Equation de Laplace 2D

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100,$$
$$T(0, y) = 60,$$
$$T(a, y) = 20.$$

```
> Digits := 4 :
```

Maillage et Conditions aux limites:

```
> ndx := 3; ndy := 3;
```

$$ndx := 3$$
$$ndy := 3 \qquad (1.1)$$

```
> i_max := ndx + 1 : j_max := ndy + 1 :
```

Nombre d'équations:

```
> N := (i_max - 2)·(j_max - 2);
```

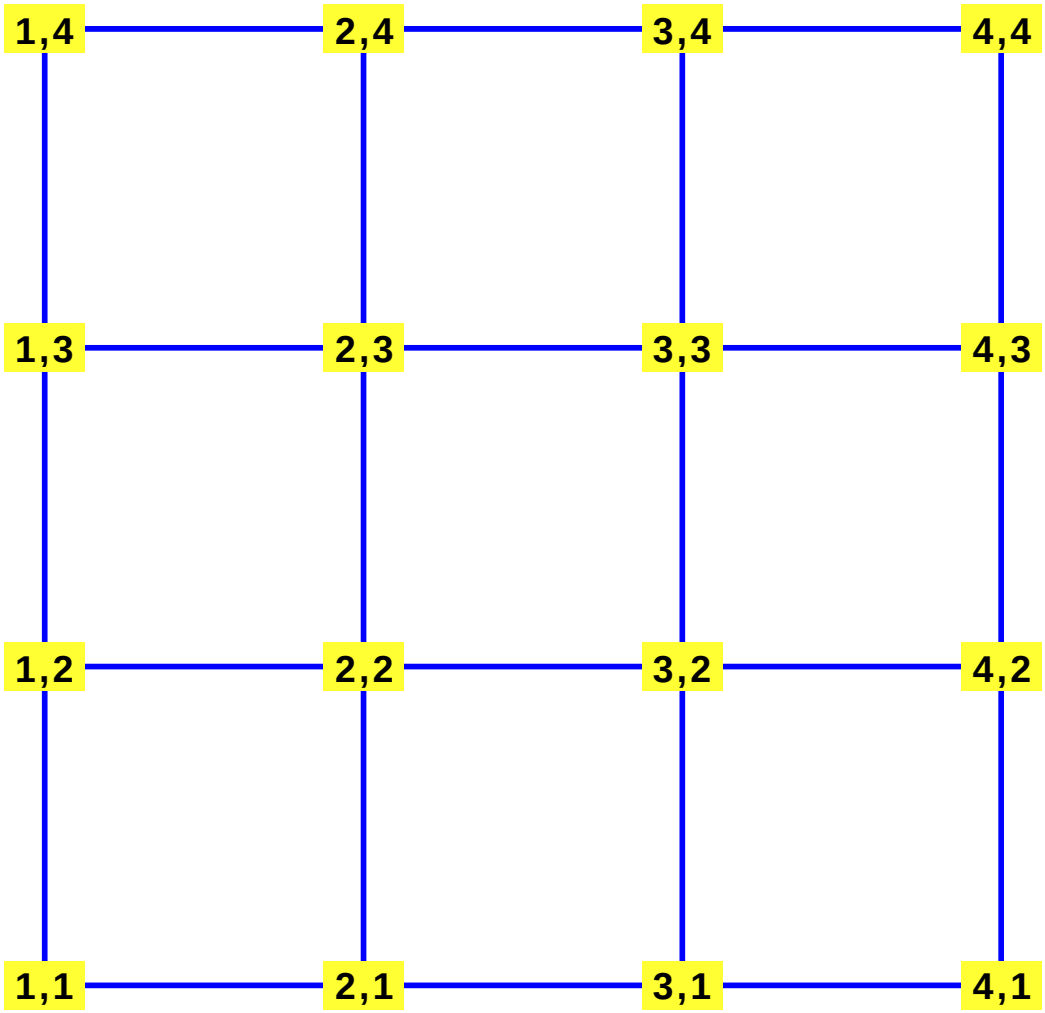
$$N := 4 \qquad (1.2)$$

Maillage:

```
> with(GraphTheory) : with(SpecialGraphs) :
> G := GridGraph( i_max, j_max )
```

$$G := \text{Graph 1: an undirected unweighted graph with 16 vertices and 24 edge(s)} \quad \textbf{(1.3)}$$

```
> DrawGraph(G)
```



Conditions aux Limites:

```
> Tb := 0.; Tg := 60; Td := 20; Th := 100.;
```

$$Tb := 0.$$
$$Tg := 60$$
$$Td := 20$$
$$Th := 100. \quad \textbf{(1.4)}$$

```
> for i from 2 to i_max − 1 do T[i, 1] := Tb end do; T[1, 1] := 0.5·(Tb + Tg); T[ i_max, 1 ]
      := 0.5·(Tb + Td);
```

$$T_{2,1} := 0.$$
$$T_{3,1} := 0.$$
$$T_{1,1} := 30.0$$
$$T_{4,1} := 10.0 \quad \textbf{(1.5)}$$

```
> for i from 2 to i_max − 1 do T[i, j_max] := Th end do; T[1, j_max] := 0.5·(Tg + Th); T[ i_max,
      j_max] := 0.5·(Td + Th);
```

