Equation de Laplace 2D

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Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 200,$$
$$T(x, b) = 0,$$
$$T(0, y) = 0,$$
$$T(a, y) = 100$$

Solution discrétisée (formulation en 5 points):

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> Restart:
> a := 5; b := 3; ndx := 5; ndy := 3
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$$a := 5$$
$$b := 3$$
$$ndx := 5$$
$$ndy := 3 \qquad \textbf{(1.1)}$$

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> Δx := a/ndx ; Δy := b/ndy; β := Δx/Δy;
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$$\Delta x := 1$$
$$\Delta y := 1$$
$$\beta := 1 \qquad \textbf{(1.2)}$$

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> i_max := ndx + 1; j_max := ndy + 1;
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$$i_{max} := 6$$
$$j_{max} := 4 \qquad \textbf{(1.3)}$$

Nombre d'équations:

$> N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 8 \qquad \textbf{(1.4)}$$

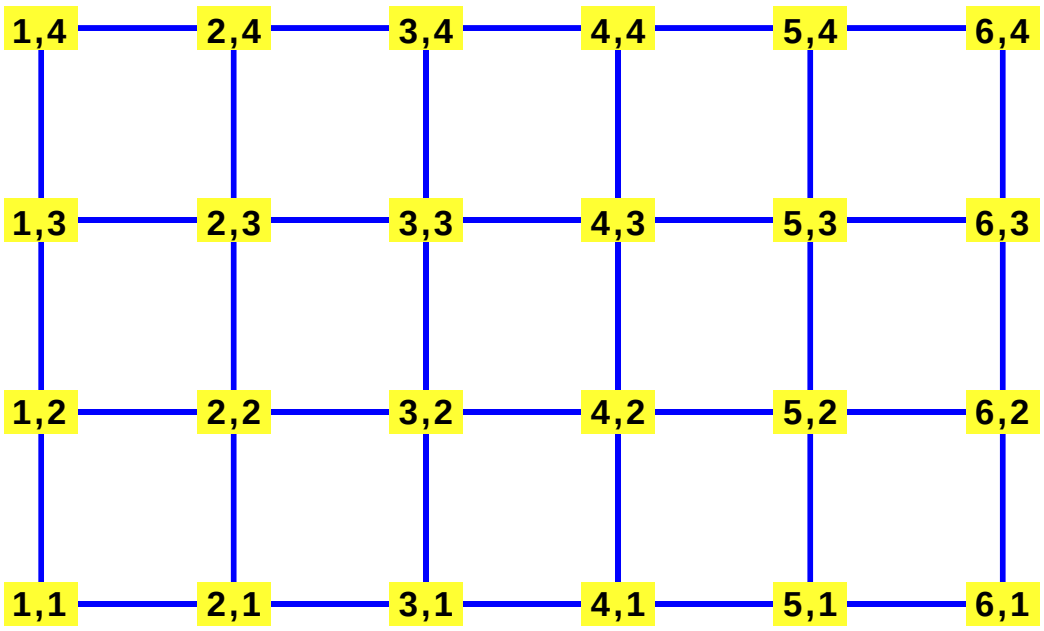
Maillage:

$> with(GraphTheory) : with(SpecialGraphs) :$

$> G := GridGraph(i_{max}, j_{max})$

$$G := Graph\ 1:\ an\ undirected\ unweighted\ graph\ with\ 24\ vertices\ and\ 38\ edge(s) \qquad \textbf{(1.5)}$$

$> DrawGraph(G)$



Conditions aux Limites:

$>$ **for** i **from** 1 **to** i_{max} **do** $T[i, 1] := 200$ **end do**;

$$T_{1,1} := 200$$
$$T_{2,1} := 200$$
$$T_{3,1} := 200$$
$$T_{4,1} := 200$$
$$T_{5,1} := 200$$
$$T_{6,1} := 200 \qquad \textbf{(1.6)}$$

$>$ **for** i **from** 1 **to** i_{max} **do** $T[i, j_{max}] := 0$ **end do**;

$$T_{1,4} := 0$$

$$T_{2,4} := 0$$
$$T_{3,4} := 0$$
$$T_{4,4} := 0$$
$$T_{5,4} := 0$$
$$T_{6,4} := 0 \quad \textbf{(1.7)}$$

> **for** j **from** 1 **to** j_{max} **do** $T[1, j] := 0$ **end do**;

$$T_{1,1} := 0$$
$$T_{1,2} := 0$$
$$T_{1,3} := 0$$
$$T_{1,4} := 0 \quad \textbf{(1.8)}$$

