Equation de Laplace 2D

==============================

Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

=======================

Master : Energétique

Matière : Méthodes Numériques Appliquées I

==

2011/2012

===========

Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les 3 extrémités sont soumises à des (C.L.) de Dirichlet et la quatrième à une conditions de Neumann.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 100,$$
$$T(0, y) = 0,$$
$$\frac{\partial}{\partial x} T(a, y) = 0.$$

Solution discrétisée par la formulation à 5 points:

```
> Restart :
> a := 5 : b := 15 : ndx := 5 : ndy := 15 :
> β := 1. :
> i_max := ndx + 1; j_max := ndy + 1;
```

$$6$$
$$16 \quad \textbf{(1.1)}$$

Nombre d'équations:

> $N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$56 \tag{1.2}$$

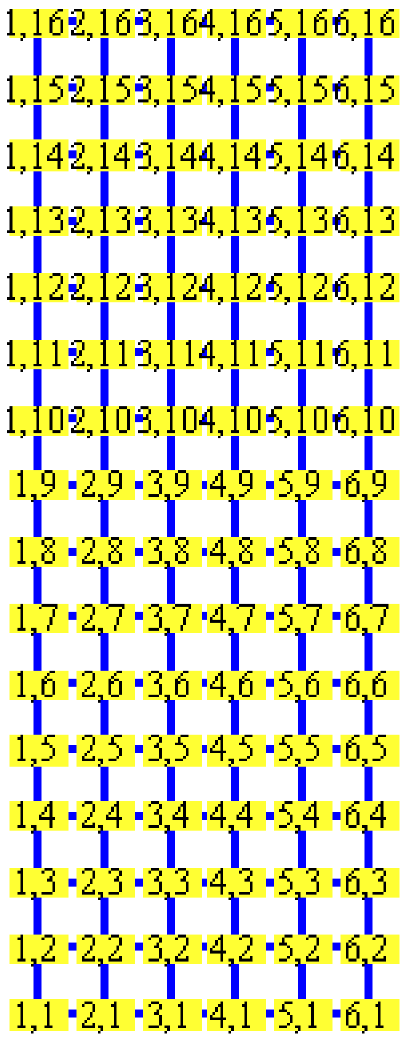
Maillage:

> *with*(*GraphTheory*) : *with*(*SpecialGraphs*) :

> $G := GridGraph(i_{max}, j_{max})$

GRAPHLN(*undirected*, *unweighted*, ["1,1", "1,2", "1,3", "1,4", "1,5", "1,6", "1,7", "1,8", "1,9", "1,10", "1,11", "1,12", "1,13", "1,14", "1,15", "1,16", "2,1", "2,2", "2,3", "2,4", "2,5", "2,6", "2,7", "2,8", "2,9", "2,10", "2,11", "2,12", "2,13", "2,14", "2,15", "2,16", "3,1", "3,2", "3,3", "3,4", "3,5", "3,6", "3,7", "3,8", "3,9", "3,10", "3,11", "3,12", "3,13", "3,14", "3,15", "3,16", "4,1", "4,2", "4,3", "4,4", "4,5", "4,6", "4,7", "4,8", "4,9", "4,10", "4,11", "4,12", "4,13", "4,14", "4,15", "4,16", "5,1", "5,2", "5,3", "5,4", "5,5", "5,6", "5,7", "5,8", "5,9", "5,10", "5,11", "5,12", "5,13", "5,14", "5,15", "5,16", "6,1", "6,2", "6,3", "6,4", "6,5", "6,6", "6,7", "6,8", "6,9", "6,10", "6,11", "6,12", "6,13", "6,14", "6,15", "6,16"], *Array*(%*id* = 173764988), *GRAPHLN/table/1*, 0) **(1.3)**

> *DrawGraph*(*G*)



Conditions aux Limites:

> **for** i **from** 1 **to** i_{max} **do** $T[i, 1] := 0$ **end do**;

0 **(1.4)**
0
0
0
0
0

```
> for i from 1 to i_max do T[i, j_max] := 100 end do;
```

$$100$$
$$100$$
$$100$$
$$100$$
$$100$$
$$100 \quad (1.5)$$

```
> for j from 1 to j_max do T[1, j] := 0 end do;
```

$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0 \quad (1.6)$$

```
> k := 1 :
```

Résolution pour les noeuds internes

```
> for i from 2 to i_max -1 do
    for j from 2 to j_max - 1 do
        Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2·(T[i, j + 1] + T[i, j-1]) - 2·(1 + β^2)
      ·T[i, j] = 0;
        Temps[k] := T[i, j];
    k := k + 1
    end do;
  end do;
```

