Equation de Laplace 2D

==========

Dr. Lad MESSAOUDI

Département de Mécanique

Université de Batna

==========

LMD : Génie Energétique

Matire : Outils Numériques

==========

2009/2010

Détermination de la temperature $T(x, y)$ travers la surface d'une plaque rectangulaire (a x b) dont les extrémités sont soumises des (C.L.) de Dirichlet.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 2.10^4 . x^2,$$
$$T(0, y) = 75,$$
$$T(a, y) = 50.$$

Solution discrétisée (formulation en 5 points):

```
> Restart:
> a := 12; b := 12; ndx := 4; ndy := 4
```

$$a := 12$$
$$b := 12$$
$$ndx := 4$$
$$ndy := 4$$

> $\Delta x := \frac{a}{ndx}$ ; $\Delta y := \frac{b}{ndy}$; $\beta := \frac{\Delta x}{\Delta y}$;

$$\Delta x := 3$$
$$\Delta y := 3$$
$$\beta := 1$$

> $i_{max} := ndx + 1$; $j_{max} := ndy + 1$;

$$i_{max} := 5$$
$$j_{max} := 5$$

Nombre d'équations:

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 9$$

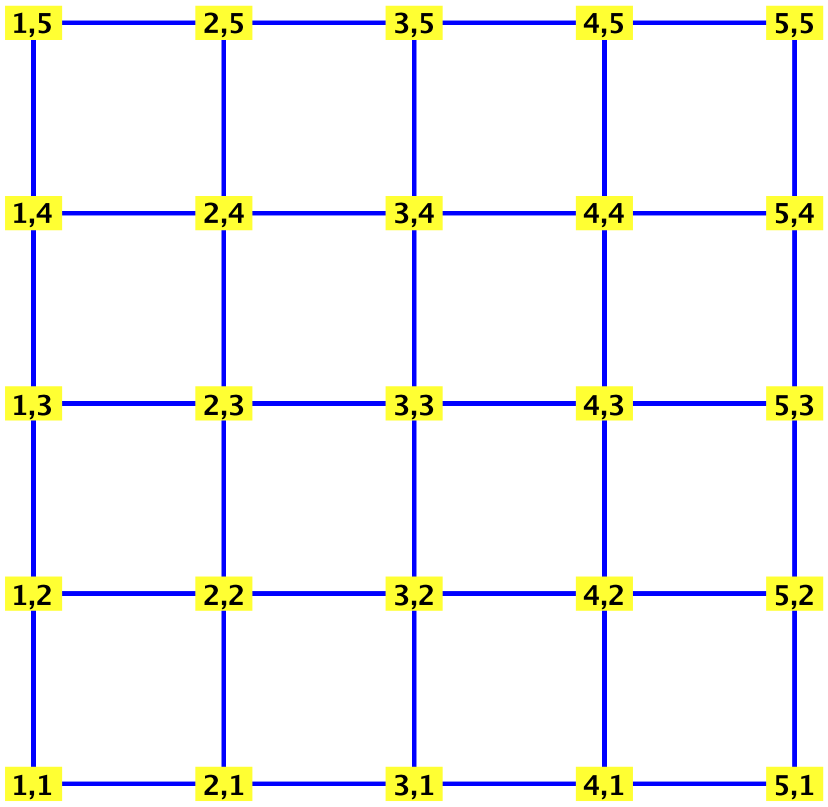
Maillage:

> *with*(*GraphTheory*) : *with*(*SpecialGraphs*) :

> $G :=$ *GridGraph*($i_{max}$, $j_{max}$)

*G := Graph 1: an undirected unweighted graph with 25 vertices and 40 edge(s)*

> *DrawGraph*(*G*)




