} do $T[i, j_{\max}] := 100 \cdot \text{evalf}\left(\sin\left(\frac{\pi \cdot (i-1) \cdot \Delta x}{a}\right)\right)$ end do;

$$\begin{aligned}
T_{1,16} &:= 0. \\
T_{2,16} &:= 30.90169944 \\
T_{3,16} &:= 58.77852524 \\
T_{4,16} &:= 80.90169944 \\
T_{5,16} &:= 95.10565165 \\
T_{6,16} &:= 100. \\
T_{7,16} &:= 95.10565163 \\
T_{8,16} &:= 80.90169941 \\
T_{9,16} &:= 58.77852522 \\
T_{10,16} &:= 30.90169936
\end{aligned}$$

$$T_{11,16} := 0. \tag{1.7}$$

> for j from 1 to j_{\max} do $T[1, j] := 0$ end do;

$$\begin{aligned}
T_{1,1} &:= 0 \\
T_{1,2} &:= 0 \\
T_{1,3} &:= 0 \\
T_{1,4} &:= 0 \\
T_{1,5} &:= 0 \\
T_{1,6} &:= 0 \\
T_{1,7} &:= 0 \\
T_{1,8} &:= 0 \\
T_{1,9} &:= 0 \\
T_{1,10} &:= 0 \\
T_{1,11} &:= 0 \\
T_{1,12} &:= 0 \\
T_{1,13} &:= 0 \\
T_{1,14} &:= 0 \\
T_{1,15} &:= 0 \\
T_{1,16} &:= 0
\end{aligned}$$

$$\tag{1.8}$$

> for j from 1 to j_{\max} do $T[i_{\max}, j] := 0$ end do;

$$\begin{aligned}
T_{11,1} &:= 0 \\
T_{11,2} &:= 0 \\
T_{11,3} &:= 0 \\
T_{11,4} &:= 0 \\
T_{11,5} &:= 0 \\
T_{11,6} &:= 0 \\
T_{11,7} &:= 0
\end{aligned}$$

$$\begin{aligned}
T_{11,8} &:= 0 \\
T_{11,9} &:= 0 \\
T_{11,10} &:= 0 \\
T_{11,11} &:= 0 \\
T_{11,12} &:= 0 \\
T_{11,13} &:= 0 \\
T_{11,14} &:= 0 \\
T_{11,15} &:= 0 \\
T_{11,16} &:= 0
\end{aligned} \tag{1.9}$$

$$\begin{aligned}
&> k := 1 \\
& \hspace{15em} k := 1
\end{aligned} \tag{1.1.1}$$

Résolution pour les noeuds internes:

$$\begin{aligned}
&> \text{for } j \text{ from } 2 \text{ to } j_{\max} - 1 \text{ do} \\
& \quad \text{for } i \text{ from } 2 \text{ to } i_{\max} - 1 \text{ do} \\
& \quad \quad Eq[k] := T[i+1, j+1] + T[i+1, j-1] + T[i-1, j+1] + T[i-1, j-1] \\
& \quad \quad + 2 \cdot \frac{5 - \beta^2}{1 + \beta^2} \cdot (T[i+1, j] + T[i-1, j]) + 2 \cdot \frac{5 \cdot \beta^2 - 1}{1 + \beta^2} \cdot (T[i, j+1] + T[i, j] \\
& \quad \quad - 1) - 20 \cdot T[i, j] = 0; \\
& \quad \quad Temps[k] := T[i, j]; \\
& \quad \quad k := k + 1 \\
& \quad \text{end do;} \\
& \text{end do;}
\end{aligned}$$

Ecriture du système d'équations:

$$\begin{aligned}
&> \text{for } k \text{ from } 1 \text{ to } N \text{ do } Eq[k] \text{ end do;} \\
& \quad T_{3,3} + 4.000000000 T_{3,2} + 4.000000000 T_{2,3} - 20 T_{2,2} = 0 \\
& T_{4,3} + T_{2,3} + 4.000000000 T_{4,2} + 4.000000000 T_{2,2} + 4.000000000 T_{3,3} - 20 T_{3,2} \\
& \quad = 0 \\
& T_{5,3} + T_{3,3} + 4.000000000 T_{5,2} + 4.000000000 T_{3,2} + 4.000000000 T_{4,3} - 20 T_{4,2} \\
& \quad = 0 \\
& T_{6,3} + T_{4,3} + 4.000000000 T_{6,2} + 4.000000000 T_{4,2} + 4.000000000 T_{5,3} - 20 T_{5,2} \\
& \quad = 0 \\
& T_{7,3} + T_{5,3} + 4.000000000 T_{7,2} + 4.000000000 T_{5,2} + 4.000000000 T_{6,3} - 20 T_{6,2} \\
& \quad = 0 \\
& T_{8,3} + T_{6,3} + 4.000000000 T_{8,2} + 4.000000000 T_{6,2} + 4.000000000 T_{7,3} - 20 T_{7,2} \\
& \quad = 0 \\
& T_{9,3} + T_{7,3} + 4.000000000 T_{9,2} + 4.000000000 T_{7,2} + 4.000000000 T_{8,3} - 20 T_{8,2} \\
& \quad = 0 \\
& T_{10,3} + T_{8,3} + 4.000000000 T_{10,2} + 4.000000000 T_{8,2} + 4.000000000 T_{9,3} - 20 T_{9,2} \\
& \quad = 0 \\
& \quad T_{9,3} + 4.000000000 T_{9,2} + 4.000000000 T_{10,3} - 20 T_{10,2} = 0 \\
& T_{3,4} + T_{3,2} + 4.000000000 T_{3,3} + 4.000000000 T_{2,4} + 4.000000000 T_{2,2} - 20 T_{2,3} \\
& \quad = 0 \\
& T_{4,4} + T_{4,2} + T_{2,4} + T_{2,2} + 4.000000000 T_{4,3} + 4.000000000 T_{2,3} \\
& \quad + 4.000000000 T_{3,4} + 4.000000000 T_{3,2} - 20 T_{3,3} = 0
\end{aligned}$$