Conditions de Neumann:

```
> for j from 2 to j_max - 1 do
  Eq[k] := 2·(1 + β^2)·T[i_max, j] = 2·T[i_max - 1, j] + β^2·(T[i_max, j - 1] + T[i_max, j
        + 1]);
  Temps[k] := T[i_max, j] :
  k := k + 1 :
   end do:
> N := k - 1 :
```

Ecriture du système d'équations:

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)];
```

$$[T_{2,2}, T_{2,3}, T_{2,4}, T_{2,5}, T_{2,6}, T_{2,7}, T_{2,8}, T_{2,9}, T_{2,10}, T_{2,11}, T_{2,12}, T_{2,13}, T_{2,14}, T_{2,15}, T_{3,2}, T_{3,3}, T_{3,4}, T_{3,5}, T_{3,6}, T_{3,7}, T_{3,8}, T_{3,9}, T_{3,10}, T_{3,11}, T_{3,12}, T_{3,13}, T_{3,14}, T_{3,15}, T_{4,2}, T_{4,3}, T_{4,4}, T_{4,5}, T_{4,6}, T_{4,7}, T_{4,8}, T_{4,9}, T_{4,10}, T_{4,11}, T_{4,12}, T_{4,13}, T_{4,14}, T_{4,15}, T_{5,2}, \quad (1.1.1)$$

$T_{5,3}, T_{5,4}, T_{5,5}, T_{5,6}, T_{5,7}, T_{5,8}, T_{5,9}, T_{5,10}, T_{5,11}, T_{5,12}, T_{5,13}, T_{5,14}, T_{5,15}, T_{6,2}, T_{6,3}, T_{6,4}, T_{6,5}, T_{6,6}, T_{6,7}, T_{6,8}, T_{6,9}, T_{6,10}, T_{6,11}, T_{6,12}, T_{6,13}, T_{6,14}, T_{6,15}]$

> $SolT := solve(Eqs, Tmps);$

$[[T_{2,2} = 0.2308165911, T_{2,3} = 0.4843134281, T_{2,4} = 0.7854624321, T_{2,5} = 1.164103058, T_{2,6} = 1.658165424, T_{2,7} = 2.318115057, T_{2,8} = 3.213694640, T_{2,9} = 4.445250378, T_{2,10} = 6.165014061, T_{2,11} = 8.622046920, T_{2,12} = 12.26869607, T_{2,13} = 18.04191713, T_{2,14} = 28.18854039, T_{2,15} = 48.93901393, T_{3,2} = 0.4389529364, T_{3,3} = 0.9209746891, T_{3,4} = 1.493433242, T_{3,5} = 2.212784376, T_{3,6} = 3.150443582, T_{3,7} = 4.400600164, T_{3,8} = 6.091413124, T_{3,9} = 8.402292812, T_{3,10} = 11.59275895, T_{3,11} = 16.05447755, T_{3,12} = 22.41082021, T_{3,13} = 31.71043207, T_{3,14} = 45.77323051, T_{3,15} = 67.56751532, T_{4,2} = 0.6040204653, T_{4,3} = 1.267199150, T_{4,4} = 2.054511472, T_{4,5} = 3.043157622, T_{4,6} = 4.330224364, T_{4,7} = 