

```
> restart
> with(LinearAlgebra) :
> with(GraphTheory) : with(SpecialGraphs) :
```

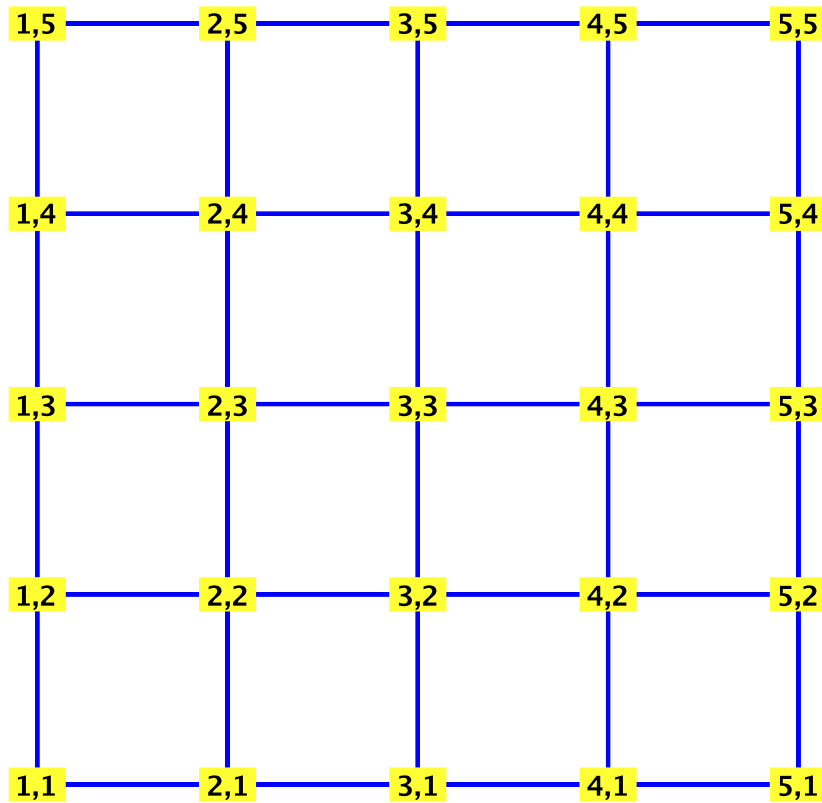
```
> imax := 5
```

```
imax:= 5
```

```
> jmax := 5
```

```
jmax:= 5
```

```
> G := GridGraph(imax, jmax) : DrawGraph(G);
```



```
> N := (imax - 3) * (jmax - 2)
```

```
N:= 6
```

```
> for j from 2 to jmax - 1 do T[1, j] := 500 end do;
```

$$T_{1,2} := 500$$

$$T_{1,3} := 500$$

$$T_{1,4} := 500$$

> for j from 2 to $jmax - 1$ do $T[imax, j] := 500$ end do;

$$T_{5,2} := 500$$

$$T_{5,3} := 500$$

$$T_{5,4} := 500$$

> for i from 2 to $imax - 1$ do $T[i, jmax] := 500$ end do;

$$T_{2,5} := 500$$

$$T_{3,5} := 500$$

$$T_{4,5} := 500$$

>

> $k := 3$

$$k := 3$$

> for j from 2 to $jmax - 1$ do;

 for i from 2 to $imax - 2$ do;

$$Eq[k] := -4 \cdot T[i, j] + T[i-1, j] + T[i+1, j] + T[i, j-1] + T[i, j+1] = 0;$$

$$T[k] := T[i, j];$$

$$k := k + 1;$$

 end do;

end do;

> seq($Eq[k]$, $k = 3..N+2$)

$$\begin{aligned} -4 T_{2,2} + 500 + T_{3,2} + T_{2,1} + T_{2,3} = 0, & -4 T_{3,2} + T_{2,2} + T_{4,2} + T_{3,1} + T_{3,3} = 0, & -4 T_{2,3} \\ & + 500 + T_{3,3} + T_{2,2} + T_{2,4} = 0, & -4 T_{3,3} + T_{2,3} + T_{4,3} + T_{3,2} + T_{3,4} = 0, & -4 T_{2,4} + 1000 \\ & + T_{3,4} + T_{2,3} = 0, & -4 T_{3,4} + T_{2,4} + T_{4,4} + T_{3,3} + 500 = 0 \end{aligned}$$