$$\begin{aligned}
& T_{5,4} + T_{5,2} + T_{3,4} + T_{3,2} + 4.000000000 T_{5,3} + 4.000000000 T_{3,3} \\
& \quad + 4.000000000 T_{4,4} + 4.000000000 T_{4,2} - 20 T_{4,3} = 0 \\
& T_{6,4} + T_{6,2} + T_{4,4} + T_{4,2} + 4.000000000 T_{6,3} + 4.000000000 T_{4,3} \\
& \quad + 4.000000000 T_{5,4} + 4.000000000 T_{5,2} - 20 T_{5,3} = 0 \\
& T_{7,4} + T_{7,2} + T_{5,4} + T_{5,2} + 4.000000000 T_{7,3} + 4.000000000 T_{5,3} \\
& \quad + 4.000000000 T_{6,4} + 4.000000000 T_{6,2} - 20 T_{6,3} = 0 \\
& T_{8,4} + T_{8,2} + T_{6,4} + T_{6,2} + 4.000000000 T_{8,3} + 4.000000000 T_{6,3} \\
& \quad + 4.000000000 T_{7,4} + 4.000000000 T_{7,2} - 20 T_{7,3} = 0 \\
& T_{9,4} + T_{9,2} + T_{7,4} + T_{7,2} + 4.000000000 T_{9,3} + 4.000000000 T_{7,3} \\
& \quad + 4.000000000 T_{8,4} + 4.000000000 T_{8,2} - 20 T_{8,3} = 0 \\
& T_{10,4} + T_{10,2} + T_{8,4} + T_{8,2} + 4.000000000 T_{10,3} + 4.000000000 T_{8,3} \\
& \quad + 4.000000000 T_{9,4} + 4.000000000 T_{9,2} - 20 T_{9,3} = 0 \\
& T_{9,4} + T_{9,2} + 4.000000000 T_{9,3} + 4.000000000 T_{10,4} + 4.000000000 T_{10,2} \\
& \quad - 20 T_{10,3} = 0 \\
& T_{3,5} + T_{3,3} + 4.000000000 T_{3,4} + 4.000000000 T_{2,5} + 4.000000000 T_{2,3} - 20 T_{2,4} \\
& \quad = 0 \\
& T_{4,5} + T_{4,3} + T_{2,5} + T_{2,3} + 4.000000000 T_{4,4} + 4.000000000 T_{2,4} \\
& \quad + 4.000000000 T_{3,5} + 4.000000000 T_{3,3} - 20 T_{3,4} = 0 \\
& T_{5,5} + T_{5,3} + T_{3,5} + T_{3,3} + 4.000000000 T_{5,4} + 4.000000000 T_{3,4} \\
& \quad + 4.000000000 T_{4,5} + 4.000000000 T_{4,3} - 20 T_{4,4} = 0 \\
& T_{6,5} + T_{6,3} + T_{4,5} + T_{4,3} + 4.000000000 T_{6,4} + 4.000000000 T_{4,4} \\
& \quad + 4.000000000 T_{5,5} + 4.000000000 T_{5,3} - 20 T_{5,4} = 0 \\
& T_{7,5} + T_{7,3} + T_{5,5} + T_{5,3} + 4.000000000 T_{7,4} + 4.000000000 T_{5,4} \\
& \quad + 4.000000000 T_{6,5} + 4.000000000 T_{6,3} - 20 T_{6,4} = 0 \\
& T_{8,5} + T_{8,3} + T_{6,5} + T_{6,3} + 4.000000000 T_{8,4} + 4.000000000 T_{6,4} \\
& \quad + 4.000000000 T_{7,5} + 4.000000000 T_{7,3} - 20 T_{7,4} = 0 \\
& T_{9,5} + T_{9,3} + T_{7,5} + T_{7,3} + 4.000000000 T_{9,4} + 4.000000000 T_{7,4} \\
& \quad + 4.000000000 T_{8,5} + 4.000000000 T_{8,3} - 20 T_{8,4} = 0 \\
& T_{10,5} + T_{10,3} + T_{8,5} + T_{8,3} + 4.000000000 T_{10,4} + 4.000000000 T_{8,4} \\
& \quad + 4.000000000 T_{9,5} + 4.000000000 T_{9,3} - 20 T_{9,4} = 0 \\
& T_{9,5} + T_{9,3} + 4.000000000 T_{9,4} + 4.000000000 T_{10,5} + 4.000000000 T_{10,3} \\
& \quad - 20 T_{10,4} = 0 \\
& T_{3,6} + T_{3,4} + 4.000000000 T_{3,5} + 4.000000000 T_{2,6} + 4.000000000 T_{2,4} - 20 T_{2,5} \\
& \quad = 0 \\
& T_{4,6} + T_{4,4} + T_{2,6} + T_{2,4} + 4.000000000 T_{4,5} + 4.000000000 T_{2,5} \\
& \quad + 4.000000000 T_{3,6} + 4.000000000 T_{3,4} - 20 T_{3,5} = 0 \\
& T_{5,6} + T_{5,4} + T_{3,6} + T_{3,4} + 4.000000000 T_{5,5} + 4.000000000 T_{3,5} \\
& \quad + 4.000000000 T_{4,6} + 4.000000000 T_{4,4} - 20 T_{4,5} = 0 \\
& T_{6,6} + T_{6,4} + T_{4,6} + T_{4,4} + 4.000000000 T_{6,5} + 4.000000000 T_{4,5} \\
& \quad + 4.000000000 T_{5,6} + 4.000000000 T_{5,4} - 20 T_{5,5} = 0 \\
& T_{7,6} + T_{7,4} + T_{5,6} + T_{5,4} + 4.000000000 T_{7,5} + 4.000000000 T_{5,5} \\
& \quad + 4.000000000 T_{6,6} + 4.000000000 T_{6,4} - 20 T_{6,5} = 0 \\
& T_{8,6} + T_{8,4} + T_{6,6} + T_{6,4} + 4.000000000 T_{8,5} + 4.000000000 T_{6,5}
\end{aligned}$$

