



$$T_{3,4} := 100$$

$$T_{4,4} := 100$$

$k := 1 :$

**for j from 2 to  $j_{\max} - 1$  do**

**for i from 2 to  $i_{\max} - 1$  do**

$$Eq[k] := -2 \cdot (1 + \beta^2) \cdot T[i, j] + T[i + 1, j] + T[i - 1, j] + \beta^2 \cdot (T[i, j + 1] + T[i, j - 1]) = 0 :$$

$$Temps[k] := T[i, j] :$$

$k := k + 1 :$

**end do:**

$$Eq[k] := -2 \cdot (1 + \beta^2) \cdot T[i_{\max}, j] + 2 \cdot T[i_{\max} - 1, j] + \beta^2 \cdot (T[i_{\max}, j - 1] + T[i_{\max}, j + 1]) + 2 \cdot \alpha \cdot \Delta x = 0 :$$

$$Temps[k] := T[i_{\max}, j] :$$

$k := k + 1 :$

**end do:**

**> for k from 1 to N do Eq[k] end do;**

$$-4 T_{2,2} + T_{3,2} + T_{2,3} = 0$$

$$-4 T_{3,2} + T_{4,2} + T_{2,2} + T_{3,3} = 0$$

$$-4 T_{4,2} + 2 T_{3,2} + T_{4,3} = 0$$

$$-4 T_{2,3} + T_{3,3} + 100 + T_{2,2} = 0$$

$$-4 T_{3,3} + T_{4,3} + T_{2,3} + 100 + T_{3,2} = 0$$

$$-4 T_{4,3} + 2 T_{3,3} + T_{4,2} + 100 = 0$$

**> N := k - 1;**

$$N := 6$$

**> Eqs := {seq(Eq[k], k = 1..N)};**

$$Eqs := \{-4 T_{2,2} + T_{3,2} + T_{2,3} = 0, -4 T_{4,2} + 2 T_{3,2} + T_{4,3} = 0, -4 T_{2,3} + T_{3,3} + 100 + T_{2,2} = 0, -4 T_{3,2} + T_{4,2} + T_{2,2} + T_{3,3} = 0, -4 T_{4,3} + 2 T_{3,3} + T_{4,2} + 100 = 0, -4 T_{3,3} + T_{4,3} + T_{2,3} + 100 + T_{3,2} = 0\}$$

**> Tmps := [seq(Temps[k], k = 1..N)];**

$$Tmps := [T_{2,2}, T_{3,2}, T_{4,2}, T_{2,3}, T_{3,3}, T_{4,3}]$$

**> SolT := evalf(solve(Eqs, Tmps));**

$$SolT := [[T_{2,2} = 17.37373737, T_{3,2} = 25.75757576, T_{4,2} = 28.08080808, T_{2,3} = 43.73737374, T_{3,3} = 57.57575758, T_{4,3} = 60.80808081]]$$

```
> Eqs := [seq(Eq[k], k = 1..N)];
Eqs := [-4 T2,2 + T3,2 + T2,3 = 0, -4 T3,2 + T4,2 + T2,2 + T3,3 = 0, -4 T4,2 + 2 T3,2
+ T4,3 = 0, -4 T2,3 + T3,3 + 100 + T2,2 = 0, -4 T3,3 + T4,3 + T2,3 + 100 + T3,2 = 0,
-4 T4,3 + 2 T3,3 + T4,2 + 100 = 0]
```

```
> M, R := GenerateMatrix(Eqs, Tmps)
```

$$M, R := \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 2 & -4 & 0 & 0 & 1 \\ 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -100 \\ -100 \\ -100 \end{bmatrix}$$

