Equation de Laplace 2D

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100,$$
$$T(0, y) = 60,$$
$$T(a, y) = 20.$$

Solution discrétisée (formulation en 9 points):

> $Restart:$

> $a := 1; b := 1; ndx := 3; ndy := 3$

$$a := 1$$
$$b := 1$$
$$ndx := 3$$
$$ndy := 3 \qquad (1.1)$$

> $\Delta x := \frac{a}{ndx}; \Delta y := \frac{b}{ndy}; \beta := \frac{\Delta x}{\Delta y};$

$$\Delta x := \frac{1}{3}$$
$$\Delta y := \frac{1}{3}$$

(1.2)

$$\beta := 1 \qquad (1.2)$$

> $i_{\max} := ndx + 1;\ j_{\max} := ndy + 1;$

$$i_{max} := 4$$
$$j_{max} := 4 \qquad (1.3)$$

Nombre d'équations:

> $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$

$$N := 4 \qquad (1.4)$$

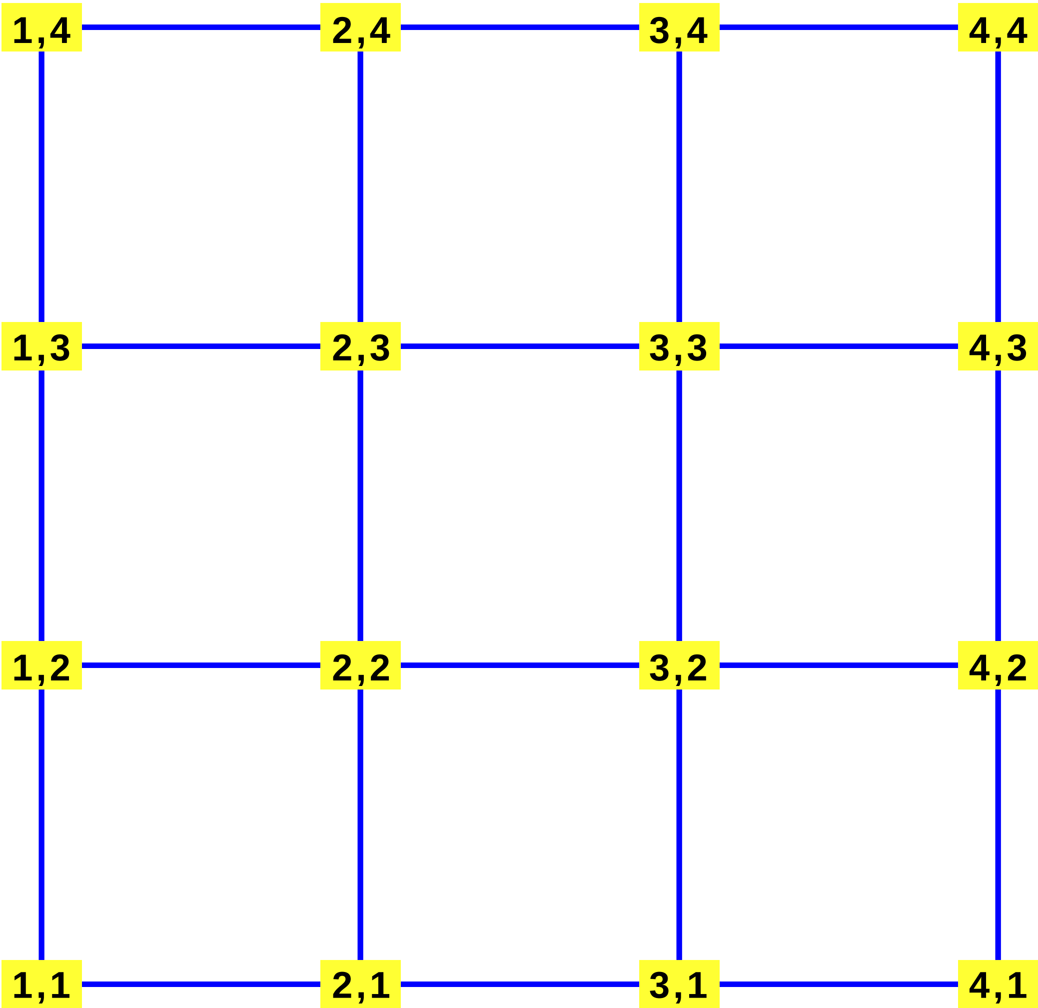
Maillage:

> $with(GraphTheory) : with(SpecialGraphs) :$

> $G := GridGraph(i_{\max}, j_{\max})$

$G :=$ *Graph 1: an undirected unweighted graph with 16 vertices and 24 edge(s)* (1.5)

> $DrawGraph(G)$




Conditions aux Limites:

> $Tb := 0.;\ Tg := 60;\ Td := 20;\ Th := 100.;$

$$Tb := 0.$$
$$Tg := 60$$
$$Td := 20$$
$$Th := 100. \qquad (1.6)$$

> **for** i **from** 2 **to** $i_{\max} - 1$ **do** $T[i, 1] := Tb$ **end do**; $T[1, 1] := 0.5 \cdot (Tb + Tg);\ T[i_{\max}, 1] := 0.5 \cdot (Tb + Td);$

$$T_{2,1} := 0.$$

$$T_{3,1} := 0.$$
$$T_{1,1} := 30.0$$
$$T_{4,1} := 10.0 \quad \textbf{(1.7)}$$

> **for** i **from** 2 **to** $i_{max} - 1$ **do** $T[i, j_{max}] := Th$ **end do**; $T[1, j_{max}] := 0.5 \cdot (Tg + Th)$; $T[i_{max}, j_{max}] := 0.5 \cdot (Td + Th)$;

$$T_{2,4} := 100.$$
$$T_{3,4} := 100.$$
$$T_{1,4} := 80.0$$
$$T_{4,4} := 60.0 \quad \textbf{(1.8)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[1, j] := Tg$ **end do**;

$$T_{1,2} := 60$$
$$T_{1,3} := 60 \quad \textbf{(1.9)}$$

> **for** j **from** 2 **to** $j_{max} - 1$ **do** $T[i_{max}, j] := Td$ **end do**;

$$T_{4,2} := 20$$
$$T_{4,3} := 20 \quad \textbf{(1.10)}$$

> $k := 1$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

> **for** j **from** 2 **to** $j_{max} - 1$ **do**
> **for** i **from** 2 **to** $i_{max} - 1$ **do**
> $Eq[k] := T[i+1, j+1] + T[i+1, j-1] + T[i-1, j+1] + T[i-1, j-1] + 2 \cdot \frac{5 - \beta^2}{1 + \beta^2} \cdot (T[i+1, j] + T[i-1, j]) + 2 \cdot \frac{5 \cdot \beta^2 - 1}{1 + \beta^2} \cdot (T[i, j+1] + T[i, j-1]) - 20 \cdot T[i, j] = 0;$
> $Temps[k] := T[i, j];$
> $k := k + 1$
> **end do**;
> **end do**;

Ecriture du système d'équations:

> **for** $k$ **from** 1 **to** $N$ **do** $Eq[k]$ **end do**;

$$T_{3,3} + 330.0 + 4\,T_{3,2} + 4\,T_{2,3} - 20\,T_{2,2} = 0$$
$$110.0 + T_{2,3} + 4\,T_{2,2} + 4\,T_{3,3} - 20\,T_{3,2} = 0$$
$$880.0 + T_{3,2} + 4\,T_{3,3} + 4\,T_{2,2} - 20\,T_{2,3} = 0$$
$$660.0 + T_{2,2} + 4\,T_{2,3} + 4\,T_{3,2} - 20\,T_{3,3} = 0 \quad \textbf{(1.1.2)}$$

> $Eqs := \{seq(Eq[i], i = 1..N)\}$ :

> $Tmps := [seq(Temps[i], i = 1..N)]$;

$$Tmps := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}] \quad \textbf{(1.1.3)}$$

> $SolT := solve(Eqs, Tmps)$;

$$SolT := [[T_{2,2} = 37.14285714, T_{3,2} = 26.66666667, T_{2,3} = 63.33333333, T_{3,3} = 52.85714286]] \quad \textbf{(1.1.4)}$$

> $Solution := evalf(SolT)$;

$$Solution := [[T_{2,2} = 37.14285714, T_{3,2} = 26.66666667, T_{2,3} = 63.33333333, T_{3,3} \quad \textbf{(1.1.5)}$$

$= 52.85714286]]$

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> Sys := [seq(Eq[i], i = 1 ..N)];
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$$Sys := [T_{3,3} + 330.0 + 4\,T_{3,2} + 4\,T_{2,3} - 20\,T_{2,2} = 0, 110.0 + T_{2,3} + 4\,T_{2,2} + 4\,T_{3,3} - 20\,T_{3,2} = 0, 880.0 + T_{3,2} + 4\,T_{3,3} + 4\,T_{2,2} - 20\,T_{2,3} = 0, 660.0 + T_{2,2} + 4\,T_{2,3} + 4\,T_{3,2} - 20\,T_{3,3} = 0] \quad \textbf{(1.1.6)}$$

```
> Var := [seq(Temps[i], i = 1 ..N)];
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$$Var := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}] \quad \textbf{(1.1.7)}$$

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> with(LinearAlgebra) :
> A, b := GenerateMatrix(Sys, Var);
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$$A, b := \begin{bmatrix} -20 & 4 & 4 & 1 \\ 4 & -20 & 1 & 4 \\ 4 & 1 & -20 & 4 \\ 1 & 4 & 4 & -20 \end{bmatrix}, \begin{bmatrix} -330.0 \\ -110.0 \\ -880.0 \\ -660.0 \end{bmatrix} \quad \textbf{(1.1.8)}$$