$$\begin{aligned}
& + 4.000000000 T_{7,6} + 4.000000000 T_{7,4} - 20 T_{7,5} = 0 \\
T_{9,6} + T_{9,4} + T_{7,6} + T_{7,4} + 4.000000000 T_{9,5} + 4.000000000 T_{7,5} \\
& + 4.000000000 T_{8,6} + 4.000000000 T_{8,4} - 20 T_{8,5} = 0 \\
T_{10,6} + T_{10,4} + T_{8,6} + T_{8,4} + 4.000000000 T_{10,5} + 4.000000000 T_{8,5} \\
& + 4.000000000 T_{9,6} + 4.000000000 T_{9,4} - 20 T_{9,5} = 0 \\
T_{9,6} + T_{9,4} + 4.000000000 T_{9,5} + 4.000000000 T_{10,6} + 4.000000000 T_{10,4} \\
& - 20 T_{10,5} = 0 \\
T_{3,7} + T_{3,5} + 4.000000000 T_{3,6} + 4.000000000 T_{2,7} + 4.000000000 T_{2,5} - 20 T_{2,6} \\
& = 0 \\
T_{4,7} + T_{4,5} + T_{2,7} + T_{2,5} + 4.000000000 T_{4,6} + 4.000000000 T_{2,6} \\
& + 4.000000000 T_{3,7} + 4.000000000 T_{3,5} - 20 T_{3,6} = 0 \\
T_{5,7} + T_{5,5} + T_{3,7} + T_{3,5} + 4.000000000 T_{5,6} + 4.000000000 T_{3,6} \\
& + 4.000000000 T_{4,7} + 4.000000000 T_{4,5} - 20 T_{4,6} = 0 \\
T_{6,7} + T_{6,5} + T_{4,7} + T_{4,5} + 4.000000000 T_{6,6} + 4.000000000 T_{4,6} \\
& + 4.000000000 T_{5,7} + 4.000000000 T_{5,5} - 20 T_{5,6} = 0 \\
T_{7,7} + T_{7,5} + T_{5,7} + T_{5,5} + 4.000000000 T_{7,6} + 4.000000000 T_{5,6} \\
& + 4.000000000 T_{6,7} + 4.000000000 T_{6,5} - 20 T_{6,6} = 0 \\
T_{8,7} + T_{8,5} + T_{6,7} + T_{6,5} + 4.000000000 T_{8,6} + 4.000000000 T_{6,6} \\
& + 4.000000000 T_{7,7} + 4.000000000 T_{7,5} - 20 T_{7,6} = 0 \\
T_{9,7} + T_{9,5} + T_{7,7} + T_{7,5} + 4.000000000 T_{9,6} + 4.000000000 T_{7,6} \\
& + 4.000000000 T_{8,7} + 4.000000000 T_{8,5} - 20 T_{8,6} = 0 \\
T_{10,7} + T_{10,5} + T_{8,7} + T_{8,5} + 4.000000000 T_{10,6} + 4.000000000 T_{8,6} \\
& + 4.000000000 T_{9,7} + 4.000000000 T_{9,5} - 20 T_{9,6} = 0 \\
T_{9,7} + T_{9,5} + 4.000000000 T_{9,6} + 4.000000000 T_{10,7} + 4.000000000 T_{10,5} \\
& - 20 T_{10,6} = 0 \\
T_{3,8} + T_{3,6} + 4.000000000 T_{3,7} + 4.000000000 T_{2,8} + 4.000000000 T_{2,6} - 20 T_{2,7} \\
& = 0 \\
T_{4,8} + T_{4,6} + T_{2,8} + T_{2,6} + 4.000000000 T_{4,7} + 4.000000000 T_{2,7} \\
& + 4.000000000 T_{3,8} + 4.000000000 T_{3,6} - 20 T_{3,7} = 0 \\
T_{5,8} + T_{5,6} + T_{3,8} + T_{3,6} + 4.000000000 T_{5,7} + 4.000000000 T_{3,7} \\
& + 4.000000000 T_{4,8} + 4.000000000 T_{4,6} - 20 T_{4,7} = 0 \\
T_{6,8} + T_{6,6} + T_{4,8} + T_{4,6} + 4.000000000 T_{6,7} + 4.000000000 T_{4,7} \\
& + 4.000000000 T_{5,8} + 4.000000000 T_{5,6} - 20 T_{5,7} = 0 \\
T_{7,8} + T_{7,6} + T_{5,8} + T_{5,6} + 4.000000000 T_{7,7} + 4.000000000 T_{5,7} \\
& + 4.000000000 T_{6,8} + 4.000000000 T_{6,6} - 20 T_{6,7} = 0 \\
T_{8,8} + T_{8,6} + T_{6,8} + T_{6,6} + 4.000000000 T_{8,7} + 4.000000000 T_{6,7} \\
& + 4.000000000 T_{7,8} + 4.000000000 T_{7,6} - 20 T_{7,7} = 0 \\
T_{9,8} + T_{9,6} + T_{7,8} + T_{7,6} + 4.000000000 T_{9,7} + 4.000000000 T_{7,7} \\
& + 4.000000000 T_{8,8} + 4.000000000 T_{8,6} - 20 T_{8,7} = 0 \\
T_{10,8} + T_{10,6} + T_{8,8} + T_{8,6} + 4.000000000 T_{10,7} + 4.000000000 T_{8,7} \\
& + 4.000000000 T_{9,8} + 4.000000000 T_{9,6} - 20 T_{9,7} = 0 \\
T_{9,8} + T_{9,6} + 4.000000000 T_{9,7} + 4.000000000 T_{10,8} + 4.000000000 T_{10,6} \\
& - 20 T_{10,7} = 0
\end{aligned}$$

