Equation de Laplace 2D

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Détermination de la temperature $T(x, \mathrm{y})$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, \mathrm{y}) + \frac{\partial^2}{\partial y^2} T(x, \mathrm{y}) = 0$$

Conditions aux limites (C.L):

$$T(\mathrm{x}, 0) = 200,$$
$$T(\mathrm{x}, \mathrm{b}) = 0,$$
$$T(0, \mathrm{y}) = 0,$$
$$T(\mathrm{a}, \mathrm{y}) = 100$$

Solution discrétisée (formulation en 5 points):

> *Restart* :

> $a := 1; b := 1; ndx := 3; ndy := 3$

$$a := 1$$
$$b := 1$$
$$ndx := 3$$
$$ndy := 3 \qquad \textbf{(1.1)}$$

> $\Delta x := \frac{a}{ndx}; \Delta y := \frac{b}{ndy}; \beta := \frac{\Delta x}{\Delta y};$

$$\Delta x := \frac{1}{3}$$
$$\Delta y := \frac{1}{3} \qquad \textbf{(1.2)}$$

$$\beta := 1 \qquad (1.2)$$

> $i_{\max} := ndx + 1;\ j_{\max} := ndy + 1;$

$$i_{max} := 4$$
$$j_{max} := 4 \qquad (1.3)$$

Nombre d'équations:

> $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$

$$N := 4 \qquad (1.4)$$

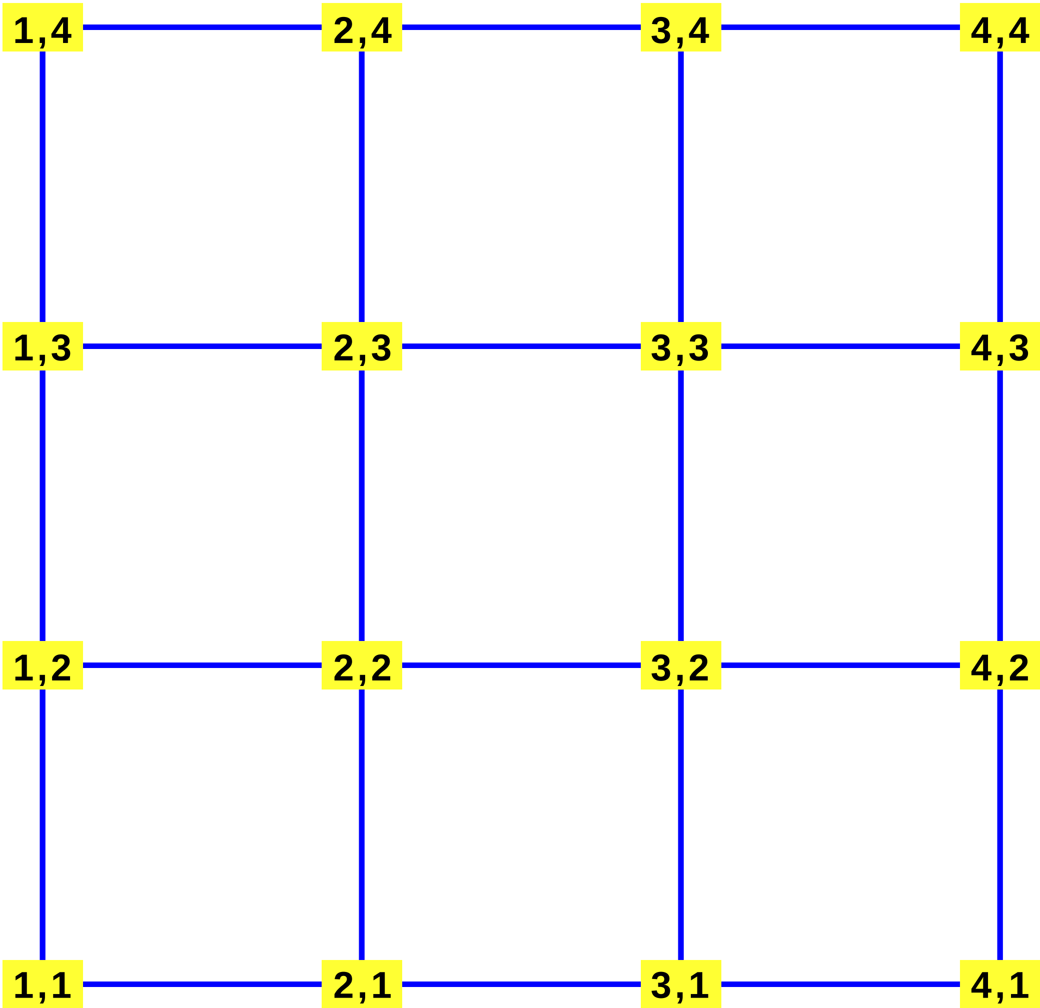
Maillage:

