```
> restart:
> with(plots):
```

Données:

```
> Δt := 0.05; L := 1.0; c := 1;
```

$$\Delta t := 0.05$$

$$L := 1.0$$

$$c := 1$$

```
> i_max := 21;
```

$$i_{max} := 21$$

```
> Δx := L/(i_max-1);
```

$$\Delta x := 0.05000000000$$

```
> λ := c^2·(Δt/Δx)^2;
```

$$\lambda := 1.000000000$$

```
> n_max := 20;
```

$$n_{max} := 20$$

```
> α := 0; β := 0;
```

$$\alpha := 0$$

$$\beta := 0$$

```
> f := unapply(2·sin(π·x/L), x, numeric):
> g := unapply(-sin(2·π·x/L), x, numeric):
```

Condition initiale:

```
> for i from 2 to i_max - 1 do u[i, 0] := evalf(f((i-1)·Δx)) end do;
```

$$u_{2,0} := 0.3128689302$$

$$u_{3,0} := 0.6180339888$$

$$u_{4,0} := 0.9079809996$$

$$u_{5,0} := 1.175570505$$

$$u_{6,0} := 1.414213563$$

$$u_{7,0} := 1.618033989$$

$$u_{8,0} := 1.782013049$$

$$u_{9,0} := 1.902113033$$

$$u_{10,0} := 1.975376681$$
$$u_{11,0} := 2.$$
$$u_{12,0} := 1.975376681$$
$$u_{13,0} := 1.902113033$$
$$u_{14,0} := 1.782013048$$
$$u_{15,0} := 1.618033988$$
$$u_{16,0} := 1.414213563$$
$$u_{17,0} := 1.175570504$$
$$u_{18,0} := 0.9079809986$$
$$u_{19,0} := 0.6180339872$$
$$u_{20,0} := 0.3128689300$$

Condition limite gauche:

> **for** n **from** 0 **to** n_{max} **do** $u[1, n] := \alpha$ **end do**;

$$u_{1,0} := 0$$
$$u_{1,1} := 0$$
$$u_{1,2} := 0$$
$$u_{1,3} := 0$$
$$u_{1,4} := 0$$
$$u_{1,5} := 0$$
$$u_{1,6} := 0$$
$$u_{1,7} := 0$$
$$u_{1,8} := 0$$
$$u_{1,9} := 0$$
$$u_{1,10} := 0$$
$$u_{1,11} := 0$$
$$u_{1,12} := 0$$
$$u_{1,13} := 0$$
$$u_{1,14} := 0$$
$$u_{1,15} := 0$$
$$u_{1,16} := 0$$
$$u_{1,17} := 0$$
$$u_{1,18} := 0$$

$$u_{1,\,19} := 0$$

$$u_{1,\,20} := 0$$

Condition limite droite:

> **for** n **from** 0 **to** n_{max} **do** $u[i_{max},\, n] := \beta$ **end do**;

$$u_{21,\,0} := 0$$

$$u_{21,\,1} := 0$$

$$u_{21,\,2} := 0$$

$$u_{21,\,3} := 0$$

$$u_{21,\,4} := 0$$

$$u_{21,\,5} := 0$$

$$u_{21,\,6} := 0$$

$$u_{21,\,7} := 0$$

$$u_{21,\,8} := 0$$

$$u_{21,\,9} := 0$$

$$u_{21,\,10} := 0$$

$$u_{21,\,11} := 0$$

$$u_{21,\,12} := 0$$

$$u_{21,\,13} := 0$$

$$u_{21,\,14} := 0$$

$$u_{21,\,15} := 0$$

$$u_{21,\,16} := 0$$

$$u_{21,\,17} := 0$$

$$u_{21,\,18} := 0$$

$$u_{21,\,19} := 0$$

$$u_{21,\,20} := 0$$

Calcul des u[i,1]:

> **for** i **from** 2 **to** $i_{max} - 1$ **do**

$$u[i,\,1] := (1 - \lambda)\cdot evalf(f((i-1)\cdot\Delta x)) + \frac{\lambda}{2}\cdot(evalf(f(i\cdot\Delta x)) + evalf(f((i-2)\cdot\Delta x))) + \Delta t\cdot evalf(g((i-1)\cdot\Delta x));$$

end do;

$$u_{2,\,1} := 0.2935661447$$

$$u_{3,\,1} := 0.5810357024$$

$$u_{4,1} := 0.8563513973$$
$$u_{5,1} := 1.113544456$$
$$u_{6,1} := 1.346802247$$
$$u_{7,1} := 1.550560480$$
$$u_{8,1} := 1.719622661$$
$$u_{9,1} := 1.849305602$$
$$u_{10,1} := 1.935605666$$
$$u_{11,1} := 1.975376681$$
$$u_{12,1} := 1.966507366$$
$$u_{13,1} := 1.908084127$$
$$u_{14,1} := 1.800524360$$
$$u_{15,1} := 1.645666132$$
$$u_{16,1} := 1.446802246$$
$$u_{17,1} := 1.208650107$$
$$u_{18,1} := 0.9372530952$$
$$u_{19,1} := 0.6398142271$$
$$u_{20,1} := 0.3244678433$$

Boucle principale:

```
> for n from 1 to n_max do
    for i from 2 to i_max - 1 do
        u[i, n+1] := 2·(1 - λ)·u[i, n] + λ·(u[i+1, n] + u[i-1, n]) - u[i, n-1]
    end do
  end do;
```

