Equation de Laplace 2D

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Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

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LMD : Energétique

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100 * \sin\left(\frac{\pi \cdot x}{a} \right),$$
$$T(0, y) = 0,$$
$$T(a, y) = 0.$$

Solution discrétisée (formulation en 9 points):

```
> Restart : with(plots) :
> Digits := 4;
```

$$Digits := 4 \quad \textbf{(1.1)}$$

```
> NbIso := 15;
```

$$NbIso := 15 \quad \textbf{(1.2)}$$

```
> a := 0.1; b := 0.15; ndx := 10; ndy := 15
```

$$a := 0.1$$
$$b := 0.15$$
$$ndx := 10$$
$$ndy := 15 \quad \textbf{(1.3)}$$

$> \Delta x := \frac{a}{ndx} ; \Delta y := \frac{b}{ndy}; \beta := \frac{\Delta x}{\Delta y};$

$$\Delta x := 0.01000$$
$$\Delta y := 0.01000$$
$$\beta := 1.000 \quad \textbf{(1.4)}$$

$> i_{max} := ndx + 1; j_{max} := ndy + 1;$

$$i_{max} := 11$$
$$j_{max} := 16 \quad \textbf{(1.5)}$$

$> Tg := 0;$
$Td := 0;$
$Tb := 0;$
$Th := 100 \cdot \sin\left(\frac{\pi \cdot (i-1) \cdot \Delta x}{a} \right); unapply(Th, i)$

$$Tg := 0$$
$$Td := 0$$
$$Tb := 0$$
$$Th := 100 \sin(0.1000\, \pi\, (i-1))$$
$$i \rightarrow 100 \sin(0.1000\, \pi\, (i-1)) \quad \textbf{(1.6)}$$

Nombre d'équations:

$> N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 126 \quad \textbf{(1.7)}$$

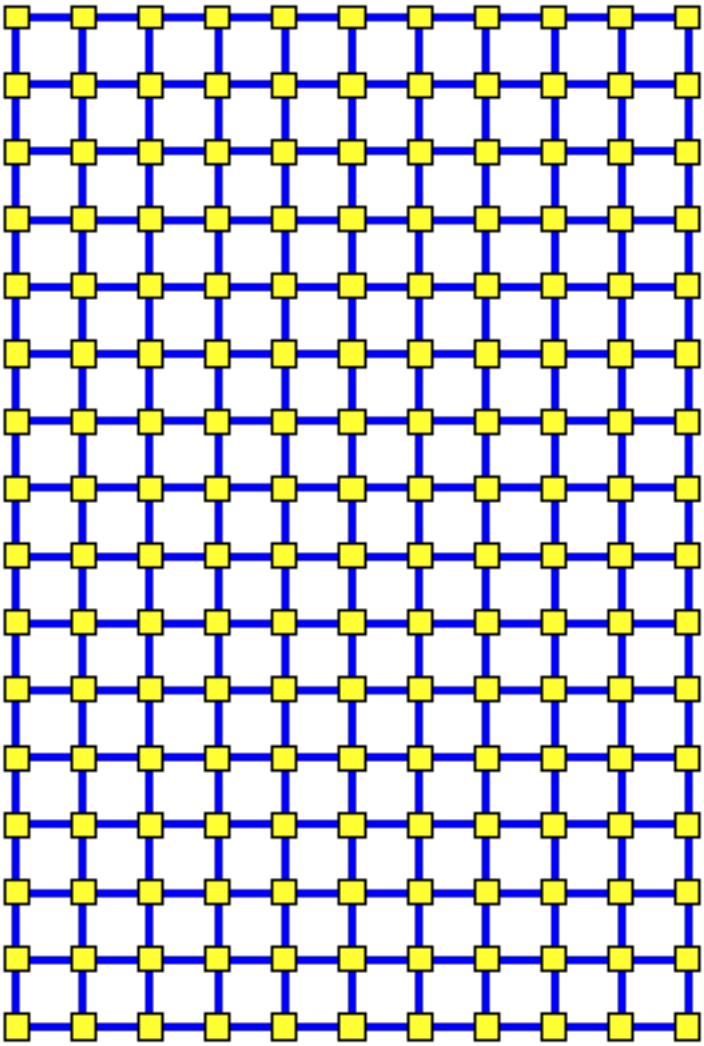
Maillage:

$> with(GraphTheory) : with(SpecialGraphs) :$

$> G := GridGraph(i_{max}, j_{max})$

$$G := Graph\ 1:\ an\ undirected\ unweighted\ graph\ with\ 176\ vertices\ and\ 325\ edge(s) \quad \textbf{(1.8)}$$

$> DrawGraph(G)$



Conditions aux Limites:

```
> for i from 2 to i_max − 1 do T[i, 1] := Tb end do;
```

$$T_{2,1} := 0$$
$$T_{3,1} := 0$$
$$T_{4,1} := 0$$
$$T_{5,1} := 0$$
$$T_{6,1} := 0$$
$$T_{7,1} := 0$$
$$T_{8,1} := 0$$
$$T_{9,1} := 0$$
$$T_{10,1} := 0 \qquad \textbf{(1.9)}$$

```
> for i from 2 to i_max − 1 do T[i, j_max] := evalf(Th(i)) end do;
```

$$T_{2,16} := 30.91$$
$$T_{3,16} := 58.79$$
$$T_{4,16} := 80.91$$
$$T_{5,16} := 95.12$$
$$T_{6,16} := 100.$$
$$T_{7,16} := 95.10$$
$$T_{8,16} := 80.91$$

$$T_{9,\,16} := 58.72$$
$$T_{10,\,16} := 30.85 \qquad \textbf{(1.10)}$$

> **for** j **from** 2 **to** $j_{\max} - 1$ **do** $T[1, \mathrm{j}] := Tg$ **end do**;

$$T_{1,\,2} := 0$$
$$T_{1,\,3} := 0$$
$$T_{1,\,4} := 0$$
$$T_{1,\,5} := 0$$
$$T_{1,\,6} := 0$$
$$T_{1,\,7} := 0$$
$$T_{1,\,8} := 0$$
$$T_{1,\,9} := 0$$
$$T_{1,\,10} := 0$$
$$T_{1,\,11} := 0$$
$$T_{1,\,12} := 0$$
$$T_{1,\,13} := 0$$
$$T_{1,\,14} := 0$$
$$T_{1,\,15} := 0 \qquad \textbf{(1.11)}$$

