Equation de Laplace 2D

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = T_1,$$
$$T(x, b) = T_2,$$
$$T(0, y) = T_3,$$
$$T(a, y) = T_4.$$

Solution discrétisée (formulation en 5 points):

```
> Restart:
> a := 1; b := 1; ndx := 4; ndy := 4
```

$$a := 1$$
$$b := 1$$
$$ndx := 4$$
$$ndy := 4 \quad \textbf{(1.1)}$$

```
>
> i_max := ndx + 1; j_max := ndy + 1;
```

$$i_{max} := 5$$
$$j_{max} := 5 \quad \textbf{(1.2)}$$

Nombre d'équations:

$> N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 9 \qquad \textbf{(1.3)}$$

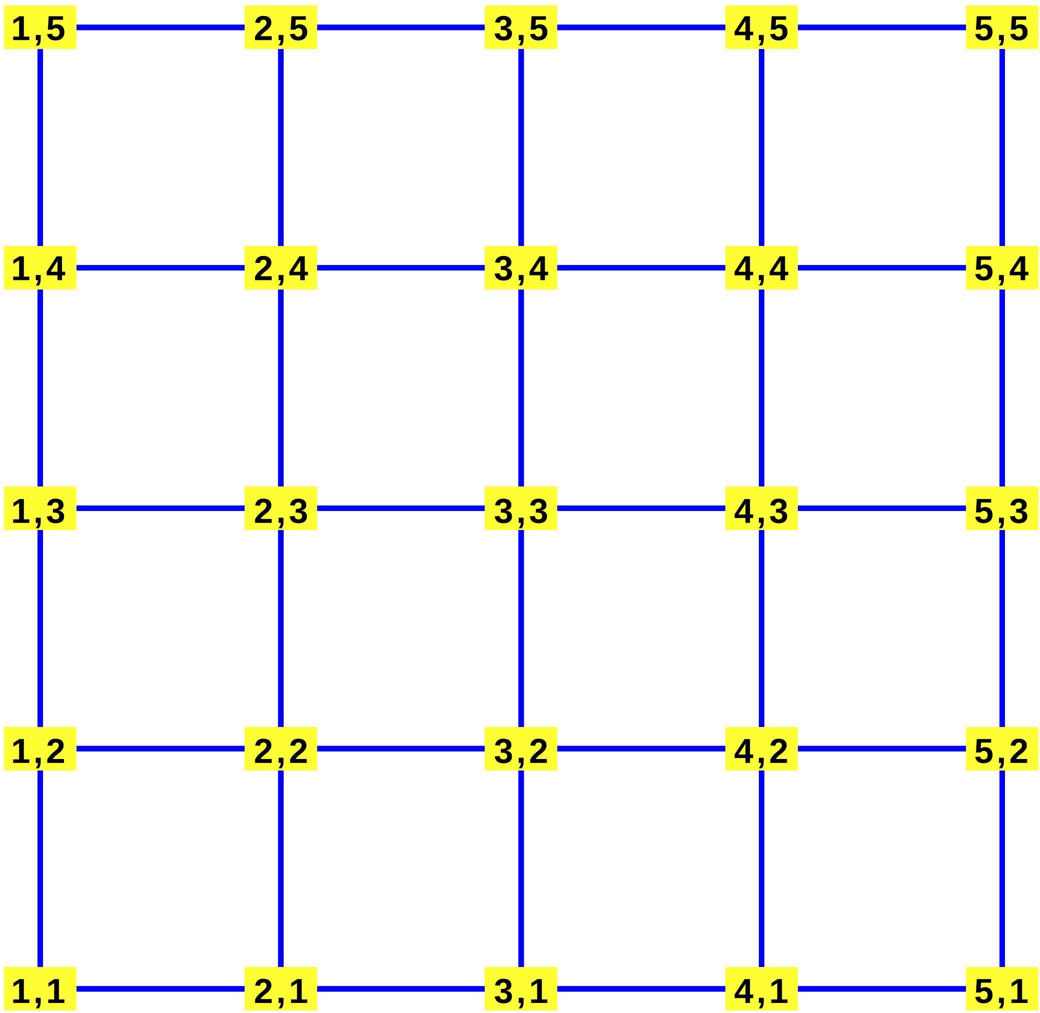
Maillage:

$> with(GraphTheory) : with(SpecialGraphs) :$

$> G := GridGraph(i_{max}, j_{max})$

G := Graph 1: an undirected unweighted graph with 25 vertices and 40 edge(s) **(1.4)**