$$T_{3,9} + T_{3,7} + 4.000000000 T_{3,8} + 4.000000000 T_{2,9} + 4.000000000 T_{2,7} - 20 T_{2,8} = 0$$

$$T_{4,9} + T_{4,7} + T_{2,9} + T_{2,7} + 4.000000000 T_{4,8} + 4.000000000 T_{2,8} + 4.000000000 T_{3,9} + 4.000000000 T_{3,7} - 20 T_{3,8} = 0$$

$$T_{5,9} + T_{5,7} + T_{3,9} + T_{3,7} + 4.000000000 T_{5,8} + 4.000000000 T_{3,8} + 4.000000000 T_{4,9} + 4.000000000 T_{4,7} - 20 T_{4,8} = 0$$

$$T_{6,9} + T_{6,7} + T_{4,9} + T_{4,7} + 4.000000000 T_{6,8} + 4.000000000 T_{4,8} + 4.000000000 T_{5,9} + 4.000000000 T_{5,7} - 20 T_{5,8} = 0$$

$$T_{7,9} + T_{7,7} + T_{5,9} + T_{5,7} + 4.000000000 T_{7,8} + 4.000000000 T_{5,8} + 4.000000000 T_{6,9} + 4.000000000 T_{6,7} - 20 T_{6,8} = 0$$

$$T_{8,9} + T_{8,7} + T_{6,9} + T_{6,7} + 4.000000000 T_{8,8} + 4.000000000 T_{6,8} + 4.000000000 T_{7,9} + 4.000000000 T_{7,7} - 20 T_{7,8} = 0$$

$$T_{9,9} + T_{9,7} + T_{7,9} + T_{7,7} + 4.000000000 T_{9,8} + 4.000000000 T_{7,8} + 4.000000000 T_{8,9} + 4.000000000 T_{8,7} - 20 T_{8,8} = 0$$