> **for** j **from** 2 **to** $j_{\max} - 1$ **do** $T[i_{\max}, \mathrm{j}] := Td$ **end do**;

$$T_{11,\,2} := 0$$
$$T_{11,\,3} := 0$$
$$T_{11,\,4} := 0$$
$$T_{11,\,5} := 0$$
$$T_{11,\,6} := 0$$
$$T_{11,\,7} := 0$$
$$T_{11,\,8} := 0$$
$$T_{11,\,9} := 0$$
$$T_{11,\,10} := 0$$
$$T_{11,\,11} := 0$$
$$T_{11,\,12} := 0$$
$$T_{11,\,13} := 0$$
$$T_{11,\,14} := 0$$
$$T_{11,\,15} := 0 \qquad \textbf{(1.12)}$$

> $T[1, 1] := \dfrac{Tg + Tb}{2}$

$$T_{1,\,1} := 0 \qquad \textbf{(1.13)}$$

> $T[i_{\max}, j_{\max}] := \dfrac{Th(i_{\max}) + Td}{2}$

$$T_{11,\,16} := 0 \qquad \textbf{(1.14)}$$

> $T[i_{\max}, 1] := \dfrac{Tb + Td}{2}$

$$T_{11,\,1} := 0 \qquad \textbf{(1.15)}$$

> $T[1, j_{\max}] := \dfrac{Tg + Th(1)}{2}$

(1.16)

$$T_{1,16} := 0 \qquad \textbf{(1.16)}$$

```
> k := 1 :
```

$$k := 1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 2 to i_max - 1 do
    for j from 2 to j_max - 1 do
      Eq[k] := T[i+1,j+1] + T[i+1,j-1] + T[i-1,j+1] + T[i-1,j-1]
      + 2·((5-β^2)/(1+β^2))·(T[i+1,j] + T[i-1,j]) + 2·((5·β^2-1)/(1+β^2))·(T[i,j+1] + T[i,j
      -1]) - 20·T[i,j] = 0;
      Temps[k] := T[i,j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,3} + 4.000\,T_{3,2} + 4.000\,T_{2,3} - 20\,T_{2,2} = 0$$

$$T_{3,4} + T_{3,2} + 4.000\,T_{3,3} + 4.000\,T_{2,4} + 4.000\,T_{2,2} - 20\,T_{2,3} = 0$$

$$T_{3,5} + T_{3,3} + 4.000\,T_{3,4} + 4.000\,T_{2,5} + 4.000\,T_{2,3} - 20\,T_{2,4} = 0$$

$$T_{3,6} + T_{3,4} + 4.000\,T_{3,5} + 4.000\,T_{2,6} + 4.000\,T_{2,4} - 20\,T_{2,5} = 0$$

$$T_{3,7} + T_{3,5} + 4.000\,T_{3,6} + 4.000\,T_{2,7} + 4.000\,T_{2,5} - 20\,T_{2,6} = 0$$

$$T_{3,8} + T_{3,6} + 4.000\,T_{3,7} + 4.000\,T_{2,8} + 4.000\,T_{2,6} - 20\,T_{2,7} = 0$$

$$T_{3,9} + T_{3,7} + 4.000\,T_{3,8} + 4.000\,T_{2,9} + 4.000\,T_{2,7} - 20\,T_{2,8} = 0$$

$$T_{3,10} + T_{3,8} + 4.000\,T_{3,9} + 4.000\,T_{2,10} + 4.000\,T_{2,8} - 20\,T_{2,9} = 0$$

$$T_{3,11} + T_{3,9} + 4.000\,T_{3,10} + 4.000\,T_{2,11} + 4.000\,T_{2,9} - 20\,T_{2,10} = 0$$

$$T_{3,12} + T_{3,10} + 4.000\,T_{3,11} + 4.000\,T_{2,12} + 4.000\,T_{2,10} - 20\,T_{2,11} = 0$$

$$T_{3,13} + T_{3,11} + 4.000\,T_{3,12} + 4.000\,T_{2,13} + 4.000\,T_{2,11} - 20\,T_{2,12} = 0$$

$$T_{3,14} + T_{3,12} + 4.000\,T_{3,13} + 4.000\,T_{2,14} + 4.000\,T_{2,12} - 20\,T_{2,13} = 0$$

$$T_{3,15} + T_{3,13} + 4.000\,T_{3,14} + 4.000\,T_{2,15} + 4.000\,T_{2,13} - 20\,T_{2,14} = 0$$

$$182.4 + T_{3,14} + 4.000\,T_{3,15} + 4.000\,T_{2,14} - 20\,T_{2,15} = 0$$

$$T_{4,3} + T_{2,3} + 4.000\,T_{4,2} + 4.000\,T_{2,2} + 4.000\,T_{3,3} - 20\,T_{3,2} = 0$$

$$T_{4,4} + T_{4,2} + T_{2,4} + T_{2,2} + 4.000\,T_{4,3} + 4.000\,T_{2,3} + 4.000\,T_{3,4} + 4.000\,T_{3,2} - 20\,T_{3,3} = 0$$

$$T_{4,5} + T_{4,3} + T_{2,5} + T_{2,3} + 4.000\,T_{4,4} + 4.000\,T_{2,4} + 4.000\,T_{3,5} + 4.000\,T_{3,3} - 20\,T_{3,4} = 0$$

$$T_{4,6} + T_{4,4} + T_{2,6} + T_{2,4} + 4.000\,T_{4,5} + 4.000\,T_{2,5} + 4.000\,T_{3,6} + 4.000\,T_{3,4} - 20\,T_{3,5} = 0$$

$$T_{4,7} + T_{4,5} + T_{2,7} + T_{2,5} + 4.000\,T_{4,6} + 4.000\,T_{2,6} + 4.000\,T_{3,7} + 4.000\,T_{3,5} - 20\,T_{3,6} = 0$$

$$T_{4,8} + T_{4,6} + T_{2,8} + T_{2,6} + 4.000\,T_{4,7} + 4.000\,T_{2,7} + 4.000\,T_{3,8} + 4.000\,T_{3,6} - 20\,T_{3,7} = 0$$

