

## Equation de Laplace 2D

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Détermination de la température  $T(x, y)$  à travers la surface d'une plaque rectangulaire ( $a \times b$ ) dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L.):

$$\begin{aligned} T(x, 0) &= 0, \\ T(x, b) &= 100, \\ T(0, y) &= 75, \\ T(a, y) &= 50. \end{aligned}$$

Solution discrétisée (formulation en 9 points):

> *Restart :*

>  $a := 1; b := 1; ndx := 4; ndy := 4$

$a := 1$   
 $b := 1$   
 $ndx := 4$   
 $ndy := 4$

(1.1)

>  $\Delta x := \frac{a}{ndx}; \Delta y := \frac{b}{ndy}; \beta := \frac{\Delta x}{\Delta y};$

$\Delta x := \frac{1}{4}$

$\Delta y := \frac{1}{4}$

(1.2)

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β := 1 (1.2)
> i_max := ndx + 1; j_max := ndy + 1;

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i_max := 5 (1.3)
j_max := 5

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Nombre d'équations:

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> N := (i_max - 2) · (j_max - 2) (1.4)
N := 9

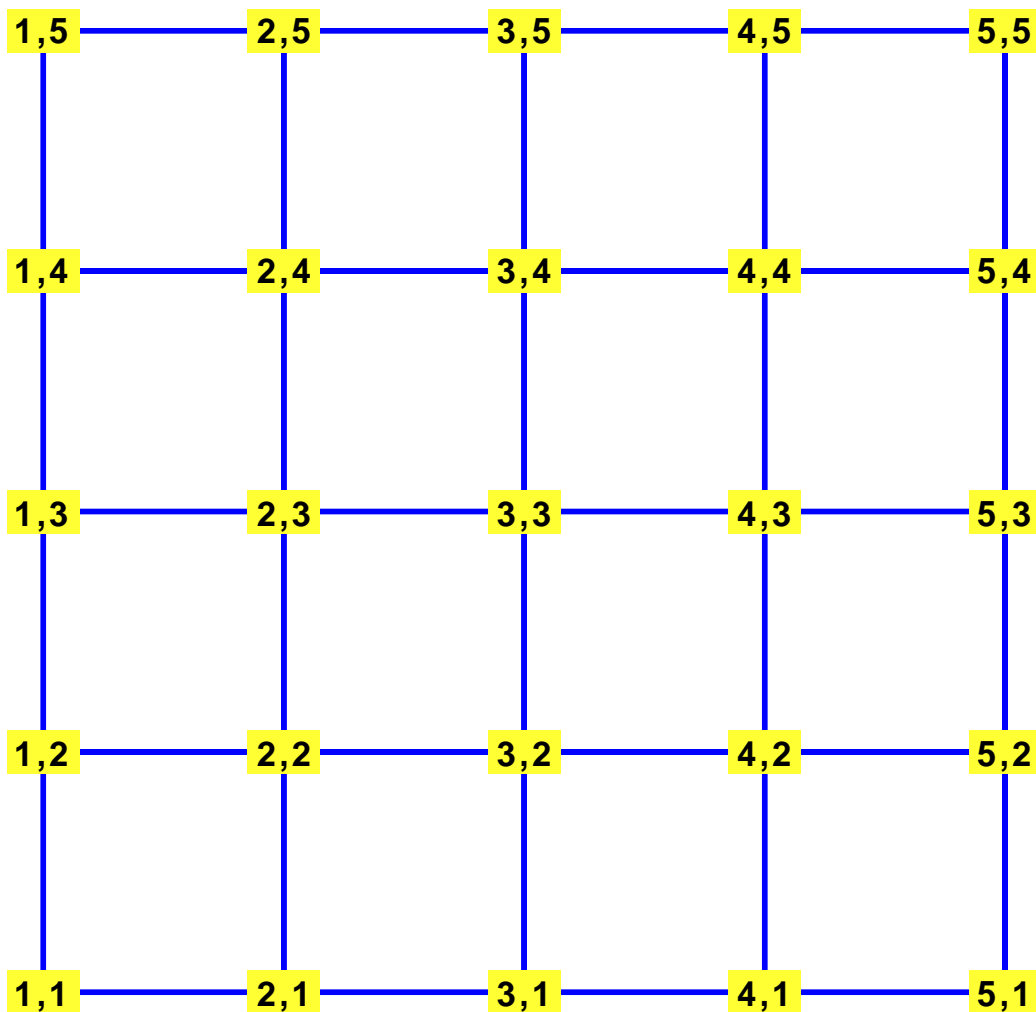
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Maillage:

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> with(GraphTheory) : with(SpecialGraphs) :
> G := GridGraph(i_max, j_max) (1.5)
G := Graph 1: an undirected unweighted graph with 25 vertices and 40 edge(s)
> DrawGraph(G)

```



Conditions aux Limites:

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> Tb := 0.; Tg := 75; Td := 50; Th := 100.; (1.6)
Tb := 0.
Tg := 75
Td := 50
Th := 100.
> for i from 2 to i_max - 1 do T[i, 1] := Tb end do; T[1, 1] := 0.5 · (Tb + Tg); T[i_max, 1] := 0.5 · (Tb + Td);
T_{2,1} := 0.

```

$$\begin{aligned}
T_{3,1} &:= 0. \\
T_{4,1} &:= 0. \\
T_{1,1} &:= 37.5 \\
T_{5,1} &:= 25.0
\end{aligned} \tag{1.7}$$

> for i from 2 to  $i_{\max} - 1$  do  $T[i, j_{\max}] := Th$  end do;  $T[1, j_{\max}] := 0.5 \cdot (Tg + Th)$ ;  $T[i_{\max}, j_{\max}] := 0.5 \cdot (Td + Th)$ ;

$$\begin{aligned}
T_{2,5} &:= 100. \\
T_{3,5} &:= 100. \\
T_{4,5} &:= 100. \\
T_{1,5} &:= 87.5 \\
T_{5,5} &:= 75.0
\end{aligned} \tag{1.8}$$

> for j from 2 to  $j_{\max} - 1$  do  $T[1, j] := Tg$  end do;

$$\begin{aligned}
T_{1,2} &:= 75 \\
T_{1,3} &:= 75 \\
T_{1,4} &:= 75
\end{aligned} \tag{1.9}$$

> for j from 2 to  $j_{\max} - 1$  do  $T[i_{\max}, j] := Td$  end do;

$$\begin{aligned}
T_{5,2} &:= 50 \\
T_{5,3} &:= 50 \\
T_{5,4} &:= 50
\end{aligned} \tag{1.10}$$

> k := 1

$$k := 1 \tag{1.1.1}$$

Résolution pour les noeuds internes:

> for j from 2 to  $j_{\max} - 1$  do

for i from 2 to  $i_{\max} - 1$  do

$$\begin{aligned}
Eq[k] &:= T[i+1, j+1] + T[i+1, j-1] + T[i-1, j+1] + T[i-1, j-1] \\
&+ 2 \cdot \frac{5 - \beta^2}{1 + \beta^2} \cdot (T[i+1, j] + T[i-1, j]) + 2 \cdot \frac{5 \cdot \beta^2 - 1}{1 + \beta^2} \cdot (T[i, j+1] + T[i, j \\
&- 1]) - 20 \cdot T[i, j] = 0; \\
Temps[k] &:= T[i, j];
\end{aligned}$$

k := k + 1

end do;

end do;

Ecriture du système d'équations:

> for k from 1 to N do  $Eq[k]$  end do;

$$\begin{aligned}
T_{3,3} + 412.5 + 4 T_{3,2} + 4 T_{2,3} - 20 T_{2,2} &= 0 \\
T_{4,3} + T_{2,3} + 4 T_{4,2} + 4 T_{2,2} + 4 T_{3,3} - 20 T_{3,2} &= 0 \\
275.0 + T_{3,3} + 4 T_{3,2} + 4 T_{4,3} - 20 T_{4,2} &= 0 \\
T_{3,4} + T_{3,2} + 450 + 4 T_{3,3} + 4 T_{2,4} + 4 T_{2,2} - 20 T_{2,3} &= 0 \\
T_{4,4} + T_{4,2} + T_{2,4} + T_{2,2} + 4 T_{4,3} + 4 T_{2,3} + 4 T_{3,4} + 4 T_{3,2} - 20 T_{3,3} &= 0 \\
300 + T_{3,4} + T_{3,2} + 4 T_{3,3} + 4 T_{4,4} + 4 T_{4,2} - 20 T_{4,3} &= 0 \\
962.5 + T_{3,3} + 4 T_{3,4} + 4 T_{2,3} - 20 T_{2,4} &= 0 \\
600. + T_{4,3} + T_{2,3} + 4 T_{4,4} + 4 T_{2,4} + 4 T_{3,3} - 20 T_{3,4} &= 0
\end{aligned}$$

$$825.0 + T_{3,3} + 4 T_{3,4} + 4 T_{4,3} - 20 T_{4,4} = 0 \quad (1.1.2)$$

> *Eqs* := {seq(*Eq*[*i*], *i* = 1 ..*N*) } :