## Tracé des Iso

```
[> with(plottools) : with(plots) :
```

```
> Schema5d := proc(Tg, Nd, Tb, Th, NbIso)
  local a, b, ndx, ndy, Δx, β, i, j, imax, jmax, N, T, k, Eq, Temps, Eqs, Tmps, SolT,
    LT, Ns, GTemps, Iso, TMax, TMin, DeltaT;
  a := 15 : b := 15 : ndx := 15 : ndy := 15 : β := 1. : imax := ndx + 1 : jmax := ndy
    + 1 : Δx :=  $\frac{a}{ndx}$  :
  for j from 2 to jmax - 1 do T[1, j] := Tg end do:
  for i from 2 to imax - 1 do T[i, 1] := Tb end do:
  for i from 2 to imax - 1 do T[i, jmax] := Th end do:
  T[1, 1] :=  $\frac{Tg + Tb}{2}$  :
  T[imax, jmax] := Th :
  T[imax, 1] := Tb :
  T[1, jmax] :=  $\frac{Tg + Th}{2}$  :
  k := 1 :
  for i from 2 to imax - 1 do
    for j from 2 to jmax - 1 do
      Eq[k] := T[i + 1, j] + T[i - 1, j] + β2 · (T[i, j + 1] + T[i, j - 1]) - 2 · (1
        + β2) · T[i, j] = 0 :
      Temps[k] := T[i, j] :
    k := k + 1 :
  end do:
```

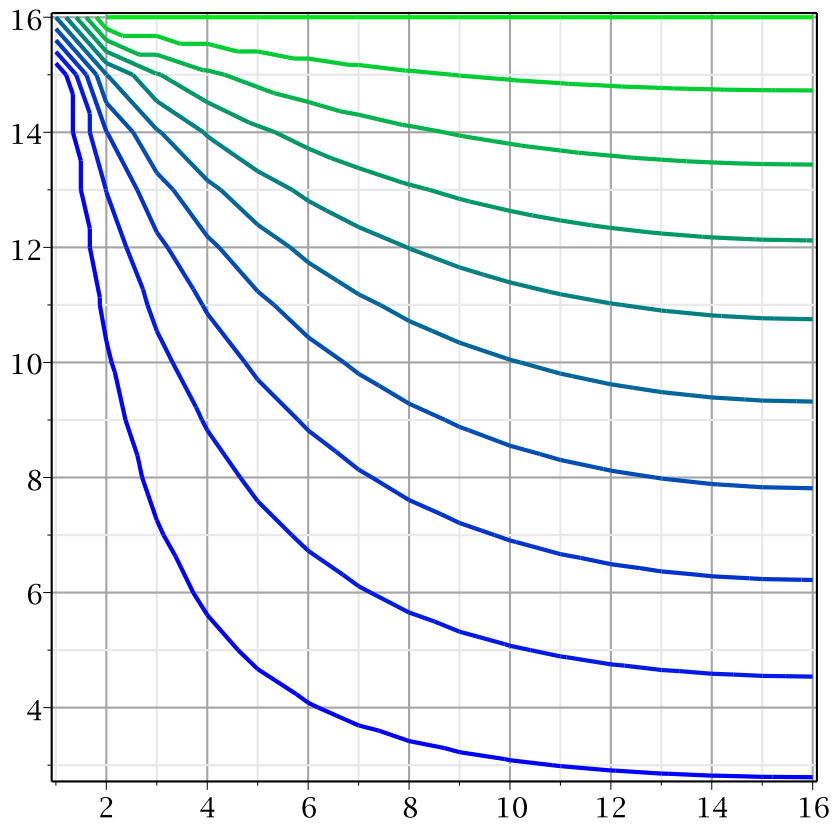
```

end do:
for j from 2 to  $j_{\max} - 1$  do
   $Eq[k] := 2 \cdot (1 + \beta^2) \cdot T[i_{\max}, j] = 2 \cdot T[i_{\max} - 1, j] + \beta^2 \cdot (T[i_{\max}, j - 1] + T[i_{\max}, j$ 
     $+ 1]) + 2 \cdot Nd \cdot \Delta x :$ 
   $Temps[k] := T[i_{\max}, j] :$ 
   $k := k + 1 :$ 
end do:
 $N := k - 1 :$ 
 $Eqs := \{seq(Eq[i], i = 1 .. N)\} :$ 
 $Tmps := [seq(Temps[i], i = 1 .. N)] :$ 
 $SolT := solve(Eqs, Tmps) :$ 
 $k := 1 :$ 
for i from 2 to  $i_{\max}$  do
  for j from 2 to  $j_{\max} - 1$  do
     $T[i, j] := rhs(SolT_{1, k}) :$ 
     $k := k + 1 :$ 
  end do:
end do:
 $GTemps := [seq([seq(T[i, j], j = 1 ..  $j_{\max}$ )], i = 1 ..  $i_{\max}$ )] :$ 
 $TMax := \max(seq(seq(T[i, j], j = 1 ..  $j_{\max}$ ), i = 1 ..  $i_{\max}$ )) :$ 
 $TMin := \min(seq(seq(T[i, j], j = 1 ..  $j_{\max}$ ), i = 1 ..  $i_{\max}$ )) :$ 
 $DeltaT := evalf\left(\frac{TMax - TMin}{NbIso}\right) :$ 
for k from 1 to  $NbIso$  do  $Iso[k] := k \cdot DeltaT$  end do:
 $listcontplot(GTemps, axes = boxed, gridlines = true, thickness = 2, coloring$ 
   $= [blue, green], contours = [seq(Iso[k], k = 1 ..  $NbIso$ )], legend$ 
   $= [seq(convert(Iso[k], string), k = 1 ..  $NbIso$ )]$ 
end proc:

```

### Tracé des isothermes:

```
> Schema5d(0, 0, 0, 100, 10)
```



**Condition Limite droite de Neumann discrétisée par un schéma décentré d'ordre 1:**

```

[> restart: with(LinearAlgebra) :
> L := 15; H := 15; ndx := 3; ndy := 3;
                                     L:= 15
                                     H:= 15
                                     ndx:= 3
                                     ndy:= 3
> Tg := 0; Tb := 0; Th := 100; alpha := 0
                                     Tg:= 0
                                     Tb:= 0
                                     Th:= 100
                                     alpha:= 0

```

>  $\Delta x := \frac{L}{ndx}$ ;  $\Delta y := \frac{H}{ndy}$ ;  $\beta := \frac{\Delta x}{\Delta y}$

$\Delta x := 5$

$\Delta y := 5$

$\beta := 1$

>  $i_{\max} := ndx + 1$ ;  $j_{\max} := ndy + 1$ ;

$i_{\max} := 4$

$j_{\max} := 4$

>  $N := (i_{\max} - 2) \cdot (j_{\max} - 2)$ ;

$N := 4$

> **for j from 2 to  $j_{\max} - 1$  do  $T[1, j] := Tg$  end do;**

$T_{1,2} := 0$

$T_{1,3} := 0$

> **for i from 2 to  $i_{\max}$  do  $T[i, 1] := Tb$  end do;**

$T_{2,1} := 0$

$T_{3,1} := 0$

$T_{4,1} := 0$

> **for i from 2 to  $i_{\max}$  do  $T[i, j_{\max}] := Th$  end do;**

$T_{2,4} := 100$

$T_{3,4} := 100$

$T_{4,4} := 100$

>  $k := 1$  :

**for j from 2 to  $j_{\max} - 1$  do**

$T[i_{\max}, j] := T[i_{\max} - 1, j] + \alpha \cdot \Delta x$ ;

**for i from 2 to  $i_{\max} - 1$  do**

$Eq[k] := -2 \cdot (1 + \beta^2) \cdot T[i, j] + T[i + 1, j] + T[i - 1, j] + \beta^2 \cdot (T[i, j + 1]$   
 $+ T[i, j - 1]) = 0$ ;

$Temps[k] := T[i, j]$ ;

$k := k + 1$  :