$$T_{4,9} + T_{4,7} + T_{2,9} + T_{2,7} + 4.000\,T_{4,8} + 4.000\,T_{2,8} + 4.000\,T_{3,9} + 4.000\,T_{3,7}$$

$$-20\,T_{3,8}=0$$

$$T_{4,10}+T_{4,8}+T_{2,10}+T_{2,8}+4.000\,T_{4,9}+4.000\,T_{2,9}+4.000\,T_{3,10}+4.000\,T_{3,8}-20\,T_{3,9}=0$$

$$T_{4,11}+T_{4,9}+T_{2,11}+T_{2,9}+4.000\,T_{4,10}+4.000\,T_{2,10}+4.000\,T_{3,11}+4.000\,T_{3,9}-20\,T_{3,10}=0$$

$$T_{4,12}+T_{4,10}+T_{2,12}+T_{2,10}+4.000\,T_{4,11}+4.000\,T_{2,11}+4.000\,T_{3,12}+4.000\,T_{3,10}-20\,T_{3,11}=0$$

$$T_{4,13}+T_{4,11}+T_{2,13}+T_{2,11}+4.000\,T_{4,12}+4.000\,T_{2,12}+4.000\,T_{3,13}+4.000\,T_{3,11}-20\,T_{3,12}=0$$

$$T_{4,14}+T_{4,12}+T_{2,14}+T_{2,12}+4.000\,T_{4,13}+4.000\,T_{2,13}+4.000\,T_{3,14}+4.000\,T_{3,12}-20\,T_{3,13}=0$$

$$T_{4,15}+T_{4,13}+T_{2,15}+T_{2,13}+4.000\,T_{4,14}+4.000\,T_{2,14}+4.000\,T_{3,15}+4.000\,T_{3,13}-20\,T_{3,14}=0$$

$$347.0+T_{4,14}+T_{2,14}+4.000\,T_{4,15}+4.000\,T_{2,15}+4.000\,T_{3,14}-20\,T_{3,15}=0$$

$$T_{5,3}+T_{3,3}+4.000\,T_{5,2}+4.000\,T_{3,2}+4.000\,T_{4,3}-20\,T_{4,2}=0$$

$$T_{5,4}+T_{5,2}+T_{3,4}+T_{3,2}+4.000\,T_{5,3}+4.000\,T_{3,3}+4.000\,T_{4,4}+4.000\,T_{4,2}-20\,T_{4,3}=0$$

$$T_{5,5}+T_{5,3}+T_{3,5}+T_{3,3}+4.000\,T_{5,4}+4.000\,T_{3,4}+4.000\,T_{4,5}+4.000\,T_{4,3}-20\,T_{4,4}=0$$

$$T_{5,6}+T_{5,4}+T_{3,6}+T_{3,4}+4.000\,T_{5,5}+4.000\,T_{3,5}+4.000\,T_{4,6}+4.000\,T_{4,4}-20\,T_{4,5}=0$$

$$T_{5,7}+T_{5,5}+T_{3,7}+T_{3,5}+4.000\,T_{5,6}+4.000\,T_{3,6}+4.000\,T_{4,7}+4.000\,T_{4,5}-20\,T_{4,6}=0$$

$$T_{5,8}+T_{5,6}+T_{3,8}+T_{3,6}+4.000\,T_{5,7}+4.000\,T_{3,7}+4.000\,T_{4,8}+4.000\,T_{4,6}-20\,T_{4,7}=0$$

$$T_{5,9}+T_{5,7}+T_{3,9}+T_{3,7}+4.000\,T_{5,8}+4.000\,T_{3,8}+4.000\,T_{4,9}+4.000\,T_{4,7}-20\,T_{4,8}=0$$

$$T_{5,10}+T_{5,8}+T_{3,10}+T_{3,8}+4.000\,T_{5,9}+4.000\,T_{3,9}+4.000\,T_{4,10}+4.000\,T_{4,8}-20\,T_{4,9}=0$$

$$T_{5,11}+T_{5,9}+T_{3,11}+T_{3,9}+4.000\,T_{5,10}+4.000\,T_{3,10}+4.000\,T_{4,11}+4.000\,T_{4,9}-20\,T_{4,10}=0$$

$$T_{5,12}+T_{5,10}+T_{3,12}+T_{3,10}+4.000\,T_{5,11}+4.000\,T_{3,11}+4.000\,T_{4,12}+4.000\,T_{4,10}-20\,T_{4,11}=0$$

$$T_{5,13}+T_{5,11}+T_{3,13}+T_{3,11}+4.000\,T_{5,12}+4.000\,T_{3,12}+4.000\,T_{4,13}+4.000\,T_{4,11}-20\,T_{4,12}=0$$

$$T_{5,14}+T_{5,12}+T_{3,14}+T_{3,12}+4.000\,T_{5,13}+4.000\,T_{3,13}+4.000\,T_{4,14}+4.000\,T_{4,12}-20\,T_{4,13}=0$$

$$T_{5,15}+T_{5,13}+T_{3,15}+T_{3,13}+4.000\,T_{5,14}+4.000\,T_{3,14}+4.000\,T_{4,15}+4.000\,T_{4,13}-20\,T_{4,14}=0$$

$$477.5+T_{5,14}+T_{3,14}+4.000\,T_{5,15}+4.000\,T_{3,15}+4.000\,T_{4,14}-20\,T_{4,15}=0$$

$$T_{6,3}+T_{4,3}+4.000\,T_{6,2}+4.000\,T_{4,2}+4.000\,T_{5,3}-20\,T_{5,2}=0$$

$$T_{6,4}+T_{6,2}+T_{4,4}+T_{4,2}+4.000\,T_{6,3}+4.000\,T_{4,3}+4.000\,T_{5,4}+4.000\,T_{5,2}-20\,T_{5,3}=0$$

$$T_{6,5} + T_{6,3} + T_{4,5} + T_{4,3} + 4.000\,T_{6,4} + 4.000\,T_{4,4} + 4.000\,T_{5,5} + 4.000\,T_{5,3} - 20\,T_{5,4} = 0$$

$$T_{6,6} + T_{6,4} + T_{4,6} + T_{4,4} + 4.000\,T_{6,5} + 4.000\,T_{4,5} + 4.000\,T_{5,6} + 4.000\,T_{5,4} - 20\,T_{5,5} = 0$$

$$T_{6,7} + T_{6,5} + T_{4,7} + T_{4,5} + 4.000\,T_{6,6} + 4.000\,T_{4,6} + 4.000\,T_{5,7} + 4.000\,T_{5,5} - 20\,T_{5,6} = 0$$

$$T_{6,8} + T_{6,6} + T_{4,8} + T_{4,6} + 4.000\,T_{6,7} + 4.000\,T_{4,7} + 4.000\,T_{5,8} + 4.000\,T_{5,6} - 20\,T_{5,7} = 0$$