> $T[4, 2] := T[2, 2]$; $T[4, 3] := T[2, 3]$; $T[4, 4] := T[2, 4]$;

$$T_{4,2} := T_{2,2}$$

$$T_{4,3} := T_{2,3}$$

$$T_{4,4} := T_{2,4}$$

> seq($Eq[k]$, $k = 3..N+2$)

$$\begin{aligned} -4 T_{2,2} + 500 + T_{3,2} + T_{2,1} + T_{2,3} = 0, & -4 T_{3,2} + 2 T_{2,2} + T_{3,1} + T_{3,3} = 0, & -4 T_{2,3} + 500 \\ & + T_{3,3} + T_{2,2} + T_{2,4} = 0, & -4 T_{3,3} + 2 T_{2,3} + T_{3,2} + T_{3,4} = 0, & -4 T_{2,4} + 1000 + T_{3,4} \\ & + T_{2,3} = 0, & -4 T_{3,4} + 2 T_{2,4} + T_{3,3} + 500 = 0 \end{aligned}$$

> $Eq[1] := -9 \cdot T[2, 1] + 2000 \cdot T[3, 1] + 2 \cdot T[2, 2] = 0$;

$$Eq_1 := -9 T_{2,1} + 2000. + T_{3,1} + 2 T_{2,2} = 0$$

$$> Eq[2] := -9 \cdot T[3, 1] + 1500. + 2 \cdot T[3, 2] + 2 \cdot T[2, 1] = 0;$$

$$Eq_2 := -9 T_{3,1} + 1500. + 2 T_{3,2} + 2 T_{2,1} = 0$$

$$> Eqs := \{seq(Eq[k], k = 1..N + 2)\}$$

$$Eqs := \{-9 T_{2,1} + 2000. + T_{3,1} + 2 T_{2,2} = 0, -4 T_{2,4} + 1000 + T_{3,4} + T_{2,3} = 0, -9 T_{3,1} + 1500. + 2 T_{3,2} + 2 T_{2,1} = 0, -4 T_{3,2} + 2 T_{2,2} + T_{3,1} + T_{3,3} = 0, -4 T_{3,3} + 2 T_{2,3} + T_{3,2} + T_{3,4} = 0, -4 T_{3,4} + 2 T_{2,4} + T_{3,3} + 500 = 0, -4 T_{2,2} + 500 + T_{3,2} + T_{2,1} + T_{2,3} = 0, -4 T_{2,3} + 500 + T_{3,3} + T_{2,2} + T_{2,4} = 0\}$$

$$> Vars := \{T[2, 1], T[3, 1], seq(T[k], k = 3..N + 2)\}$$

$$Vars := \{T_{2,1}, T_{2,2}, T_{2,3}, T_{2,4}, T_{3,1}, T_{3,2}, T_{3,3}, T_{3,4}\}$$

$$> solve(Eqs, Vars);$$

$$\{T_{2,1} = 356.9946105, T_{2,2} = 436.9497540, T_{2,3} = 472.0650755, T_{2,4} = 489.3047233, T_{3,1} = 339.0519867, T_{3,2} = 418.7393298, T_{3,3} = 462.0058247, T_{3,4} = 485.1538178\}$$

$$> (A, b) := GenerateMatrix(Eqs, Vars);$$

$$A, b := \begin{bmatrix} -9 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & -9 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 & -4 \\ 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & -4 & 1 \end{bmatrix}, \begin{bmatrix} -2000. \\ -1000 \\ -1500. \\ -500 \\ -500 \\ -500 \\ 0 \\ 0 \end{bmatrix}$$

$$> A$$

$$\begin{bmatrix} -9 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & -9 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 & -4 \\ 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & -4 & 1 \end{bmatrix}$$