**end do;**

**end do;**

> **for k from 1 to  $N$  do  $Eq[k]$  end do;**

$-4 T_{2,2} + T_{3,2} + T_{2,3} = 0$

```

-3 T3,2 + T2,2 + T3,3 = 0
-4 T2,3 + T3,3 + 100 + T2,2 = 0
-3 T3,3 + T2,3 + 100 + T3,2 = 0
> N := k - 1;
N := 4
> Eqs := {seq(Eq[k], k = 1 ..N)};
Eqs := {-4 T2,2 + T3,2 + T2,3 = 0, -3 T3,2 + T2,2 + T3,3 = 0, -4 T2,3 + T3,3 + 100 + T2,2
= 0, -3 T3,3 + T2,3 + 100 + T3,2 = 0}
> Tmps := [seq(Temps[k], k = 1 ..N)];
Tmps := [T2,2, T3,2, T2,3, T3,3]
> SolT := evalf(solve(Eqs, Tmps));
SolT := [[ T2,2 = 16.84210526, T3,2 = 24.21052632, T2,3 = 43.15789474, T3,3
= 55.78947368]]
> Eqs := [seq(Eq[k], k = 1 ..N)];
Eqs := [-4 T2,2 + T3,2 + T2,3 = 0, -3 T3,2 + T2,2 + T3,3 = 0, -4 T2,3 + T3,3 + 100 + T2,2
= 0, -3 T3,3 + T2,3 + 100 + T3,2 = 0]
> M, R := GenerateMatrix(Eqs, Tmps)
M, R :=  $\begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -3 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -100 \\ -100 \end{bmatrix}$ 
>

```

## ▼ Condition Limite droite de Neumann discrétisée par un schéma décentré d'ordre 1 (Version2):

```

> restart: with(LinearAlgebra) :
> L := 15; H := 15; ndx := 3; ndy := 3;
L := 15
H := 15
ndx := 3
ndy := 3
> Tg := 0; Tb := 0; Th := 100; alpha := 0
Tg := 0
Tb := 0

```

```

Th := 100
α := 0
> Δx :=  $\frac{L}{ndx}$ ; Δy :=  $\frac{H}{ndy}$ ; β :=  $\frac{\Delta x}{\Delta y}$ 
Δx := 5
Δy := 5
β := 1
> i_max := ndx + 1; j_max := ndy + 1;
i_max := 4
j_max := 4
> N := (i_max - 2) · (j_max - 2);
N := 4

> for j from 2 to j_max - 1 do T[1, j] := Tg end do;
T1,2 := 0
T1,3 := 0
> for i from 2 to i_max do T[i, 1] := Tb end do;
T2,1 := 0
T3,1 := 0
T4,1 := 0
> for i from 2 to i_max do T[i, j_max] := Th end do;
T2,4 := 100
T3,4 := 100
T4,4 := 100
> k := 1:
for j from 2 to j_max - 1 do
for i from 2 to i_max - 2 do
Eq[k] := -2 · (1 + β2) · T[i, j] + T[i + 1, j] + T[i - 1, j] + β2 · (T[i, j + 1]
+ T[i, j - 1]) = 0:
Temps[k] := T[i, j]:
k := k + 1:
end do:
Eq[k] := - (1 + 2 · β2) · T[i_max - 1, j] + T[i_max - 2, j] + β2 · (T[i_max - 1, j + 1]
+ T[i_max - 1, j - 1]) + α · Δx = 0:
Temps[k] := T[i_max - 1, j]:

```



```

    k := k + 1 :
end do:

> for k from 1 to N do Eq[k] end do;
    -4 T2,2 + T3,2 + T2,3 = 0
    -3 T3,2 + T2,2 + T3,3 = 0
    -4 T2,3 + T3,3 + 100 + T2,2 = 0
    -3 T3,3 + T2,3 + 100 + T3,2 = 0

> N := k - 1;
    N := 4

> Eqs := {seq(Eq[k], k = 1 ..N) };
Eqs := { -4 T2,2 + T3,2 + T2,3 = 0, -3 T3,2 + T2,2 + T3,3 = 0, -4 T2,3 + T3,3 + 100 + T2,2
= 0, -3 T3,3 + T2,3 + 100 + T3,2 = 0 }

> Tmps := [seq(Temps[k], k = 1 ..N) ];
    Tmps := [ T2,2, T3,2, T2,3, T3,3 ]

> SolT := evalf(solve(Eqs, Tmps));
SolT := [[ T2,2 = 16.84210526, T3,2 = 24.21052632, T2,3 = 43.15789474, T3,3
= 55.78947368 ]]

> Eqs := [seq(Eq[k], k = 1 ..N) ];
Eqs := [ -4 T2,2 + T3,2 + T2,3 = 0, -3 T3,2 + T2,2 + T3,3 = 0, -4 T2,3 + T3,3 + 100 + T2,2
= 0, -3 T3,3 + T2,3 + 100 + T3,2 = 0 ]

> M, R := GenerateMatrix(Eqs, Tmps)
    M, R :=  $\begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -3 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -100 \\ -100 \end{bmatrix}$ 