$$T_{6,9} + T_{6,7} + T_{4,9} + T_{4,7} + 4.000\,T_{6,8} + 4.000\,T_{4,8} + 4.000\,T_{5,9} + 4.000\,T_{5,7} - 20\,T_{5,8} = 0$$

$$T_{6,10} + T_{6,8} + T_{4,10} + T_{4,8} + 4.000\,T_{6,9} + 4.000\,T_{4,9} + 4.000\,T_{5,10} + 4.000\,T_{5,8} - 20\,T_{5,9} = 0$$

$$T_{6,11} + T_{6,9} + T_{4,11} + T_{4,9} + 4.000\,T_{6,10} + 4.000\,T_{4,10} + 4.000\,T_{5,11} + 4.000\,T_{5,9} - 20\,T_{5,10} = 0$$

$$T_{6,12} + T_{6,10} + T_{4,12} + T_{4,10} + 4.000\,T_{6,11} + 4.000\,T_{4,11} + 4.000\,T_{5,12} + 4.000\,T_{5,10} - 20\,T_{5,11} = 0$$

$$T_{6,13} + T_{6,11} + T_{4,13} + T_{4,11} + 4.000\,T_{6,12} + 4.000\,T_{4,12} + 4.000\,T_{5,13} + 4.000\,T_{5,11} - 20\,T_{5,12} = 0$$

$$T_{6,14} + T_{6,12} + T_{4,14} + T_{4,12} + 4.000\,T_{6,13} + 4.000\,T_{4,13} + 4.000\,T_{5,14} + 4.000\,T_{5,12} - 20\,T_{5,13} = 0$$

$$T_{6,15} + T_{6,13} + T_{4,15} + T_{4,13} + 4.000\,T_{6,14} + 4.000\,T_{4,14} + 4.000\,T_{5,15} + 4.000\,T_{5,13} - 20\,T_{5,14} = 0$$

$$561.3 + T_{6,14} + T_{4,14} + 4.000\,T_{6,15} + 4.000\,T_{4,15} + 4.000\,T_{5,14} - 20\,T_{5,15} = 0$$

$$T_{7,3} + T_{5,3} + 4.000\,T_{7,2} + 4.000\,T_{5,2} + 4.000\,T_{6,3} - 20\,T_{6,2} = 0$$

$$T_{7,4} + T_{7,2} + T_{5,4} + T_{5,2} + 4.000\,T_{7,3} + 4.000\,T_{5,3} + 4.000\,T_{6,4} + 4.000\,T_{6,2} - 20\,T_{6,3} = 0$$

$$T_{7,5} + T_{7,3} + T_{5,5} + T_{5,3} + 4.000\,T_{7,4} + 4.000\,T_{5,4} + 4.000\,T_{6,5} + 4.000\,T_{6,3} - 20\,T_{6,4} = 0$$

$$T_{7,6} + T_{7,4} + T_{5,6} + T_{5,4} + 4.000\,T_{7,5} + 4.000\,T_{5,5} + 4.000\,T_{6,6} + 4.000\,T_{6,4} - 20\,T_{6,5} = 0$$

$$T_{7,7} + T_{7,5} + T_{5,7} + T_{5,5} + 4.000\,T_{7,6} + 4.000\,T_{5,6} + 4.000\,T_{6,7} + 4.000\,T_{6,5} - 20\,T_{6,6} = 0$$

$$T_{7,8} + T_{7,6} + T_{5,8} + T_{5,6} + 4.000\,T_{7,7} + 4.000\,T_{5,7} + 4.000\,T_{6,8} + 4.000\,T_{6,6} - 20\,T_{6,7} = 0$$

$$T_{7,9} + T_{7,7} + T_{5,9} + T_{5,7} + 4.000\,T_{7,8} + 4.000\,T_{5,8} + 4.000\,T_{6,9} + 4.000\,T_{6,7} - 20\,T_{6,8} = 0$$

$$T_{7,10} + T_{7,8} + T_{5,10} + T_{5,8} + 4.000\,T_{7,9} + 4.000\,T_{5,9} + 4.000\,T_{6,10} + 4.000\,T_{6,8} - 20\,T_{6,9} = 0$$

$$T_{7,11} + T_{7,9} + T_{5,11} + T_{5,9} + 4.000\,T_{7,10} + 4.000\,T_{5,10} + 4.000\,T_{6,11} + 4.000\,T_{6,9} - 20\,T_{6,10} = 0$$

$$T_{7,12} + T_{7,10} + T_{5,12} + T_{5,10} + 4.000\,T_{7,11} + 4.000\,T_{5,11} + 4.000\,T_{6,12} + 4.000\,T_{6,10} - 20\,T_{6,11} = 0$$