$$T_{2,4} := 100.$$
$$T_{3,4} := 100.$$
$$T_{1,4} := 80.0$$

(1.6)

$$T_{4,4} := 60.0 \quad (1.6)$$

```
> for j from 2 to j_max - 1 do T[1, j] := Tg end do;
```

$$T_{1,2} := 60$$
$$T_{1,3} := 60 \quad (1.7)$$

```
> for j from 2 to j_max - 1 do T[i_max, j] := Td end do;
```

$$T_{4,2} := 20$$
$$T_{4,3} := 20 \quad (1.8)$$

```
>
```

Solution discrétisée par la formulation à 9 points et β=1:

```
> Restart:
> β := 1;
```

$$\beta := 1 \quad (2.1)$$

```
> k := 1:
```

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Eq[k] := T[i+1, j+1] + T[i+1, j-1] + T[i-1, j+1] + T[i-1, j-1]
      + 2·(5 - β^2)/(1 + β^2)·(T[i+1, j] + T[i-1, j]) + 2·(5·β^2 - 1)/(1 + β^2)·(T[i, j+1] + T[i, j-1])
      - 20·T[i, j] = 0;
      Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,3} + 330.0 + 4\,T_{3,2} + 4\,T_{2,3} - 20\,T_{2,2} = 0$$
$$110.0 + T_{2,3} + 4\,T_{2,2} + 4\,T_{3,3} - 20\,T_{3,2} = 0$$
$$880.0 + T_{3,2} + 4\,T_{3,3} + 4\,T_{2,2} - 20\,T_{2,3} = 0$$
$$660.0 + T_{2,2} + 4\,T_{2,3} + 4\,T_{3,2} - 20\,T_{3,3} = 0 \quad (2.2)$$

```
> Eqs := {seq(Eq[i], i = 1..N)}:
> Tmps := [seq(Temps[i], i = 1..N)]:
```

Solution:

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 37.14,\ T_{3,2} = 26.67,\ T_{2,3} = 63.33,\ T_{3,3} = 52.86]] \quad (2.3)$$

Solution discrétisée par la formulation à 9 points et β=0.25:

```
> Restart:
> β := 0.25;
```

$$\beta := 0.25 \quad (3.1)$$

```
> k := 1:
```

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Eq[k] := T[i+1,j+1] + T[i+1,j-1] + T[i-1,j+1] + T[i-1,j-1]
      + 2·(5 - β^2)/(1 + β^2)·(T[i+1,j] + T[i-1,j]) + 2·(5·β^2 - 1)/(1 + β^2)·(T[i,j+1] + T[i,j-1])
      - 20·T[i,j] = 0;
      Temps[k] := T[i,j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,3} + 648.0 + 9.300\,T_{3,2} - 1.295\,T_{2,3} - 20\,T_{2,2} = 0$$
$$216.0 + T_{2,3} + 9.300\,T_{2,2} - 1.295\,T_{3,3} - 20\,T_{3,2} = 0$$
$$668.5 + T_{3,2} + 9.300\,T_{3,3} - 1.295\,T_{2,2} - 20\,T_{2,3} = 0$$
$$236.5 + T_{2,2} + 9.300\,T_{2,3} - 1.295\,T_{3,2} - 20\,T_{3,3} = 0 \qquad \textbf{(3.2)}$$

```
> Eqs := {seq(Eq[i], i = 1..N)}:
> Tmps := [seq(Temps[i], i = 1..N)]:
```

Solution:

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 46.07,\ T_{3,2} = 32.40,\ T_{2,3} = 48.04,\ T_{3,3} = 34.37]] \qquad \textbf{(3.3)}$$

```
>
```

Solution discrétisée par la formulation à 9 points et β=4:

```
> Restart:
> β := 4;
```

$$\beta := 4 \qquad \textbf{(4.1)}$$

```
> k := 1:
```

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Eq[k] := T[i+1,j+1] + T[i+1,j-1] + T[i-1,j+1] + T[i-1,j-1]
      + 2·(5 - β^2)/(1 + β^2)·(T[i+1,j] + T[i-1,j]) + 2·(5·β^2 - 1)/(1 + β^2)·(T[i,j+1] + T[i,j-1])
      - 20·T[i,j] = 0;
      Temps[k] := T[i,j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,3} + 12.35 - \frac{22}{17}\,T_{3,2} + \frac{158}{17}\,T_{2,3} - 20\,T_{2,2} = 0$$
$$4.12 + T_{2,3} - \frac{22}{17}\,T_{2,2} + \frac{158}{17}\,T_{3,3} - 20\,T_{3,2} = 0$$

$$1092. + T_{3,2} - \frac{22}{17} T_{3,3} + \frac{158}{17} T_{2,2} - 20\, T_{2,3} = 0$$

$$1084. + T_{2,2} - \frac{22}{17} T_{2,3} + \frac{158}{17} T_{3,2} - 20\, T_{3,3} = 0 \qquad \textbf{(4.2)}$$

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)] :
```

Solution:

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 33.13, T_{3,2} = 32.35, T_{2,3} = 67.31, T_{3,3} = 66.53]] \qquad \textbf{(4.3)}$$

```
>
```