6.042428891, T_{4,8} = 8.349064881, T_{4,9} = 11.47974880, T_{4,10} = 15.74925136, T_{4,11} = 21.59228413, T_{4,12} = 29.60967517, T_{4,13} = 40.61576042, T_{4,14} = 55.62643425, T_{4,15} = 75.55781684, T_{5,2} = 0.7099297748, T_{5,3} = 1.489289973, T_{5,4} = 2.414255873, T_{5,5} = 3.575110276, T_{5,6} = 5.084867363, T_{5,7} = 7.089826155, T_{5,8} = 9.782668710, T_{5,9} = 13.41838615, T_{5,10} = 18.33221355, T_{5,11} = 24.95573246, T_{5,12} = 33.81983591, T_{5,13} = 45.51650020, T_{5,14} = 60.55892923, T_{5,15} = 79.03731778, T_{6,2} = 0.7464086611, T_{6,3} = 1.565775095, T_{6,4} = 2.538111771, T_{6,5} = 3.758160245, T_{6,6} = 5.344308656, T_{6,7} = 7.449339655, T_{6,8} = 10.27339765, T_{6,9} = 14.07891354, T_{6,10} = 19.20548422, T_{6,11} = 26.07859624, T_{6,12} = 35.19743582, T_{6,13} = 47.07147523, T_{6,14} = 62.05546469, T_{6,15} = 80.03252506]]$ **(1.1.2)**

> $LT := [seq(T_{1,j}, j = 1..j_{max}), seq(rhs(SolT_{1,i}), i = 1..N)]:$

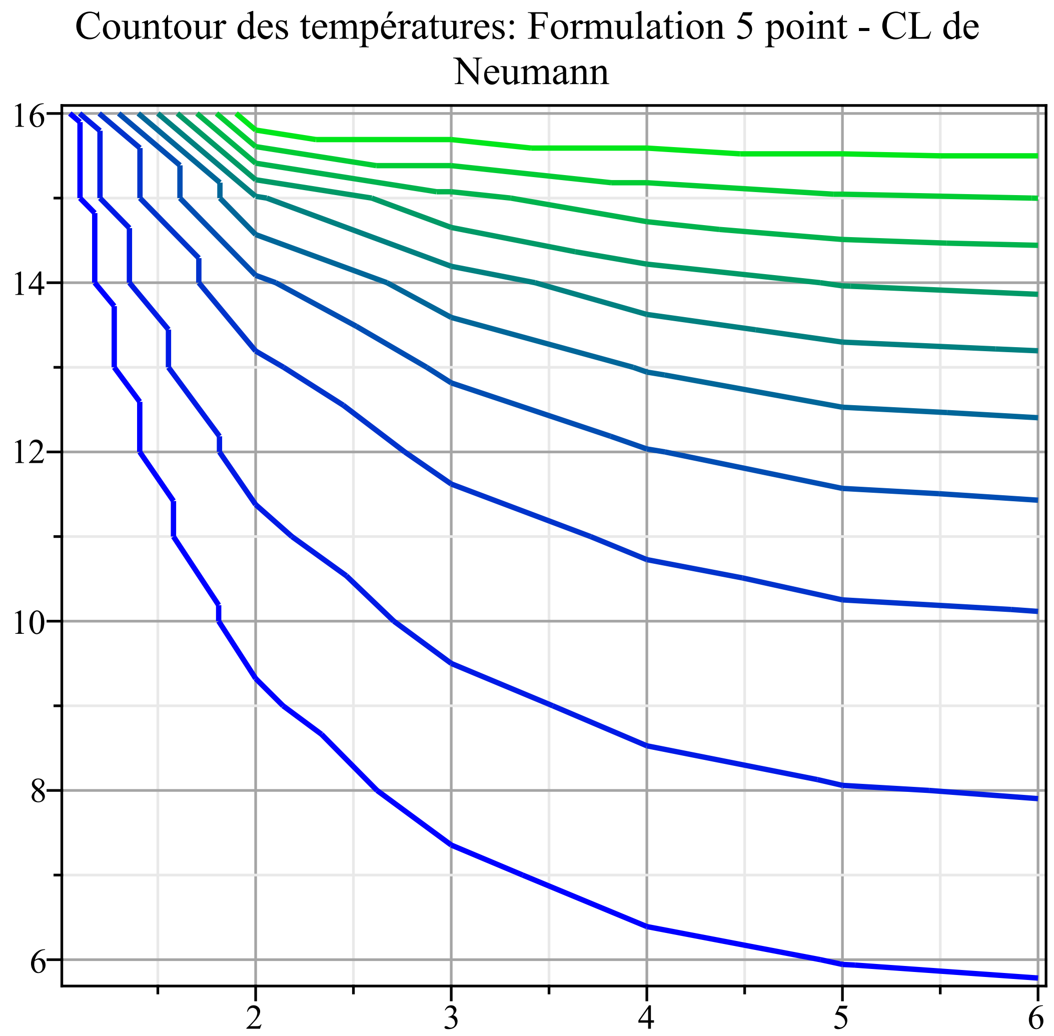
> $with(plots):$

> **for** i **from** 1 **to** $i_{max} - 2$ **do** $Ns[i] := i \cdot \frac{N-14}{i_{max}-2}$ **end do**:

> $GTemps := [[seq(T_{1,j}, j = 1..j_{max})], [T_{2,1}, seq(rhs(SolT_{1,i}), i = 1..Ns_1), T_{2,j_{max}}], [T_{3,1}, seq(rhs(SolT_{1,i}), i = Ns_1 + 1..Ns_2), T_{3,j_{max}}], [T_{4,1}, seq(rhs(SolT_{1,i}), i = Ns_2 + 1..Ns_3), T_{4,j_{max}}], [T_{5,1}, seq(rhs(SolT_{1,i}), i = Ns_3 + 1..Ns_4), T_{5,j_{max}}], [T_{6,1}, seq(rhs(SolT_{1,j}), j = N - 13..N), T_{6,j_{max}}]]:$

Tracé des isothermes:

> *listcontplot*(*GTemps*, *title* = "Countour des températures: Formulation 5 point - CL de Neumann", *axes* = *boxed*, *gridlines* = *true*, *thickness* = 2, *coloring* = [*blue*, *green*], *contours* = [5, 10, 20, 30, 40, 50, 60, 70, 80, 90])


Countour des températures: Formulation 5 point - CL de Neumann
16
14
12
10
8
6
2
3
4
5
6

> ?