$$T_{7,13} + T_{7,11} + T_{5,13} + T_{5,11} + 4.000\,T_{7,12} + 4.000\,T_{5,12} + 4.000\,T_{6,13}$$

$$
\begin{aligned}
&+4.000\,T_{6,11}-20\,T_{6,12}=0\\
&T_{7,14}+T_{7,12}+T_{5,14}+T_{5,12}+4.000\,T_{7,13}+4.000\,T_{5,13}+4.000\,T_{6,14}\\
&+4.000\,T_{6,12}-20\,T_{6,13}=0\\
&T_{7,15}+T_{7,13}+T_{5,15}+T_{5,13}+4.000\,T_{7,14}+4.000\,T_{5,14}+4.000\,T_{6,15}\\
&+4.000\,T_{6,13}-20\,T_{6,14}=0\\
&590.2+T_{7,14}+T_{5,14}+4.000\,T_{7,15}+4.000\,T_{5,15}+4.000\,T_{6,14}-20\,T_{6,15}=0\\
&T_{8,3}+T_{6,3}+4.000\,T_{8,2}+4.000\,T_{6,2}+4.000\,T_{7,3}-20\,T_{7,2}=0\\
&T_{8,4}+T_{8,2}+T_{6,4}+T_{6,2}+4.000\,T_{8,3}+4.000\,T_{6,3}+4.000\,T_{7,4}+4.000\,T_{7,2}\\
&-20\,T_{7,3}=0\\
&T_{8,5}+T_{8,3}+T_{6,5}+T_{6,3}+4.000\,T_{8,4}+4.000\,T_{6,4}+4.000\,T_{7,5}+4.000\,T_{7,3}\\
&-20\,T_{7,4}=0\\
&T_{8,6}+T_{8,4}+T_{6,6}+T_{6,4}+4.000\,T_{8,5}+4.000\,T_{6,5}+4.000\,T_{7,6}+4.000\,T_{7,4}\\
&-20\,T_{7,5}=0\\
&T_{8,7}+T_{8,5}+T_{6,7}+T_{6,5}+4.000\,T_{8,6}+4.000\,T_{6,6}+4.000\,T_{7,7}+4.000\,T_{7,5}\\
&-20\,T_{7,6}=0\\
&T_{8,8}+T_{8,6}+T_{6,8}+T_{6,6}+4.000\,T_{8,7}+4.000\,T_{6,7}+4.000\,T_{7,8}+4.000\,T_{7,6}\\
&-20\,T_{7,7}=0\\
&T_{8,9}+T_{8,7}+T_{6,9}+T_{6,7}+4.000\,T_{8,8}+4.000\,T_{6,8}+4.000\,T_{7,9}+4.000\,T_{7,7}\\
&-20\,T_{7,8}=0\\
&T_{8,10}+T_{8,8}+T_{6,10}+T_{6,8}+4.000\,T_{8,9}+4.000\,T_{6,9}+4.000\,T_{7,10}+4.000\,T_{7,8}\\
&-20\,T_{7,9}=0\\
&T_{8,11}+T_{8,9}+T_{6,11}+T_{6,9}+4.000\,T_{8,10}+4.000\,T_{6,10}+4.000\,T_{7,11}+4.000\,T_{7,9}\\
&-20\,T_{7,10}=0\\
&T_{8,12}+T_{8,10}+T_{6,12}+T_{6,10}+4.000\,T_{8,11}+4.000\,T_{6,11}+4.000\,T_{7,12}\\
&+4.000\,T_{7,10}-20\,T_{7,11}=0\\
&T_{8,13}+T_{8,11}+T_{6,13}+T_{6,11}+4.000\,T_{8,12}+4.000\,T_{6,12}+4.000\,T_{7,13}\\
&+4.000\,T_{7,11}-20\,T_{7,12}=0\\
&T_{8,14}+T_{8,12}+T_{6,14}+T_{6,12}+4.000\,T_{8,13}+4.000\,T_{6,13}+4.000\,T_{7,14}\\
&+4.000\,T_{7,12}-20\,T_{7,13}=0\\
&T_{8,15}+T_{8,13}+T_{6,15}+T_{6,13}+4.000\,T_{8,14}+4.000\,T_{6,14}+4.000\,T_{7,15}\\
&+4.000\,T_{7,13}-20\,T_{7,14}=0\\
&561.3+T_{8,14}+T_{6,14}+4.000\,T_{8,15}+4.000\,T_{6,15}+4.000\,T_{7,14}-20\,T_{7,15}=0\\
&T_{9,3}+T_{7,3}+4.000\,T_{9,2}+4.000\,T_{7,2}+4.000\,T_{8,3}-20\,T_{8,2}=0\\
&T_{9,4}+T_{9,2}+T_{7,4}+T_{7,2}+4.000\,T_{9,3}+4.000\,T_{7,3}+4.000\,T_{8,4}+4.000\,T_{8,2}\\
&-20\,T_{8,3}=0\\
&T_{9,5}+T_{9,3}+T_{7,5}+T_{7,3}+4.000\,T_{9,4}+4.000\,T_{7,4}+4.000\,T_{8,5}+4.000\,T_{8,3}\\
&-20\,T_{8,4}=0\\
&T_{9,6}+T_{9,4}+T_{7,6}+T_{7,4}+4.000\,T_{9,5}+4.000\,T_{7,5}+4.000\,T_{8,6}+4.000\,T_{8,4}\\
&-20\,T_{8,5}=0\\
&T_{9,7}+T_{9,5}+T_{7,7}+T_{7,5}+4.000\,T_{9,6}+4.000\,T_{7,6}+4.000\,T_{8,7}+4.000\,T_{8,5}\\
&-20\,T_{8,6}=0\\
&T_{9,8}+T_{9,6}+T_{7,8}+T_{7,6}+4.000\,T_{9,7}+4.000\,T_{7,7}+4.000\,T_{8,8}+4.000\,T_{8,6}\\
&-20\,T_{8,7}=0
\end{aligned}
$$