Conditions aux Limites:

```
> for i from 2 to i_max - 1 do T[i, 1] := 0 end do;
```

$$T_{2,1} := 0$$
$$T_{3,1} := 0$$
$$T_{4,1} := 0$$

```
> for j from 2 to j_max - 1 do T[1, j] := 75 end do;
```

$$T_{1,2} := 75$$
$$T_{1,3} := 75$$
$$T_{1,4} := 75$$

```
> for j from 2 to j_max - 1 do T[i_max, j] := 50 end do;
```

$$T_{5,2} := 50$$
$$T_{5,3} := 50$$
$$T_{5,4} := 50$$

```
> for i from 2 to i_max - 1 do T[i, j_max] := 2.((i - 1)·Δx)^2 end do;
```

$$T_{2,5} := 18.$$
$$T_{3,5} := 72.$$
$$T_{4,5} := 162.$$

```
> k := 1
```

$$k := 1$$

Résolution pour les noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2·(T[i, j + 1] + T[i, j - 1]) - 2·(1
      + β^2)·T[i, j] = 0;
      Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Ecriture du systme d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$T_{3,2} + 75 + T_{2,3} - 4\,T_{2,2} = 0$$
$$T_{4,2} + T_{2,2} + T_{3,3} - 4\,T_{3,2} = 0$$
$$50 + T_{3,2} + T_{4,3} - 4\,T_{4,2} = 0$$
$$T_{3,3} + 75 + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0$$
$$T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0$$
$$50 + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0$$
$$T_{3,4} + 93. + T_{2,3} - 4\,T_{2,4} = 0$$
$$T_{4,4} + T_{2,4} + 72. + T_{3,3} - 4\,T_{3,4} = 0$$
$$212. + T_{3,4} + T_{4,3} - 4\,T_{4,4} = 0$$

```
> Eqs := {seq(Eq[i], i = 1..N)}:
> Tmps := [seq(Temps[i], i = 1..N)];
```

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}]$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 40.37500000, T_{3,2} = 31.47321429, T_{4,2} = 34.01785714, T_{2,3} = 55.02678571, T_{3,3} = 51.50000000, T_{4,3} = 54.59821429, T_{2,4} = 53.23214286, T_{3,4} = 64.90178571, T_{4,4} = 82.87500000]]$$

```
> Solution := evalf(SolT);
```

$$Solution := [[T_{2,2} = 40.37500000, T_{3,2} = 31.47321429, T_{4,2} = 34.01785714, T_{2,3} = 55.02678571, T_{3,3} = 51.50000000, T_{4,3} = 54.59821429, T_{2,4} = 53.23214286, T_{3,4} = 64.90178571, T_{4,4} = 82.87500000]]$$

```
> Sys := [seq(Eq[i], i = 1..N)];
```

$$Sys := [T_{3,2} + 75 + T_{2,3} - 4\,T_{2,2} = 0,\ T_{4,2} + T_{2,2} + T_{3,3} - 4\,T_{3,2} = 0,\ 50 + T_{3,2} + T_{4,3} - 4\,T_{4,2} = 0,\ T_{3,3} + 75 + T_{2,4} + T_{2,2} - 4\,T_{2,3} = 0,\ T_{4,3} + T_{2,3} + T_{3,4} + T_{3,2} - 4\,T_{3,3} = 0,\ 50 + T_{3,3} + T_{4,4} + T_{4,2} - 4\,T_{4,3} = 0,\ T_{3,4} + 93. + T_{2,3} - 4\,T_{2,4} = 0,\ T_{4,4} + T_{2,4} + 72. + T_{3,3} - 4\,T_{3,4} = 0,\ 212. + T_{3,4} + T_{4,3} - 4\,T_{4,4} = 0]$$

```
> Var := [seq(Temps[i], i = 1..N)];
```

$$Var := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}, T_{2,4}, T_{3,4}, T_{4,4}]$$

```
> with(LinearAlgebra):
> A, b := GenerateMatrix(Sys, Var);
```

$$A, b := \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix}, \begin{bmatrix} -75 \\ 0 \\ -50 \\ -75 \\ 0 \\ -50 \\ -93. \\ -72. \\ -212. \end{bmatrix}$$