```

▼ **Condition Limite droite de Neumann discrétisée par un schéma décentré d'ordre 2:**

```

> restart: with(LinearAlgebra) :
> L := 15; H := 15; ndx := 3; ndy := 3;
    L:= 15
    H:= 15
    ndx:= 3
    ndy:= 3
> Tg := 0; Tb := 0; Th := 100; α := 0
    Tg:= 0
    Tb:= 0
    Th:= 100
    α:= 0
> Δx :=  $\frac{L}{ndx}$ ; Δy :=  $\frac{H}{ndy}$ ; β :=  $\frac{\Delta x}{\Delta y}$ 
    Δx:= 5
    Δy:= 5
    β:= 1
> imax := ndx + 1; jmax := ndy + 1;
    imax:= 4
    jmax:= 4
> N := ( imax - 2 ) · ( jmax - 2 );
    N:= 4

> for j from 2 to jmax - 1 do T[1, j] := Tg end do;
    T1,2:= 0
    T1,3:= 0
> for i from 2 to imax do T[i, 1] := Tb end do;
    T2,1:= 0
    T3,1:= 0
    T4,1:= 0
> for i from 2 to imax do T[i, jmax] := Th end do;
    T2,4:= 100
    T3,4:= 100
    T4,4:= 100
> k := 1:

```

**for j from 2 to  $j_{\max} - 1$  do**

$$T[i_{\max}, j] := \frac{1}{3} \cdot (4 \cdot T[i_{\max} - 1, j] - T[i_{\max} - 2, j] + 2 \cdot \alpha \cdot \Delta x);$$

**for i from 2 to  $i_{\max} - 1$  do**

$$Eq[k] := -2 \cdot (1 + \beta^2) \cdot T[i, j] + T[i + 1, j] + T[i - 1, j] + \beta^2 \cdot (T[i, j + 1] + T[i, j - 1]) = 0;$$

$$Temps[k] := T[i, j];$$

$$k := k + 1;$$

**end do:**

**end do:**

> **for k from 1 to N do Eq[k] end do;**

$$-4 T_{2,2} + T_{3,2} + T_{2,3} = 0$$

$$-\frac{8}{3} T_{3,2} + \frac{2}{3} T_{2,2} + T_{3,3} = 0$$

$$-4 T_{2,3} + T_{3,3} + 100 + T_{2,2} = 0$$

$$-\frac{8}{3} T_{3,3} + \frac{2}{3} T_{2,3} + 100 + T_{3,2} = 0$$

>

>  $N := k - 1;$

$$N := 4$$

>  $Eqs := \{seq(Eq[k], k = 1..N)\};$

$$Eqs := \left\{ -4 T_{2,2} + T_{3,2} + T_{2,3} = 0, -\frac{8}{3} T_{3,2} + \frac{2}{3} T_{2,2} + T_{3,3} = 0, -4 T_{2,3} + T_{3,3} + 100 + T_{2,2} = 0, -\frac{8}{3} T_{3,3} + \frac{2}{3} T_{2,3} + 100 + T_{3,2} = 0 \right\}$$

>  $Tmps := [seq(Temps[k], k = 1..N)];$

$$Tmps := [T_{2,2}, T_{3,2}, T_{2,3}, T_{3,3}]$$

>  $SolT := evalf(solve(Eqs, Tmps));$

$$SolT := [[T_{2,2} = 17.56168360, T_{3,2} = 26.26995646, T_{2,3} = 43.97677794, T_{3,3} = 58.34542816]]$$

>

>  $Eqs := [seq(Eq[k], k = 1..N)];$

$$Eqs := \left[ -4 T_{2,2} + T_{3,2} + T_{2,3} = 0, -\frac{8}{3} T_{3,2} + \frac{2}{3} T_{2,2} + T_{3,3} = 0, -4 T_{2,3} + T_{3,3} + 100 + T_{2,2} = 0, -\frac{8}{3} T_{3,3} + \frac{2}{3} T_{2,3} + 100 + T_{3,2} = 0 \right]$$