$$T_{9,9}+T_{9,7}+T_{7,9}+T_{7,7}+4.000\,T_{9,8}+4.000\,T_{7,8}+4.000\,T_{8,9}+4.000\,T_{8,7}-20\,T_{8,8}=0$$
$$T_{9,10}+T_{9,8}+T_{7,10}+T_{7,8}+4.000\,T_{9,9}+4.000\,T_{7,9}+4.000\,T_{8,10}+4.000\,T_{8,8}-20\,T_{8,9}=0$$
$$T_{9,11}+T_{9,9}+T_{7,11}+T_{7,9}+4.000\,T_{9,10}+4.000\,T_{7,10}+4.000\,T_{8,11}+4.000\,T_{8,9}-20\,T_{8,10}=0$$
$$T_{9,12}+T_{9,10}+T_{7,12}+T_{7,10}+4.000\,T_{9,11}+4.000\,T_{7,11}+4.000\,T_{8,12}+4.000\,T_{8,10}-20\,T_{8,11}=0$$
$$T_{9,13}+T_{9,11}+T_{7,13}+T_{7,11}+4.000\,T_{9,12}+4.000\,T_{7,12}+4.000\,T_{8,13}+4.000\,T_{8,11}-20\,T_{8,12}=0$$
$$T_{9,14}+T_{9,12}+T_{7,14}+T_{7,12}+4.000\,T_{9,13}+4.000\,T_{7,13}+4.000\,T_{8,14}+4.000\,T_{8,12}-20\,T_{8,13}=0$$
$$T_{9,15}+T_{9,13}+T_{7,15}+T_{7,13}+4.000\,T_{9,14}+4.000\,T_{7,14}+4.000\,T_{8,15}+4.000\,T_{8,13}-20\,T_{8,14}=0$$
$$477.4+T_{9,14}+T_{7,14}+4.000\,T_{9,15}+4.000\,T_{7,15}+4.000\,T_{8,14}-20\,T_{8,15}=0$$
$$T_{10,3}+T_{8,3}+4.000\,T_{10,2}+4.000\,T_{8,2}+4.000\,T_{9,3}-20\,T_{9,2}=0$$
$$T_{10,4}+T_{10,2}+T_{8,4}+T_{8,2}+4.000\,T_{10,3}+4.000\,T_{8,3}+4.000\,T_{9,4}+4.000\,T_{9,2}-20\,T_{9,3}=0$$
$$T_{10,5}+T_{10,3}+T_{8,5}+T_{8,3}+4.000\,T_{10,4}+4.000\,T_{8,4}+4.000\,T_{9,5}+4.000\,T_{9,3}-20\,T_{9,4}=0$$
$$T_{10,6}+T_{10,4}+T_{8,6}+T_{8,4}+4.000\,T_{10,5}+4.000\,T_{8,5}+4.000\,T_{9,6}+4.000\,T_{9,4}-20\,T_{9,5}=0$$
$$T_{10,7}+T_{10,5}+T_{8,7}+T_{8,5}+4.000\,T_{10,6}+4.000\,T_{8,6}+4.000\,T_{9,7}+4.000\,T_{9,5}-20\,T_{9,6}=0$$
$$T_{10,8}+T_{10,6}+T_{8,8}+T_{8,6}+4.000\,T_{10,7}+4.000\,T_{8,7}+4.000\,T_{9,8}+4.000\,T_{9,6}-20\,T_{9,7}=0$$
$$T_{10,9}+T_{10,7}+T_{8,9}+T_{8,7}+4.000\,T_{10,8}+4.000\,T_{8,8}+4.000\,T_{9,9}+4.000\,T_{9,7}-20\,T_{9,8}=0$$
$$T_{10,10}+T_{10,8}+T_{8,10}+T_{8,8}+4.000\,T_{10,9}+4.000\,T_{8,9}+4.000\,T_{9,10}+4.000\,T_{9,8}-20\,T_{9,9}=0$$
$$T_{10,11}+T_{10,9}+T_{8,11}+T_{8,9}+4.000\,T_{10,10}+4.000\,T_{8,10}+4.000\,T_{9,11}+4.000\,T_{9,9}-20\,T_{9,10}=0$$
$$T_{10,12}+T_{10,10}+T_{8,12}+T_{8,10}+4.000\,T_{10,11}+4.000\,T_{8,11}+4.000\,T_{9,12}+4.000\,T_{9,10}-20\,T_{9,11}=0$$
$$T_{10,13}+T_{10,11}+T_{8,13}+T_{8,11}+4.000\,T_{10,12}+4.000\,T_{8,12}+4.000\,T_{9,13}+4.000\,T_{9,11}-20\,T_{9,12}=0$$
$$T_{10,14}+T_{10,12}+T_{8,14}+T_{8,12}+4.000\,T_{10,13}+4.000\,T_{8,13}+4.000\,T_{9,14}+4.000\,T_{9,12}-20\,T_{9,13}=0$$
$$T_{10,15}+T_{10,13}+T_{8,15}+T_{8,13}+4.000\,T_{10,14}+4.000\,T_{8,14}+4.000\,T_{9,15}+4.000\,T_{9,13}-20\,T_{9,14}=0$$
$$346.6+T_{10,14}+T_{8,14}+4.000\,T_{10,15}+4.000\,T_{8,15}+4.000\,T_{9,14}-20\,T_{9,15}=0$$
$$T_{9,3}+4.000\,T_{9,2}+4.000\,T_{10,3}-20\,T_{10,2}=0$$
$$T_{9,4}+T_{9,2}+4.000\,T_{9,3}+4.000\,T_{10,4}+4.000\,T_{10,2}-20\,T_{10,3}=0$$

$$T_{9,5}+T_{9,3}+4.000\,T_{9,4}+4.000\,T_{10,5}+4.000\,T_{10,3}-20\,T_{10,4}=0$$
$$T_{9,6}+T_{9,4}+4.000\,T_{9,5}+4.000\,T_{10,6}+4.000\,T_{10,4}-20\,T_{10,5}=0$$
$$T_{9,7}+T_{9,5}+4.000\,T_{9,6}+4.000\,T_{10,7}+4.000\,T_{10,5}-20\,T_{10,6}=0$$
$$T_{9,8}+T_{9,6}+4.000\,T_{9,7}+4.000\,T_{10,8}+4.000\,T_{10,6}-20\,T_{10,7}=0$$
$$T_{9,9}+T_{9,7}+4.000\,T_{9,8}+4.000\,T_{10,9}+4.000\,T_{10,7}-20\,T_{10,8}=0$$
$$T_{9,10}+T_{9,8}+4.000\,T_{9,9}+4.000\,T_{10,10}+4.000\,T_{10,8}-20\,T_{10,9}=0$$
$$T_{9,11}+T_{9,9}+4.000\,T_{9,10}+4.000\,T_{10,11}+4.000\,T_{10,9}-20\,T_{10,10}=0$$
$$T_{9,12}+T_{9,10}+4.000\,T_{9,11}+4.000\,T_{10,12}+4.000\,T_{10,10}-20\,T_{10,11}=0$$
$$T_{9,13}+T_{9,11}+4.000\,T_{9,12}+4.000\,T_{10,13}+4.000\,T_{10,11}-20\,T_{10,12}=0$$
$$T_{9,14}+T_{9,12}+4.000\,T_{9,13}+4.000\,T_{10,14}+4.000\,T_{10,12}-20\,T_{10,13}=0$$
$$T_{9,15}+T_{9,13}+4.000\,T_{9,14}+4.000\,T_{10,15}+4.000\,T_{10,13}-20\,T_{10,14}=0$$
$$182.1+T_{9,14}+4.000\,T_{9,15}+4.000\,T_{10,14}-20\,T_{10,15}=0 \quad \textbf{(1.1.2)}$$

```
> Eqs := {seq(Eq[i], i=1..N)}:
> Tmps := [seq(Temps[i], i=1..N)];
```