> **for** j **from** 1 **to** j_{max} **do** $T[i_{max}, j] := 100$ **end do**;

$$T_{6,1} := 100$$
$$T_{6,2} := 100$$
$$T_{6,3} := 100$$
$$T_{6,4} := 100 \quad \textbf{(1.9)}$$

> $k := 1$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

> **for** j **from** 2 **to** $j_{max} - 1$ **do**
for i **from** 2 **to** $i_{max} - 1$ **do**
$Eq[k] := T[i+1, j] + T[i-1, j] + \beta^2 \cdot (T[i, j+1] + T[i, j-1]) - 2 \cdot (1 + \beta^2) \cdot T[i, j] = 0$;
$Temps[k] := T[i, j]$;
$k := k + 1$
end do;
end do;

Ecriture du système d'équations:

> **for** k **from** 1 **to** N **do** $Eq[k]$ **end do**;

$$T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0$$
$$T_{4,2} + T_{2,2} + T_{3,3} + 200 - 4\,T_{3,2} = 0$$
$$T_{5,2} + T_{3,2} + T_{4,3} + 200 - 4\,T_{4,2} = 0$$
$$300 + T_{4,2} + T_{5,3} - 4\,T_{5,2} = 0$$
$$T_{3,3} + T_{2,2} - 4\,T_{2,3} = 0$$
$$T_{4,3} + T_{2,3} + T_{3,2} - 4\,T_{3,3} = 0$$
$$T_{5,3} + T_{3,3} + T_{4,2} - 4\,T_{4,3} = 0$$
$$100 + T_{4,3} + T_{5,2} - 4\,T_{5,3} = 0 \quad \textbf{(1.1.2)}$$

> $Eqs := \{seq(Eq[i], i = 1..N)\}$:

> $Tmps := [seq(Temps[i], i = 1..N)]$;

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{5,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{5,3}] \quad \textbf{(1.1.3)}$$

> $SolT := solve(Eqs, Tmps)$;

$$SolT := \left[\left[T_{2,2} = \frac{18420}{209}, T_{3,2} = \frac{24460}{209}, T_{4,2} = \frac{26360}{209}, T_{5,2} = \frac{26020}{209}, T_{2,3}\right.\right. \quad \textbf{(1.1.4)}$$

$$= \frac{7420}{209}, T_{3,3} = \frac{11260}{209}, T_{4,3} = \frac{13160}{209}, T_{5,3} = \frac{15020}{209} \Big] \Big]$$

```
> Solution := evalf(SolT);
```

$$Solution := \Big[\Big[T_{2,2} = 88.13397129, T_{3,2} = 117.0334928, T_{4,2} = 126.1244019, T_{5,2} = 124.4976077, T_{2,3} = 35.50239234, T_{3,3} = 53.87559809, T_{4,3} = 62.96650718, T_{5,3} = 71.86602871 \Big] \Big] \quad (1.1.5)$$

```
> Sys := [seq(Eq[i], i = 1 ..N)];
```

$$Sys := \Big[T_{3,2} + 200 + T_{2,3} - 4\,T_{2,2} = 0, T_{4,2} + T_{2,2} + T_{3,3} + 200 - 4\,T_{3,2} = 0, T_{5,2} + T_{3,2} + T_{4,3} + 200 - 4\,T_{4,2} = 0, 300 + T_{4,2} + T_{5,3} - 4\,T_{5,2} = 0, T_{3,3} + T_{2,2} - 4\,T_{2,3} = 0, T_{4,3} + T_{2,3} + T_{3,2} - 4\,T_{3,3} = 0, T_{5,3} + T_{3,3} + T_{4,2} - 4\,T_{4,3} = 0, 100 + T_{4,3} + T_{5,2} - 4\,T_{5,3} = 0 \Big] \quad (1.1.6)$$

```
> Var := [seq(Temps[i], i = 1 ..N)];
```

$$Var := \Big[T_{2,2}, T_{3,2}, T_{4,2}, T_{5,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{5,3} \Big] \quad (1.1.7)$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := \begin{bmatrix} -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 \end{bmatrix}, \begin{bmatrix} -200 \\ -200 \\ -200 \\ -300 \\ 0 \\ 0 \\ 0 \\ -100 \end{bmatrix} \quad (1.1.8)$$