> $with(GraphTheory) : with(SpecialGraphs) :$

> $G := GridGraph(i_{\max}, j_{\max})$

$$G := \textit{Graph 1: an undirected unweighted graph with 16 vertices and 24 edge(s)} \qquad (1.5)$$

> $DrawGraph(G)$



Conditions aux Limites:

> **for** i **from** 1 **to** $i_{\max}$ **do** $T[i, 1] := 200$ **end do**;

$$T_{1,\,1} := 200$$
$$T_{2,\,1} := 200$$
$$T_{3,\,1} := 200$$
$$T_{4,\,1} := 200 \qquad (1.6)$$

> **for** i **from** 1 **to** $i_{\max}$ **do** $T[i, j_{\max}] := 0$ **end do**;

$$T_{1,\,4} := 0$$

$$T_{2,4} := 0$$
$$T_{3,4} := 0$$
$$T_{4,4} := 0 \quad (1.7)$$

```
> for j from 1 to j_max do T[1, j] := 0 end do;
```

$$T_{1,1} := 0$$
$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0 \quad (1.8)$$

```
> for j from 1 to j_max do T[i_max, j] := 100 end do;
```

$$T_{4,1} := 100$$
$$T_{4,2} := 100$$
$$T_{4,3} := 100$$
$$T_{4,4} := 100 \quad (1.9)$$

```
> k := 1
```

$$k := 1 \quad (1.1.1)$$

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max -1 do
        Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2·(T[i, j + 1] + T[i, j-1]) - 2·(1
      + β^2)·T[i, j] = 0;
        Temps[k] := T[i, j];
    k := k + 1
   end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0$$
$$300 + T_{2,2} + T_{3,3} - 4\,T_{3,2} = 0$$
$$T_{3,3} + T_{2,2} - 4\,T_{2,3} = 0$$
$$100 + T_{2,3} + T_{3,2} - 4\,T_{3,3} = 0 \quad (1.1.2)$$

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)];
```

$$Tmps := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}] \quad (1.1.3)$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := \left[\left[T_{2,2} = \frac{175}{2}, T_{3,2} = \frac{225}{2}, T_{2,3} = \frac{75}{2}, T_{3,3} = \frac{125}{2}\right]\right] \quad (1.1.4)$$

```
> Solution := evalf(SolT);
```

$$Solution := [[T_{2,2} = 87.50000000, T_{3,2} = 112.5000000, T_{2,3} = 37.50000000, T_{3,3} = 62.50000000]] \quad (1.1.5)$$

```
> Sys := [seq(Eq[i], i = 1 ..N)];
```

$$Sys := [T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0, 300 + T_{2,2} + T_{3,3} - 4\,T_{3,2} = 0, T_{3,3} + T_{2,2} - 4\,T_{2,3} = 0, 100 + T_{2,3} + T_{3,2} - 4\,T_{3,3} = 0] \quad (1.1.6)$$

```
> Var := [seq(Temps[i], i = 1 ..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}] \quad \textbf{(1.1.7)}$$

> *with*(*LinearAlgebra*) :

> *A*, *b* := *GenerateMatrix*(*Sys*, *Var*);

$$A, b := \begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -4 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -4 \end{bmatrix}, \begin{bmatrix} -200 \\ -300 \\ 0 \\ -100 \end{bmatrix} \quad \textbf{(1.1.8)}$$