$$Tmps := [T_{2,2}, T_{2,3}, T_{2,4}, T_{2,5}, T_{2,6}, T_{2,7}, T_{2,8}, T_{2,9}, T_{2,10}, T_{2,11}, T_{2,12}, T_{2,13}, T_{2,14}, T_{2,15}, T_{3,2}, T_{3,3}, T_{3,4}, T_{3,5}, T_{3,6}, T_{3,7}, T_{3,8}, T_{3,9}, T_{3,10}, T_{3,11}, T_{3,12}, T_{3,13}, T_{3,14}, T_{3,15}, T_{4,2}, T_{4,3}, T_{4,4}, T_{4,5}, T_{4,6}, T_{4,7}, T_{4,8}, T_{4,9}, T_{4,10}, T_{4,11}, T_{4,12}, T_{4,13}, T_{4,14}, T_{4,15}, T_{5,2}, T_{5,3}, T_{5,4}, T_{5,5}, T_{5,6}, T_{5,7}, T_{5,8}, T_{5,9}, T_{5,10}, T_{5,11}, T_{5,12}, T_{5,13}, T_{5,14}, T_{5,15}, T_{6,2}, T_{6,3}, T_{6,4}, T_{6,5}, T_{6,6}, T_{6,7}, T_{6,8}, T_{6,9}, T_{6,10}, T_{6,11}, T_{6,12}, T_{6,13}, T_{6,14}, T_{6,15}, T_{7,2}, T_{7,3}, T_{7,4}, T_{7,5}, T_{7,6}, T_{7,7}, T_{7,8}, T_{7,9}, T_{7,10}, T_{7,11}, T_{7,12}, T_{7,13}, T_{7,14}, T_{7,15}, T_{8,2}, T_{8,3}, T_{8,4}, T_{8,5}, T_{8,6}, T_{8,7}, T_{8,8}, T_{8,9}, T_{8,10}, T_{8,11}, T_{8,12}, T_{8,13}, T_{8,14}, T_{8,15}, T_{9,2}, T_{9,3}, T_{9,4}, T_{9,5}, T_{9,6}, T_{9,7}, T_{9,8}, T_{9,9}, T_{9,10}, T_{9,11}, T_{9,12}, T_{9,13}, T_{9,14}, T_{9,15}, T_{10,2}, T_{10,3}, T_{10,4}, T_{10,5}, T_{10,6}, T_{10,7}, T_{10,8}, T_{10,9}, T_{10,10}, T_{10,11}, T_{10,12}, T_{10,13}, T_{10,14}, T_{10,15}] \quad \textbf{(1.1.3)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2}=0.1773, T_{2,3}=0.3722, T_{2,4}=0.6042, T_{2,5}=0.8963, T_{2,6}=1.278, T_{2,7}=1.786, T_{2,8}=2.472, T_{2,9}=3.405, T_{2,10}=4.676, T_{2,11}=6.412, T_{2,12}=8.787, T_{2,13}=12.04, T_{2,14}=16.48, T_{2,15}=22.57, T_{3,2}=0.3372, T_{3,3}=0.7080, T_{3,4}=1.149, T_{3,5}=1.705, T_{3,6}=2.430, T_{3,7}=3.397, T_{3,8}=4.703, T_{3,9}=6.476, T_{3,10}=8.894, T_{3,11}=12.20, T_{3,12}=16.71, T_{3,13}=22.89, T_{3,14}=31.35, T_{3,15}=42.93, T_{4,2}=0.4642, T_{4,3}=0.9745, T_{4,4}=1.582, T_{4,5}=2.347, T_{4,6}=3.345, T_{4,7}=4.676, T_{4,8}=6.473, T_{4,9}=8.913, T_{4,10}=12.24, T_{4,11}=16.79, T_{4,12}=23.00, T_{4,13}=31.51, T_{4,14}=43.15, T_{4,15}=59.09, T_{5,2}=0.5457, T_{5,3}=1.146, T_{5,4}=1.860, T_{5,5}=2.759, T_{5,6}=3.932, T_{5,7}=5.497, T_{5,8}=7.609, T_{5,9}=10.48, T_{5,10}=14.39, T_{5,11}=19.73, T_{5,12}=27.04, T_{5,13}=37.04, T_{5,14}=50.73, T_{5,15}=69.46, T_{6,2}=0.5737, T_{6,3}=1.205, T_{6,4}=1.955, T_{6,5}=2.901, T_{6,6}=4.135, T_{6,7}=5.780, T_{6,8}=8.000, T_{6,9}=11.02, T_{6,10}=15.13, T_{6,11}=20.75, T_{6,12}=28.43, T_{6,13}=38.94, T_{6,14}=53.33, T_{6,15}=73.03, T_{7,2}=0.5457, T_{7,3}=1.146, T_{7,4}=1.860, T_{7,5}=2.759, T_{7,6}=3.932, T_{7,7}=5.497, T_{7,8}=7.609, T_{7,9}=10.48, T_{7,10}=14.39, T_{7,11}=19.73, T_{7,12}=27.04, T_{7,13}=37.04, T_{7,14}=50.72, T_{7,15}=69.45, T_{8,2}=0.4642, T_{8,3}=0.9745, T_{8,4}=1.582, T_{8,5}=2.347, T_{8,6}=3.345, T_{8,7}=4.676, T_{8,8}=6.472, T_{8,9}=8.913, T_{8,10}=12.24, T_{8,11}=16.78, T_{8,12}=23.00, T_{8,13}=31.50, T_{8,14}=43.14, T_{8,15}=59.07, T_{9,2}=0.3372, T_{9,3}=0.7080, T_{9,4}=1.149, T_{9,5} \quad \textbf{(1.1.4)}$$

$= 1.705, T_{9,6} = 2.430, T_{9,7} = 3.397, T_{9,8} = 4.702, T_{9,9} = 6.475, T_{9,10} = 8.893, T_{9,11} = 12.19, T_{9,12} = 16.71, T_{9,13} = 22.89, T_{9,14} = 31.34, T_{9,15} = 42.90, T_{10,2} = 0.1773, T_{10,3} = 0.3722, T_{10,4} = 0.6042, T_{10,5} = 0.8963, T_{10,6} = 1.278, T_{10,7} = 1.786, T_{10,8} = 2.472, T_{10,9} = 3.404, T_{10,10} = 4.675, T_{10,11} = 6.411, T_{10,12} = 8.784, T_{10,13} = 12.03, T_{10,14} = 16.47, T_{10,15} = 22.55]]$

Extraction des valeurs des températures:

```
> k := 1 :
  for i from 2 to i_max - 1 do
   for j from 2 to j_max - 1 do
       T[i, j] := rhs(SolT_{1, k});
       k := k + 1
   end do;
  end do
```