$$T_{10,9} + T_{10,7} + T_{8,9} + T_{8,7} + 4.000000000 T_{10,8} + 4.000000000 T_{8,8} + 4.000000000 T_{9,9} + 4.000000000 T_{9,7} - 20 T_{9,8} = 0$$

$$T_{9,9} + T_{9,7} + 4.000000000 T_{9,8} + 4.000000000 T_{10,9} + 4.000000000 T_{10,7} - 20 T_{10,8} = 0$$

$$T_{3,10} + T_{3,8} + 4.000000000 T_{3,9} + 4.000000000 T_{2,10} + 4.000000000 T_{2,8} - 20 T_{2,9} = 0$$

$$T_{4,10} + T_{4,8} + T_{2,10} + T_{2,8} + 4.000000000 T_{4,9} + 4.000000000 T_{2,9} + 4.000000000 T_{3,10} + 4.000000000 T_{3,8} - 20 T_{3,9} = 0$$

$$T_{5,10} + T_{5,8} + T_{3,10} + T_{3,8} + 4.000000000 T_{5,9} + 4.000000000 T_{3,9} + 4.000000000 T_{4,10} + 4.000000000 T_{4,8} - 20 T_{4,9} = 0$$

$$T_{6,10} + T_{6,8} + T_{4,10} + T_{4,8} + 4.000000000 T_{6,9} + 4.000000000 T_{4,9} + 4.000000000 T_{5,10} + 4.000000000 T_{5,8} - 20 T_{5,9} = 0$$

$$T_{7,10} + T_{7,8} + T_{5,10} + T_{5,8} + 4.000000000 T_{7,9} + 4.000000000 T_{5,9} + 4.000000000 T_{6,10} + 4.000000000 T_{6,8} - 20 T_{6,9} = 0$$

$$T_{8,10} + T_{8,8} + T_{6,10} + T_{6,8} + 4.000000000 T_{8,9} + 4.000000000 T_{6,9} + 4.000000000 T_{7,10} + 4.000000000 T_{7,8} - 20 T_{7,9} = 0$$

$$T_{9,10} + T_{9,8} + T_{7,10} + T_{7,8} + 4.000000000 T_{9,9} + 4.000000000 T_{7,9} + 4.000000000 T_{8,10} + 4.000000000 T_{8,8} - 20 T_{8,9} = 0$$

$$T_{10,10} + T_{10,8} + T_{8,10} + T_{8,8} + 4.000000000 T_{10,9} + 4.000000000 T_{8,9} + 4.000000000 T_{9,10} + 4.000000000 T_{9,8} - 20 T_{9,9} = 0$$

$$T_{9,10} + T_{9,8} + 4.000000000 T_{9,9} + 4.000000000 T_{10,10} + 4.000000000 T_{10,8} - 20 T_{10,9} = 0$$

$$T_{3,11} + T_{3,9} + 4.000000000 T_{3,10} + 4.000000000 T_{2,11} + 4.000000000 T_{2,9} - 20 T_{2,10} = 0$$

$$T_{4,11} + T_{4,9} + T_{2,11} + T_{2,9} + 4.000000000 T_{4,10} + 4.000000000 T_{2,10} + 4.000000000 T_{3,11} + 4.000000000 T_{3,9} - 20 T_{3,10} = 0$$

$$T_{5,11} + T_{5,9} + T_{3,11} + T_{3,9} + 4.000000000 T_{5,10} + 4.000000000 T_{3,10} + 4.000000000 T_{4,11} + 4.000000000 T_{4,9} - 20 T_{4,10} = 0$$

