Equation de Laplace 2D

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 200,$$
$$T(x, b) = 0,$$
$$T(0, y) = 0,$$
$$T(a, y) = 100$$

Solution discrétisée (formulation en 5 points):

```
> Restart:
> a := 1; b := 1; ndx := 4; ndy := 4
```

$$a := 1$$
$$b := 1$$
$$ndx := 4$$
$$ndy := 4$$

(1.1)

```
> Δx := a/ndx ; Δy := b/ndy; β := Δx/Δy;
```

$$\Delta x := \frac{1}{4}$$
$$\Delta y := \frac{1}{4}$$

(1.2)

$$\beta := 1 \tag{1.2}$$

```
> i_max := ndx + 1; j_max := ndy + 1;
```

$$i_{max} := 5$$
$$j_{max} := 5 \tag{1.3}$$

Nombre d'équations:

```
> N := (i_max - 2)·(j_max - 2)
```

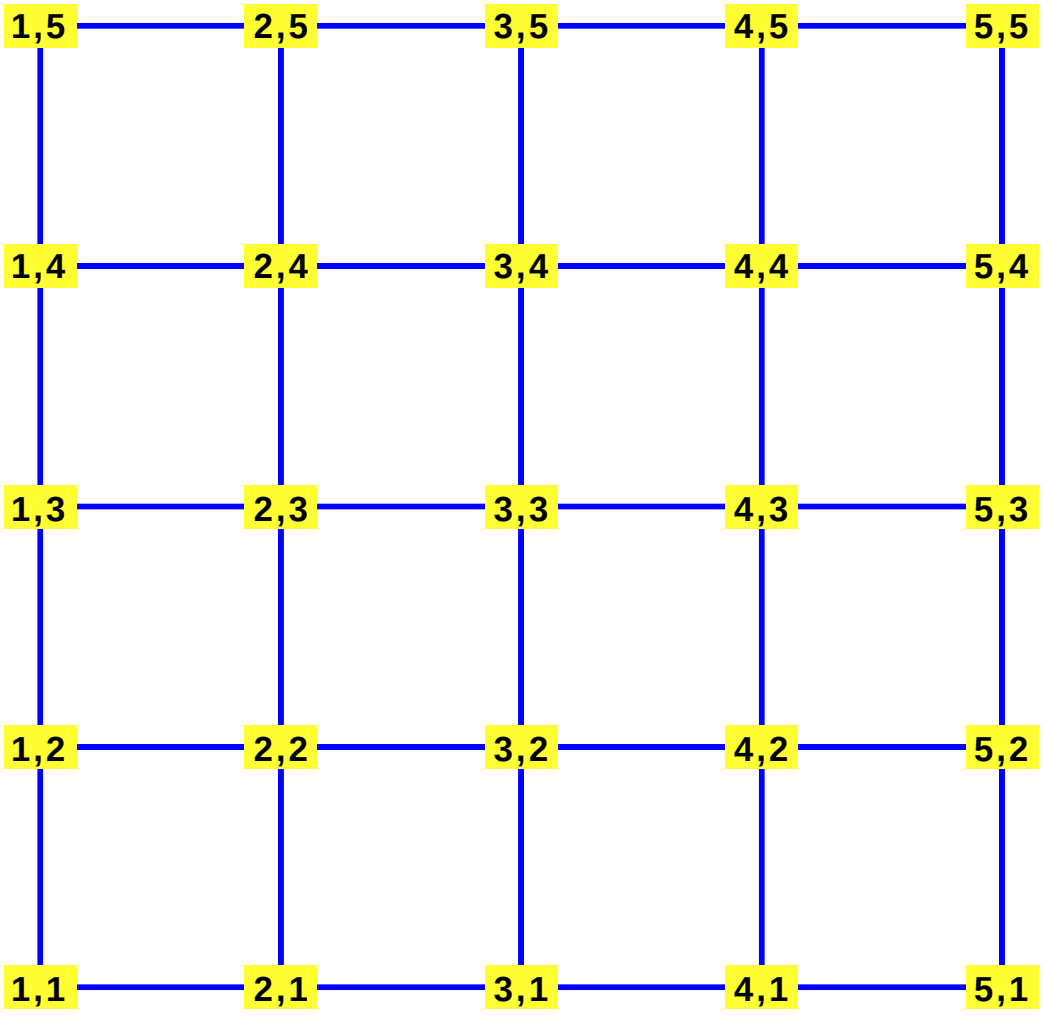
$$N := 9 \tag{1.4}$$

Maillage:

```
> with(GraphTheory) : with(SpecialGraphs) :
> G := GridGraph(i_max, j_max)
```

$$G := \text{Graph 1: an undirected unweighted graph with 25 vertices and 40 edge(s)} \tag{1.5}$$

```
> DrawGraph(G)
```