>

Calcul du nombre de listes NL et de leurs limites:

$$> NL := \frac{N}{j_{max} - 2};$$

$$NL := 9 \qquad \textbf{(1.1.5)}$$

Création des listes pour le tracé:

$$> GTemps := [seq([seq(T[i,j], j = 1..j_{max})], i = 1..i_{max})]$$

$GTemps := [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0.1773, 0.3722, 0.6042, 0.8963, 1.278, 1.786, 2.472, 3.405, 4.676, 6.412, 8.787, 12.04, 16.48, 22.57, 30.91], [0, 0.3372, 0.7080, 1.149, 1.705, 2.430, 3.397, 4.703, 6.476, 8.894, 12.20, 16.71, 22.89, 31.35, 42.93, 58.79], [0, 0.4642, 0.9745, 1.582, 2.347, 3.345, 4.676, 6.473, 8.913, 12.24, 16.79, 23.00, 31.51, 43.15, 59.09, 80.91], [0, 0.5457, 1.146, 1.860, 2.759, 3.932, 5.497, 7.609, 10.48, 14.39, 19.73, 27.04, 37.04, 50.73, 69.46, 95.12], [0, 0.5737, 1.205, 1.955, 2.901, 4.135, 5.780, 8.000, 11.02, 15.13, 20.75, 28.43, 38.94, 53.33, 73.03, 100.], [0, 0.5457, 1.146, 1.860, 2.759, 3.932, 5.497, 7.609, 10.48, 14.39, 19.73, 27.04, 37.04, 50.72, 69.45, 95.10], [0, 0.4642, 0.9745, 1.582, 2.347, 3.345, 4.676, 6.472, 8.913, 12.24, 16.78, 23.00, 31.50, 43.14, 59.07, 80.91], [0, 0.3372, 0.7080, 1.149, 1.705, 2.430, 3.397, 4.702, 6.475, 8.893, 12.19, 16.71, 22.89, 31.34, 42.90, 58.72], [0, 0.1773, 0.3722, 0.6042, 0.8963, 1.278, 1.786, 2.472, 3.404, 4.675, 6.411, 8.784, 12.03, 16.47, 22.55, 30.85], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]$ **(1.1.6)**

Calcul des isothermes à tracer:

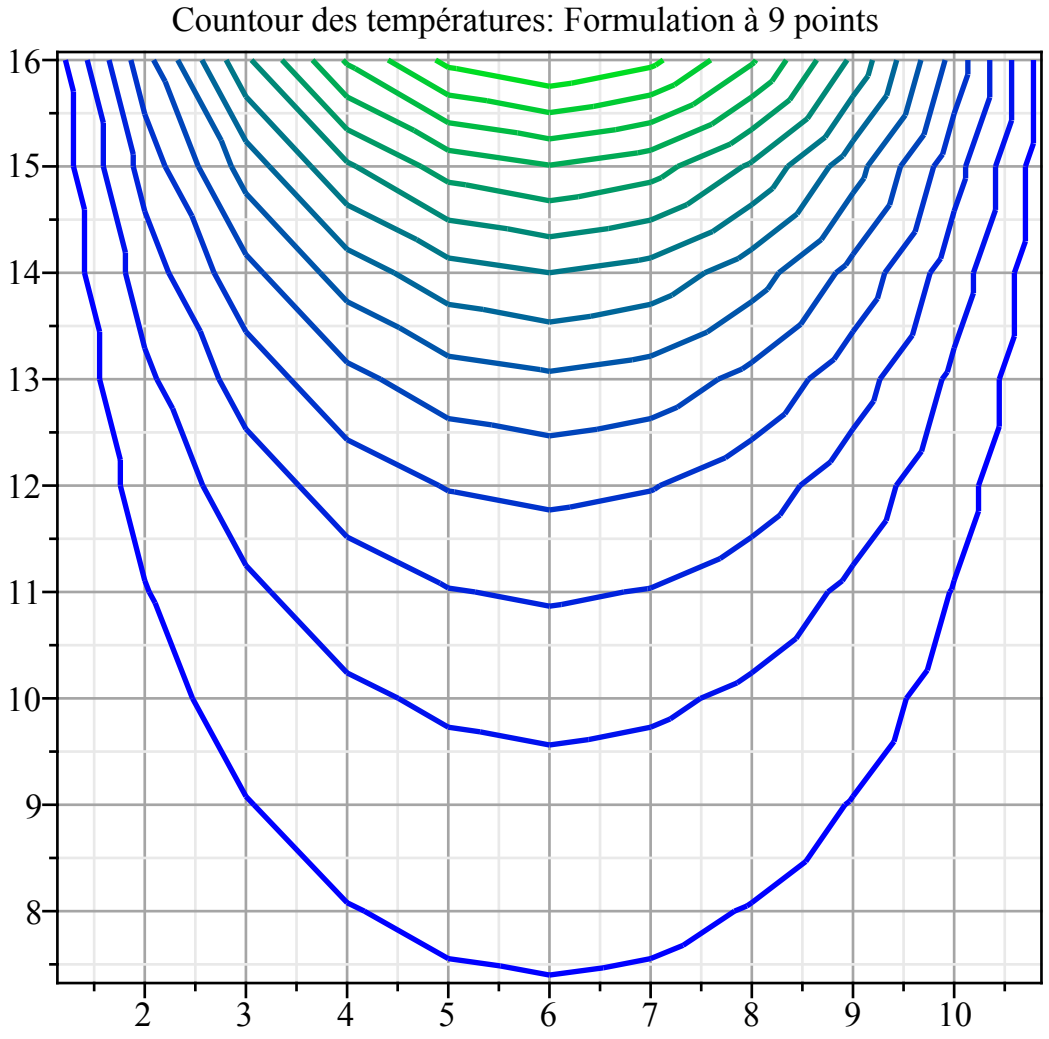
```
> TMax := max(seq(seq(T[i,j], j = 1..j_max), i = 1..i_max));
  TMin := min(seq(seq(T[i,j], j = 1..j_max), i = 1..i_max));
  DeltaT := evalf((TMax - TMin)/NbIso);
  for k from 1 to NbIso do Iso[k] := k·DeltaT end do
```

$$TMax := 100.$$
$$TMin := 0$$
$$DeltaT := 6.667$$
$$Iso_1 := 6.667$$
$$Iso_2 := 13.33$$
$$Iso_3 := 20.00$$
$$Iso_4 := 26.67$$
$$Iso_5 := 33.34$$
$$Iso_6 := 40.00$$

$$Iso_7 := 46.67$$
$$Iso_8 := 53.34$$
$$Iso_9 := 60.00$$
$$Iso_{10} := 66.67$$
$$Iso_{11} := 73.34$$
$$Iso_{12} := 80.00$$
$$Iso_{13} := 86.67$$
$$Iso_{14} := 93.34$$
$$Iso_{15} := 100.0 \qquad \textbf{(1.1.7)}$$

Tracé des isothermes:

```
> listcontplot(GTemps, title = "Countour des températures: Formulation à 9 points", axes
      = boxed, gridlines = true, thickness = 2, coloring = [blue, green], contours
      = [seq(Iso[k], k = 1 ..NbIso)])
```



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