$$T_{6,11} + T_{6,9} + T_{4,11} + T_{4,9} + 4.000000000 T_{6,10} + 4.000000000 T_{4,10}$$

$$\begin{aligned}
& + 4.000000000 T_{5,11} + 4.000000000 T_{5,9} - 20 T_{5,10} = 0 \\
T_{7,11} + T_{7,9} + T_{5,11} + T_{5,9} + 4.000000000 T_{7,10} + 4.000000000 T_{5,10} \\
& + 4.000000000 T_{6,11} + 4.000000000 T_{6,9} - 20 T_{6,10} = 0 \\
T_{8,11} + T_{8,9} + T_{6,11} + T_{6,9} + 4.000000000 T_{8,10} + 4.000000000 T_{6,10} \\
& + 4.000000000 T_{7,11} + 4.000000000 T_{7,9} - 20 T_{7,10} = 0 \\
T_{9,11} + T_{9,9} + T_{7,11} + T_{7,9} + 4.000000000 T_{9,10} + 4.000000000 T_{7,10} \\
& + 4.000000000 T_{8,11} + 4.000000000 T_{8,9} - 20 T_{8,10} = 0 \\
T_{10,11} + T_{10,9} + T_{8,11} + T_{8,9} + 4.000000000 T_{10,10} + 4.000000000 T_{8,10} \\
& + 4.000000000 T_{9,11} + 4.000000000 T_{9,9} - 20 T_{9,10} = 0 \\
T_{9,11} + T_{9,9} + 4.000000000 T_{9,10} + 4.000000000 T_{10,11} + 4.000000000 T_{10,9} \\
& - 20 T_{10,10} = 0 \\
T_{3,12} + T_{3,10} + 4.000000000 T_{3,11} + 4.000000000 T_{2,12} + 4.000000000 T_{2,10} \\
& - 20 T_{2,11} = 0 \\
T_{4,12} + T_{4,10} + T_{2,12} + T_{2,10} + 4.000000000 T_{4,11} + 4.000000000 T_{2,11} \\
& + 4.000000000 T_{3,12} + 4.000000000 T_{3,10} - 20 T_{3,11} = 0 \\
T_{5,12} + T_{5,10} + T_{3,12} + T_{3,10} + 4.000000000 T_{5,11} + 4.000000000 T_{3,11} \\
& + 4.000000000 T_{4,12} + 4.000000000 T_{4,10} - 20 T_{4,11} = 0 \\
T_{6,12} + T_{6,10} + T_{4,12} + T_{4,10} + 4.000000000 T_{6,11} + 4.000000000 T_{4,11} \\
& + 4.000000000 T_{5,12} + 4.000000000 T_{5,10} - 20 T_{5,11} = 0 \\
T_{7,12} + T_{7,10} + T_{5,12} + T_{5,10} + 4.000000000 T_{7,11} + 4.000000000 T_{5,11} \\
& + 4.000000000 T_{6,12} + 4.000000000 T_{6,10} - 20 T_{6,11} = 0 \\
T_{8,12} + T_{8,10} + T_{6,12} + T_{6,10} + 4.000000000 T_{8,11} + 4.000000000 T_{6,11} \\
& + 4.000000000 T_{7,12} + 4.000000000 T_{7,10} - 20 T_{7,11} = 0 \\
T_{9,12} + T_{9,10} + T_{7,12} + T_{7,10} + 4.000000000 T_{9,11} + 4.000000000 T_{7,11} \\
& + 4.000000000 T_{8,12} + 4.000000000 T_{8,10} - 20 T_{8,11} = 0 \\
T_{10,12} + T_{10,10} + T_{8,12} + T_{8,10} + 4.000000000 T_{10,11} + 4.000000000 T_{8,11} \\
& + 4.000000000 T_{9,12} + 4.000000000 T_{9,10} - 20 T_{9,11} = 0 \\
T_{9,12} + T_{9,10} + 4.000000000 T_{9,11} + 4.000000000 T_{10,12} + 4.000000000 T_{10,10} \\
& - 20 T_{10,11} = 0 \\
T_{3,13} + T_{3,11} + 4.000000000 T_{3,12} + 4.000000000 T_{2,13} + 4.000000000 T_{2,11} \\
& - 20 T_{2,12} = 0 \\
T_{4,13} + T_{4,11} + T_{2,13} + T_{2,11} + 4.000000000 T_{4,12} + 4.000000000 T_{2,12} \\
& + 4.000000000 T_{3,13} + 4.000000000 T_{3,11} - 20 T_{3,12} = 0 \\
T_{5,13} + T_{5,11} + T_{3,13} + T_{3,11} + 4.000000000 T_{5,12} + 4.000000000 T_{3,12} \\
& + 4.000000000 T_{4,13} + 4.000000000 T_{4,11} - 20 T_{4,12} = 0 \\
T_{6,13} + T_{6,11} + T_{4,13} + T_{4,11} + 4.000000000 T_{6,12} + 4.000000000 T_{4,12} \\
& + 4.000000000 T_{5,13} + 4.000000000 T_{5,11} - 20 T_{5,12} = 0 \\
T_{7,13} + T_{7,11} + T_{5,13} + T_{5,11} + 4.000000000 T_{7,12} + 4.000000000 T_{5,12} \\
& + 4.000000000 T_{6,13} + 4.000000000 T_{6,11} - 20 T_{6,12} = 0 \\
T_{8,13} + T_{8,11} + T_{6,13} + T_{6,11} + 4.000000000 T_{8,12} + 4.000000000 T_{6,12} \\
& + 4.000000000 T_{7,13} + 4.000000000 T_{7,11} - 20 T_{7,12} = 0 \\
T_{9,13} + T_{9,11} + T_{7,13} + T_{7,11} + 4.000000000 T_{9,12} + 4.000000000 T_{7,12} \\
& + 4.000000000 T_{8,13} + 4.000000000 T_{8,11} - 20 T_{8,12} = 0
\end{aligned}$$