>  $M, R := GenerateMatrix(Eqs, Tmps)$

$$M, R := \begin{bmatrix} -4 & 1 & 1 & 0 \\ \frac{2}{3} & -\frac{8}{3} & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & \frac{2}{3} & -\frac{8}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -100 \\ -100 \end{bmatrix}$$

## Condition Limite droite de Neumann discrétisée par un schéma décentré d'ordre 2 (Version 2):

```
> restart: with(LinearAlgebra) :
```

```
> L := 15; H := 15; ndx := 3; ndy := 3;
```

```
L:= 15
```

```
H:= 15
```

```
ndx:= 3
```

```
ndy:= 3
```

```
> Tg := 0; Tb := 0; Th := 100; alpha := 0
```

```
Tg:= 0
```

```
Tb:= 0
```

```
Th:= 100
```

```
alpha:= 0
```

```
> dx := L/ndx; dy := H/ndy; beta := dx/dy
```

```
dx:= 5
```

```
dy:= 5
```

```
beta:= 1
```

```
> imax := ndx + 1; jmax := ndy + 1;
```

```
imax:= 4
```

```
jmax:= 4
```

```
> N := (imax - 2) * (jmax - 2);
```

```
N:= 4
```

```
> for j from 2 to jmax - 1 do T[1, j] := Tg end do;
```

```

                                T1,2 := 0
                                T1,3 := 0
> for i from 2 to imax do T[i, 1] := Tb end do;
                                T2,1 := 0
                                T3,1 := 0
                                T4,1 := 0
> for i from 2 to imax do T[i, jmax] := Th end do;
                                T2,4 := 100
                                T3,4 := 100
                                T4,4 := 100
> k := 1:
  for j from 2 to jmax - 1 do
    for i from 2 to imax - 2 do
      Eq[k] := -2 · (1 + β2) · T[i, j] + T[i + 1, j] + T[i - 1, j] + β2 · (T[i, j + 1]
      + T[i, j - 1]) = 0:
      Temps[k] := T[i, j]:
      k := k + 1:
    end do:
    Eq[k] := -2 · (1 + 3 · β2) · T[imax - 1, j] + 2 · T[imax - 2, j] + 3 · β2 · (T[imax
    - 1, j + 1] + T[imax - 1, j - 1]) + 2 · α · Δx = 0:
    Temps[k] := T[imax - 1, j]:
    k := k + 1:
  end do:

> for k from 1 to N do Eq[k] end do;
                                -4 T2,2 + T3,2 + T2,3 = 0
                                -8 T3,2 + 2 T2,2 + 3 T3,3 = 0
                                -4 T2,3 + T3,3 + 100 + T2,2 = 0
                                -8 T3,3 + 2 T2,3 + 300 + 3 T3,2 = 0

> N := k - 1;
                                N := 4

> Eqs := {seq(Eq[k], k = 1..N)};
Eqs := {-4 T2,2 + T3,2 + T2,3 = 0, -8 T3,2 + 2 T2,2 + 3 T3,3 = 0, -4 T2,3 + T3,3 + 100
+ T2,2 = 0, -8 T3,3 + 2 T2,3 + 300 + 3 T3,2 = 0}

```

```

> Tmps := [seq(Temps[k], k = 1..N)];
           Tmps := [T2,2, T3,2, T2,3, T3,3]
=
> SolT := evalf(solve(Eqs, Tmps));
SolT := [[ T2,2 = 17.56168360, T3,2 = 26.26995646, T2,3 = 43.97677794, T3,3
          = 58.34542816]]

> Eqs := [seq(Eq[k], k = 1..N)];
Eqs := [-4 T2,2 + T3,2 + T2,3 = 0, -8 T3,2 + 2 T2,2 + 3 T3,3 = 0, -4 T2,3 + T3,3 + 100
         + T2,2 = 0, -8 T3,3 + 2 T2,3 + 300 + 3 T3,2 = 0]

> M, R := GenerateMatrix(Eqs, Tmps)
           M, R :=  $\begin{bmatrix} -4 & 1 & 1 & 0 \\ 2 & -8 & 0 & 3 \\ 1 & 0 & -4 & 1 \\ 0 & 3 & 2 & -8 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -100 \\ -300 \end{bmatrix}$ 
=
>

```