$> DrawGraph(G)$



$> k := 1$

$$k := 1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for j from 2 to j_max − 1 do
    for i from 2 to i_max -1 do
        Eq[k] := T[i + 1, j] + T[i − 1, j] + β^2·(T[i, j + 1] + T[i, j-1]) − 2·(1
      + β^2)·T[i, j] = 0;
        Temps[k] := T[i, j];
    k := k + 1
   end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,2} + T_{1,2} + \beta^2 (T_{2,3} + T_{2,1}) - 2(1+\beta^2) T_{2,2} = 0$$
$$T_{4,2} + T_{2,2} + \beta^2 (T_{3,3} + T_{3,1}) - 2(1+\beta^2) T_{3,2} = 0$$
$$T_{5,2} + T_{3,2} + \beta^2 (T_{4,3} + T_{4,1}) - 2(1+\beta^2) T_{4,2} = 0$$
$$T_{3,3} + T_{1,3} + \beta^2 (T_{2,4} + T_{2,2}) - 2(1+\beta^2) T_{2,3} = 0$$
$$T_{4,3} + T_{2,3} + \beta^2 (T_{3,4} + T_{3,2}) - 2(1+\beta^2) T_{3,3} = 0$$
$$T_{5,3} + T_{3,3} + \beta^2 (T_{4,4} + T_{4,2}) - 2(1+\beta^2) T_{4,3} = 0$$
$$T_{3,4} + T_{1,4} + \beta^2 (T_{2,5} + T_{2,3}) - 2(1+\beta^2) T_{2,4} = 0$$
$$T_{4,4} + T_{2,4} + \beta^2 (T_{3,5} + T_{3,3}) - 2(1+\beta^2) T_{3,4} = 0$$
$$T_{5,4} + T_{3,4} + \beta^2 (T_{4,5} + T_{4,3}) - 2(1+\beta^2) T_{4,4} = 0 \quad \textbf{(1.1.2)}$$

```
> Sys := [seq(Eq[i], i=1..N)];
```

$$Sys := [T_{3,2} + T_{1,2} + \beta^2 (T_{2,3} + T_{2,1}) - 2(1+\beta^2) T_{2,2} = 0, T_{4,2} + T_{2,2} + \beta^2 (T_{3,3} + T_{3,1}) - 2(1+\beta^2) T_{3,2} = 0, T_{5,2} + T_{3,2} + \beta^2 (T_{4,3} + T_{4,1}) - 2(1+\beta^2) T_{4,2} = 0, T_{3,3} + T_{1,3} + \beta^2 (T_{2,4} + T_{2,2}) - 2(1+\beta^2) T_{2,3} = 0, T_{4,3} + T_{2,3} + \beta^2 (T_{3,4} + T_{3,2}) - 2(1+\beta^2) T_{3,3} = 0, T_{5,3} + T_{3,3} + \beta^2 (T_{4,4} + T_{4,2}) - 2(1+\beta^2) T_{4,3} = 0, T_{3,4} + T_{1,4} + \beta^2 (T_{2,5} + T_{2,3}) - 2(1+\beta^2) T_{2,4} = 0, T_{4,4} + T_{2,4} + \beta^2 (T_{3,5} + T_{3,3}) - 2(1+\beta^2) T_{3,4} = 0, T_{5,4} + T_{3,4} + \beta^2 (T_{4,5} + T_{4,3}) - 2(1+\beta^2) T_{4,4} = 0] \quad \textbf{(1.1.3)}$$

```
> Var := [seq(Temps[i], i=1..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}] \quad \textbf{(1.1.4)}$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := [[-2 - 2\beta^2, 1, 0, \beta^2, 0, 0, 0, 0, 0], \quad \textbf{(1.1.5)}$$
$$[1, -2 - 2\beta^2, 1, 0, \beta^2, 0, 0, 0, 0],$$
$$[0, 1, -2 - 2\beta^2, 0, 0, \beta^2, 0, 0, 0],$$
$$[\beta^2, 0, 0, -2 - 2\beta^2, 1, 0, \beta^2, 0, 0],$$
$$[0, \beta^2, 0, 1, -2 - 2\beta^2, 1, 0, \beta^2, 0],$$
$$[0, 0, \beta^2, 0, 1, -2 - 2\beta^2, 0, 0, \beta^2],$$
$$[0, 0, 0, \beta^2, 0, 0, -2 - 2\beta^2, 1, 0],$$
$$[0, 0, 0, 0, \beta^2, 0, 1, -2 - 2\beta^2, 1],$$

$$\left[0, 0, 0, 0, 0, \beta^{2}, 0, 1, -2 - 2\,\beta^{2}\right]\Big], \begin{bmatrix} -T_{1,2} - \beta^{2}\,T_{2,1} \\ -\beta^{2}\,T_{3,1} \\ -T_{5,2} - \beta^{2}\,T_{4,1} \\ -T_{1,3} \\ 0 \\ -T_{5,3} \\ -T_{1,4} - \beta^{2}\,T_{2,5} \\ -\beta^{2}\,T_{3,5} \\ -T_{5,4} - \beta^{2}\,T_{4,5} \end{bmatrix}$$