$$\begin{aligned}
& T_{10,13} + T_{10,11} + T_{8,13} + T_{8,11} + 4.000000000 T_{10,12} + 4.000000000 T_{8,12} \\
& \quad + 4.000000000 T_{9,13} + 4.000000000 T_{9,11} - 20 T_{9,12} = 0 \\
& T_{9,13} + T_{9,11} + 4.000000000 T_{9,12} + 4.000000000 T_{10,13} + 4.000000000 T_{10,11} \\
& \quad - 20 T_{10,12} = 0 \\
& T_{3,14} + T_{3,12} + 4.000000000 T_{3,13} + 4.000000000 T_{2,14} + 4.000000000 T_{2,12} \\
& \quad - 20 T_{2,13} = 0 \\
& T_{4,14} + T_{4,12} + T_{2,14} + T_{2,12} + 4.000000000 T_{4,13} + 4.000000000 T_{2,13} \\
& \quad + 4.000000000 T_{3,14} + 4.000000000 T_{3,12} - 20 T_{3,13} = 0 \\
& T_{5,14} + T_{5,12} + T_{3,14} + T_{3,12} + 4.000000000 T_{5,13} + 4.000000000 T_{3,13} \\
& \quad + 4.000000000 T_{4,14} + 4.000000000 T_{4,12} - 20 T_{4,13} = 0 \\
& T_{6,14} + T_{6,12} + T_{4,14} + T_{4,12} + 4.000000000 T_{6,13} + 4.000000000 T_{4,13} \\
& \quad + 4.000000000 T_{5,14} + 4.000000000 T_{5,12} - 20 T_{5,13} = 0 \\
& T_{7,14} + T_{7,12} + T_{5,14} + T_{5,12} + 4.000000000 T_{7,13} + 4.000000000 T_{5,13} \\
& \quad + 4.000000000 T_{6,14} + 4.000000000 T_{6,12} - 20 T_{6,13} = 0 \\
& T_{8,14} + T_{8,12} + T_{6,14} + T_{6,12} + 4.000000000 T_{8,13} + 4.000000000 T_{6,13} \\
& \quad + 4.000000000 T_{7,14} + 4.000000000 T_{7,12} - 20 T_{7,13} = 0 \\
& T_{9,14} + T_{9,12} + T_{7,14} + T_{7,12} + 4.000000000 T_{9,13} + 4.000000000 T_{7,13} \\
& \quad + 4.000000000 T_{8,14} + 4.000000000 T_{8,12} - 20 T_{8,13} = 0 \\
& T_{10,14} + T_{10,12} + T_{8,14} + T_{8,12} + 4.000000000 T_{10,13} + 4.000000000 T_{8,13} \\
& \quad + 4.000000000 T_{9,14} + 4.000000000 T_{9,12} - 20 T_{9,13} = 0 \\
& T_{9,14} + T_{9,12} + 4.000000000 T_{9,13} + 4.000000000 T_{10,14} + 4.000000000 T_{10,12} \\
& \quad - 20 T_{10,13} = 0 \\
& T_{3,15} + T_{3,13} + 4.000000000 T_{3,14} + 4.000000000 T_{2,15} + 4.000000000 T_{2,13} \\
& \quad - 20 T_{2,14} = 0 \\
& T_{4,15} + T_{4,13} + T_{2,15} + T_{2,13} + 4.000000000 T_{4,14} + 4.000000000 T_{2,14} \\
& \quad + 4.000000000 T_{3,15} + 4.000000000 T_{3,13} - 20 T_{3,14} = 0 \\
& T_{5,15} + T_{5,13} + T_{3,15} + T_{3,13} + 4.000000000 T_{5,14} + 4.000000000 T_{3,14} \\
& \quad + 4.000000000 T_{4,15} + 4.000000000 T_{4,13} - 20 T_{4,14} = 0 \\
& T_{6,15} + T_{6,13} + T_{4,15} + T_{4,13} + 4.000000000 T_{6,14} + 4.000000000 T_{4,14} \\
& \quad + 4.000000000 T_{5,15} + 4.000000000 T_{5,13} - 20 T_{5,14} = 0 \\
& T_{7,15} + T_{7,13} + T_{5,15} + T_{5,13} + 4.000000000 T_{7,14} + 4.000000000 T_{5,14} \\
& \quad + 4.000000000 T_{6,15} + 4.000000000 T_{6,13} - 20 T_{6,14} = 0 \\
& T_{8,15} + T_{8,13} + T_{6,15} + T_{6,13} + 4.000000000 T_{8,14} + 4.000000000 T_{6,14} \\
& \quad + 4.000000000 T_{7,15} + 4.000000000 T_{7,13} - 20 T_{7,14} = 0 \\
& T_{9,15} + T_{9,13} + T_{7,15} + T_{7,13} + 4.000000000 T_{9,14} + 4.000000000 T_{7,14} \\
& \quad + 4.000000000 T_{8,15} + 4.000000000 T_{8,13} - 20 T_{8,14} = 0 \\
& T_{10,15} + T_{10,13} + T_{8,15} + T_{8,13} + 4.000000000 T_{10,14} + 4.000000000 T_{8,14} \\
& \quad + 4.000000000 T_{9,15} + 4.000000000 T_{9,13} - 20 T_{9,14} = 0 \\
& T_{9,15} + T_{9,13} + 4.000000000 T_{9,14} + 4.000000000 T_{10,15} + 4.000000000 T_{10,13} \\
& \quad - 20 T_{10,14} = 0 \\
& 182.3853230 + T_{3,14} + 4.000000000 T_{3,15} + 4.000000000 T_{2,14} - 20 T_{2,15} = 0 \\
& 346.9174999 + T_{4,14} + T_{2,14} + 4.000000000 T_{4,15} + 4.000000000 T_{2,15} \\
& \quad + 4.000000000 T_{3,14} - 20 T_{3,15} = 0
\end{aligned}$$