> *Tmps* := [seq(*Temps*[*i*], *i* = 1 ..*N*) ];

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}] \quad (1.1.3)$$

> *SolT* := solve(*Eqs*, *Tmps*);

$$SolT := [[T_{2,2} = 42.59510870, T_{3,2} = 32.26284585, T_{4,2} = 33.49184783, T_{2,3} = 63.52519763, T_{3,3} = 56.25000000, T_{4,3} = 52.38389328, T_{2,4} = 79.00815217, T_{3,4} = 76.82806324, T_{4,4} = 69.90489130]] \quad (1.1.4)$$

> *Solution* := evalf(*SolT*);

$$Solution := [[T_{2,2} = 42.59510870, T_{3,2} = 32.26284585, T_{4,2} = 33.49184783, T_{2,3} = 63.52519763, T_{3,3} = 56.25000000, T_{4,3} = 52.38389328, T_{2,4} = 79.00815217, T_{3,4} = 76.82806324, T_{4,4} = 69.90489130]] \quad (1.1.5)$$

> *Sys* := [seq(*Eq*[*i*], *i* = 1 ..*N*) ];

$$Sys := [T_{3,3} + 412.5 + 4 T_{3,2} + 4 T_{2,3} - 20 T_{2,2} = 0, T_{4,3} + T_{2,3} + 4 T_{4,2} + 4 T_{2,2} + 4 T_{3,3} - 20 T_{3,2} = 0, 275.0 + T_{3,3} + 4 T_{3,2} + 4 T_{4,3} - 20 T_{4,2} = 0, T_{3,4} + T_{3,2} + 450 + 4 T_{3,3} + 4 T_{2,4} + 4 T_{2,2} - 20 T_{2,3} = 0, T_{4,4} + T_{4,2} + T_{2,4} + T_{2,2} + 4 T_{4,3} + 4 T_{2,3} + 4 T_{3,4} + 4 T_{3,2} - 20 T_{3,3} = 0, 300 + T_{3,4} + T_{3,2} + 4 T_{3,3} + 4 T_{4,4} + 4 T_{4,2} - 20 T_{4,3} = 0, 962.5 + T_{3,3} + 4 T_{3,4} + 4 T_{2,3} - 20 T_{2,4} = 0, 600. + T_{4,3} + T_{2,3} + 4 T_{4,4} + 4 T_{2,4} + 4 T_{3,3} - 20 T_{3,4} = 0, 825.0 + T_{3,3} + 4 T_{3,4} + 4 T_{4,3} - 20 T_{4,4} = 0] \quad (1.1.6)$$

> *Var* := [seq(*Temps*[*i*], *i* = 1 ..*N*) ];

$$Var := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}] \quad (1.1.7)$$

> with(*LinearAlgebra*) :

> *A*, *b* := GenerateMatrix(*Sys*, *Var*);

$$A, b := \begin{bmatrix} -20 & 4 & 0 & 4 & 1 & 0 & 0 & 0 & 0 \\ 4 & -20 & 4 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & -20 & 0 & 1 & 4 & 0 & 0 & 0 \\ 4 & 1 & 0 & -20 & 4 & 0 & 4 & 1 & 0 \\ 1 & 4 & 1 & 4 & -20 & 4 & 1 & 4 & 1 \\ 0 & 1 & 4 & 0 & 4 & -20 & 0 & 1 & 4 \\ 0 & 0 & 0 & 4 & 1 & 0 & -20 & 4 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 & 4 & -20 & 4 \\ 0 & 0 & 0 & 0 & 1 & 4 & 0 & 4 & -20 \end{bmatrix}, \begin{bmatrix} -412.5 \\ 0 \\ -275.0 \\ -450 \\ 0 \\ -300 \\ -962.5 \\ -600. \\ -825.0 \end{bmatrix} \quad (1.1.8)$$