Conditions aux Limites:

```
> for i from 1 to i_max do T[i, 1] := 200 end do;
```

$$T_{1,1} := 200$$
$$T_{2,1} := 200$$
$$T_{3,1} := 200$$
$$T_{4,1} := 200$$
$$T_{5,1} := 200 \tag{1.6}$$

```
> for i from 1 to i_max do T[i, j_max] := 0 end do;
```

$$T_{1,5} := 0$$
$$T_{2,5} := 0$$
$$T_{3,5} := 0$$
$$T_{4,5} := 0$$
$$T_{5,5} := 0 \quad \textbf{(1.7)}$$

> **for** j **from** 1 **to** j_{max} **do** $T[1, j] := 0$ **end do**;

$$T_{1,1} := 0$$
$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0$$
$$T_{1,5} := 0 \quad \textbf{(1.8)}$$

> **for** j **from** 1 **to** j_{max} **do** $T[i_{max}, j] := 100$ **end do**;

$$T_{5,1} := 100$$
$$T_{5,2} := 100$$
$$T_{5,3} := 100$$
$$T_{5,4} := 100$$
$$T_{5,5} := 100 \quad \textbf{(1.9)}$$

> $k := 1$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for j from 2 to jmax - 1 do
    for i from 2 to imax - 1 do
      Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2 · (T[i, j + 1] + T[i, j - 1]) - 2 · (1
    + β^2) · T[i, j] = 0;
      Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Ecriture du système d'équations:

> **for** k **from** 1 **to** N **do** $Eq[k]$ **end do**;

$$T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0$$
$$T_{4,2} + T_{2,2} + T_{3,3} + 200 - 4\,T_{3,2} = 0$$
$$300 + T_{3,2} + T_{4,3} - 4\,T_{4,2} = 0$$
$$T_{3,3} + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0$$
$$T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0$$
$$100 + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0$$
$$T_{3,4} + T_{2,3} - 4\,T_{2,4} = 0$$
$$T_{4,4} + T_{2,4} + T_{3,3} - 4\,T_{3,4} = 0$$
$$100 + T_{3,4} + T_{4,3} - 4\,T_{4,4} = 0 \quad \textbf{(1.1.2)}$$

> $Eqs := \{seq(Eq[i], i = 1..N)\}$:

> $Tmps := [seq(Temps[i], i = 1..N)]$;

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}] \quad \textbf{(1.1.3)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := \left[\left[T_{2,2} = \frac{650}{7}, T_{3,2} = \frac{3475}{28}, T_{4,2} = \frac{900}{7}, T_{2,3} = \frac{1325}{28}, T_{3,3} = 75, T_{4,3} = \frac{2525}{28}, T_{2,4} = \frac{150}{7}, T_{3,4} = \frac{1075}{28}, T_{4,4} = \frac{400}{7}\right]\right] \quad \textbf{(1.1.4)}$$

```
> Solution := evalf(SolT);
```

$$Solution := [[T_{2,2} = 92.85714286, T_{3,2} = 124.1071429, T_{4,2} = 128.5714286, T_{2,3} = 47.32142857, T_{3,3} = 75., T_{4,3} = 90.17857143, T_{2,4} = 21.42857143, T_{3,4} = 38.39285714, T_{4,4} = 57.14285714]] \quad \textbf{(1.1.5)}$$

```
> Sys := [seq(Eq[i], i = 1 ..N)];
```

$$Sys := [T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0,\ T_{4,2} + T_{2,2} + T_{3,3} + 200 - 4\,T_{3,2} = 0,\ 300 + T_{3,2} + T_{4,3} - 4\,T_{4,2} = 0,\ T_{3,3} + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0,\ T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0,\ 100 + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0,\ T_{3,4} + T_{2,3} - 4\,T_{2,4} = 0,\ T_{4,4} + T_{2,4} + T_{3,3} - 4\,T_{3,4} = 0,\ 100 + T_{3,4} + T_{4,3} - 4\,T_{4,4} = 0] \quad \textbf{(1.1.6)}$$

```
> Var := [seq(Temps[i], i = 1 ..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}] \quad \textbf{(1.1.7)}$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := \left[\begin{matrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{matrix}\right], \left[\begin{matrix} -200 \\ -200 \\ -300 \\ 0 \\ 0 \\ -100 \\ 0 \\ 0 \\ -100 \end{matrix}\right] \quad \textbf{(1.1.8)}$$