$$> AI := MatrixInverse(A);$$

$$AI :=$$

$$\left[\begin{array}{cccccccc} -\frac{3736}{29873}, & -\frac{391}{29873}, & -\frac{536}{29873}, & -\frac{248}{29873}, & -\frac{2679}{29873}, & -\frac{1068}{29873}, & -\frac{1088}{29873}, & \\ -\frac{601}{29873} & & & & & & & \\ -\frac{2679}{59746}, & -\frac{3023}{59746}, & -\frac{544}{29873}, & -\frac{1841}{59746}, & -\frac{21935}{59746}, & -\frac{4205}{29873}, & -\frac{7113}{59746}, & -\frac{4341}{59746} \\ -\frac{534}{29873}, & -\frac{3814}{29873}, & -\frac{601}{59746}, & -\frac{3845}{59746}, & -\frac{4205}{29873}, & -\frac{11411}{29873}, & -\frac{4341}{59746}, & -\frac{3876}{29873} \\ -\frac{391}{59746}, & -\frac{19799}{59746}, & -\frac{124}{29873}, & -\frac{5911}{59746}, & -\frac{3023}{59746}, & -\frac{3814}{29873}, & -\frac{1841}{59746}, & -\frac{3845}{59746} \\ -\frac{1072}{29873}, & -\frac{496}{29873}, & -\frac{3736}{29873}, & -\frac{391}{29873}, & -\frac{2176}{29873}, & -\frac{1202}{29873}, & -\frac{2679}{29873}, & -\frac{1068}{29873} \\ -\frac{1088}{29873}, & -\frac{1841}{29873}, & -\frac{2679}{59746}, & -\frac{3023}{59746}, & -\frac{7113}{29873}, & -\frac{4341}{29873}, & -\frac{21935}{59746}, & -\frac{4205}{29873} \\ -\frac{601}{29873}, & -\frac{3845}{29873}, & -\frac{534}{29873}, & -\frac{3814}{29873}, & -\frac{4341}{29873}, & -\frac{7752}{29873}, & -\frac{4205}{29873}, & -\frac{11411}{29873} \\ -\frac{248}{29873}, & -\frac{5911}{29873}, & -\frac{391}{59746}, & -\frac{19799}{59746}, & -\frac{1841}{29873}, & -\frac{3845}{29873}, & -\frac{3023}{59746}, & -\frac{3814}{29873} \end{array} \right]$$

> $AI := \text{evalf}(\text{MatrixInverse}(A));$

$AI := [[-0.1250627657, -0.01308874234, -0.01794262377, -0.008301811000,$
 $-0.08967964383, -0.03575134737, -0.03642084826, -0.02011850166],$
 $[-0.04483982191, -0.05059752954, -0.01821042413, -0.03081377833,$
 $-0.3671375490, -0.1407625615, -0.1190539952, -0.07265758377],$
 $[-0.01787567369, -0.1276738192, -0.01005925083, -0.06435577277,$
 $-0.1407625615, -0.3819837311, -0.07265758377, -0.1297492719],$
 $[-0.006544371171, -0.3313862016, -0.004150905500, -0.09893549359,$
 $-0.05059752954, -0.1276738192, -0.03081377833, -0.06435577277],$
 $[-0.03588524755, -0.01660362200, -0.1250627657, -0.01308874234,$
 $-0.07284169652, -0.04023700331, -0.08967964383, -0.03575134737],$
 $[-0.03642084826, -0.06162755666, -0.04483982191, -0.05059752954,$
 $-0.2381079905, -0.1453151675, -0.3671375490, -0.1407625615],$
 $[-0.02011850166, -0.1287115455, -0.01787567369, -0.1276738192,$
 $-0.1453151675, -0.2594985438, -0.1407625615, -0.3819837311],$
 $[-0.008301811000, -0.1978709872, -0.006544371171, -0.3313862016,$
 $-0.06162755666, -0.1287115455, -0.05059752954, -0.1276738192]]$

> b

```
[ -2000.  
  -1000  
  -1500.  
   -500  
   -500  
   -500  
    0  
    0 ]
```

```
> MatrixVectorMultiply(AI, b);
```

```
[ 356.994610495000  
  436.949753970000  
  472.065075510000  
  489.304723357000  
  339.051986735000  
  418.739329815000  
  462.005824605000  
  485.153817836500 ]
```

```
>
```