$$\begin{aligned}
& 477.4909747 + T_{5,14} + T_{3,14} + 4.000000000 T_{5,15} + 4.000000000 T_{3,15} \\
& \quad + 4.000000000 T_{4,14} - 20 T_{4,15} = 0 \\
& 561.3243060 + T_{6,14} + T_{4,14} + 4.000000000 T_{6,15} + 4.000000000 T_{4,15} \\
& \quad + 4.000000000 T_{5,14} - 20 T_{5,15} = 0 \\
& 590.2113033 + T_{7,14} + T_{5,14} + 4.000000000 T_{7,15} + 4.000000000 T_{5,15} \\
& \quad + 4.000000000 T_{6,14} - 20 T_{6,15} = 0 \\
& 561.3243060 + T_{8,14} + T_{6,14} + 4.000000000 T_{8,15} + 4.000000000 T_{6,15} \\
& \quad + 4.000000000 T_{7,14} - 20 T_{7,15} = 0 \\
& 477.4909744 + T_{9,14} + T_{7,14} + 4.000000000 T_{9,15} + 4.000000000 T_{7,15} \\
& \quad + 4.000000000 T_{8,14} - 20 T_{8,15} = 0 \\
& 346.9174996 + T_{10,14} + T_{8,14} + 4.000000000 T_{10,15} + 4.000000000 T_{8,15} \\
& \quad + 4.000000000 T_{9,14} - 20 T_{9,15} = 0 \\
& 182.3853226 + T_{9,14} + 4.000000000 T_{9,15} + 4.000000000 T_{10,14} - 20 T_{10,15} = 0 \quad (1.1.2)
\end{aligned}$$

> *Eqs* := {seq(Eq[i], i = 1..N)} :

> *Tmps* := [seq(Temps[i], i = 1..N)];

$$\begin{aligned}
& \textit{Tmps} := [T_{2,2}, T_{3,2}, T_{4,2}, T_{5,2}, T_{6,2}, T_{7,2}, T_{8,2}, T_{9,2}, T_{10,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{5,3}, T_{6,3}, \\
& \quad T_{7,3}, T_{8,3}, T_{9,3}, T_{10,3}, T_{2,4}, T_{3,4}, T_{4,4}, T_{5,4}, T_{6,4}, T_{7,4}, T_{8,4}, T_{9,4}, T_{10,4}, T_{2,5}, T_{3,5}, \\
& \quad T_{4,5}, T_{5,5}, T_{6,5}, T_{7,5}, T_{8,5}, T_{9,5}, T_{10,5}, T_{2,6}, T_{3,6}, T_{4,6}, T_{5,6}, T_{6,6}, T_{7,6}, T_{8,6}, T_{9,6}, \\
& \quad T_{10,6}, T_{2,7}, T_{3,7}, T_{4,7}, T_{5,7}, T_{6,7}, T_{7,7}, T_{8,7}, T_{9,7}, T_{10,7}, T_{2,8}, T_{3,8}, T_{4,8}, T_{5,8}, T_{6,8}, \\
& \quad T_{7,8}, T_{8,8}, T_{9,8}, T_{10,8}, T_{2,9}, T_{3,9}, T_{4,9}, T_{5,9}, T_{6,9}, T_{7,9}, T_{8,9}, T_{9,9}, T_{10,9}, T_{2,10}, T_{3,10}, \\
& \quad T_{4,10}, T_{5,10}, T_{6,10}, T_{7,10}, T_{8,10}, T_{9,10}, T_{10,10}, T_{2,11}, T_{3,11}, T_{4,11}, T_{5,11}, T_{6,11}, T_{7,11}, \\
& \quad T_{8,11}, T_{9,11}, T_{10,11}, T_{2,12}, T_{3,12}, T_{4,12}, T_{5,12}, T_{6,12}, T_{7,12}, T_{8,12}, T_{9,12}, T_{10,12}, \\
& \quad T_{2,13}, T_{3,13}, T_{4,13}, T_{5,13}, T_{6,13}, T_{7,13}, T_{8,13}, T_{9,13}, T_{10,13}, T_{2,14}, T_{3,14}, T_{4,14}, T_{5,14}, \\
& \quad T_{6,14}, T_{7,14}, T_{8,14}, T_{9,14}, T_{10,14}, T_{2,15}, T_{3,15}, T_{4,15}, T_{5,15}, T_{6,15}, T_{7,15}, T_{8,15}, T_{9,15}, \\
& \quad T_{10,15}]
\end{aligned} \quad (1.1.3)$$

>