Affichage des résultats au temps n_{max} :

```
> for i from 1 to i_max do u[i, n_max] end do;
```

$$0$$
$$-0.3128689280$$
$$-0.6180339870$$
$$-0.9079809990$$
$$-1.175570503$$
$$-1.414213562$$
$$-1.618033987$$
$$-1.782013047$$

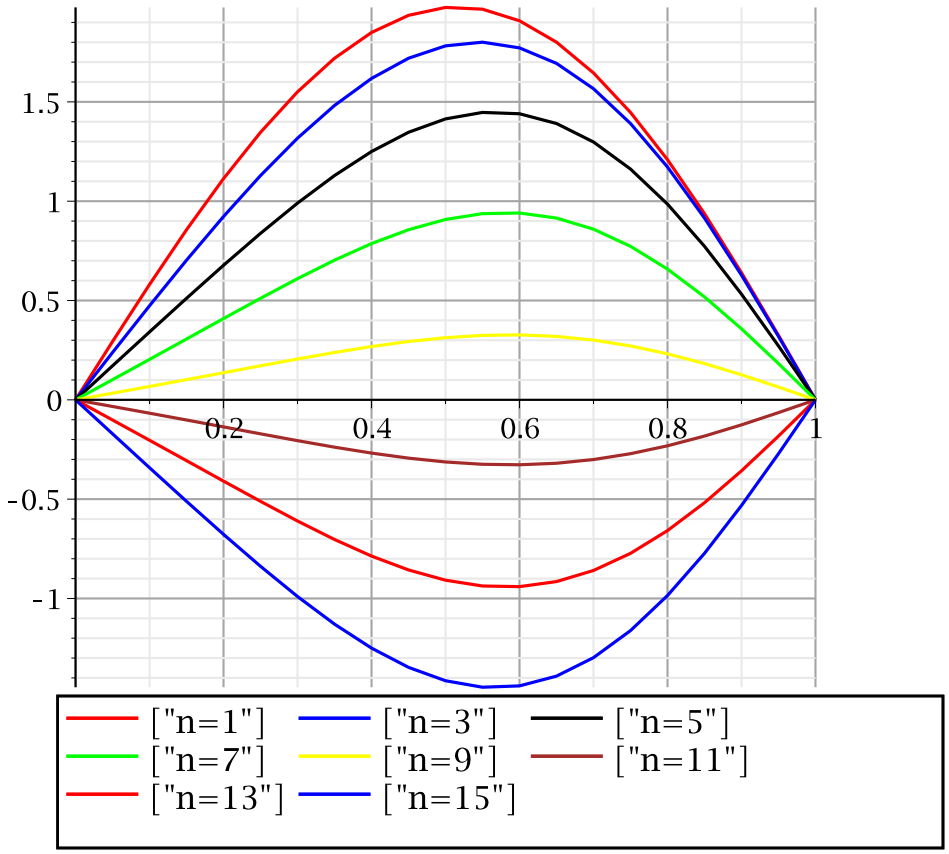
-1.902113032
-1.975376680
-1.999999999
-1.975376681
-1.902113032
-1.782013049
-1.618033988
-1.414213563
-1.175570505
-0.9079810000
-0.6180339890
-0.3128689290
0

Préparation des listes pour le tracé:

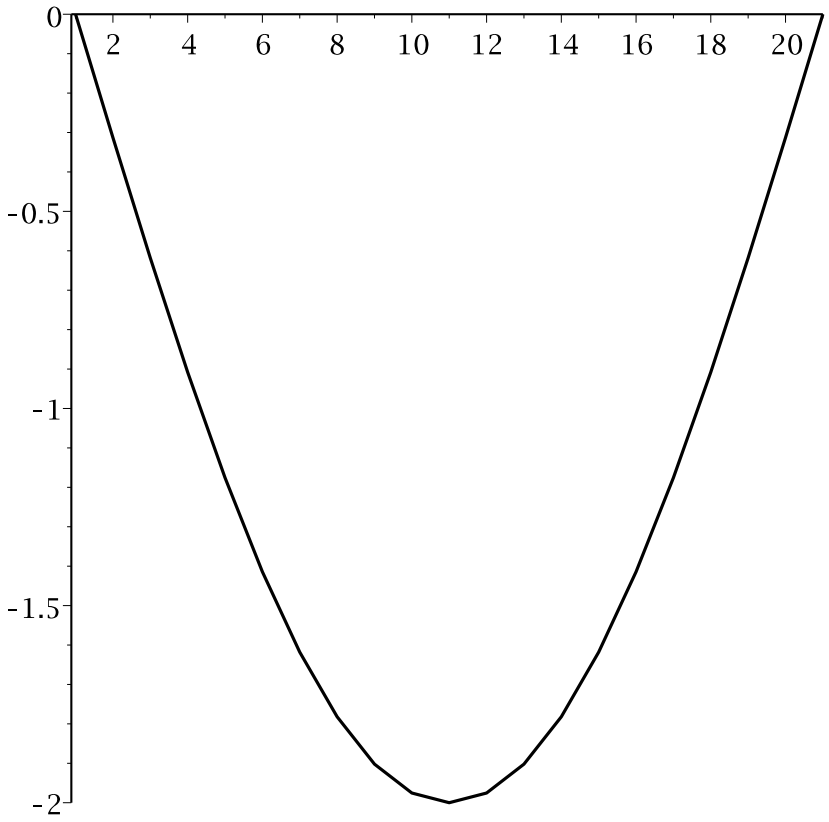
```
> for n from 1 to n_max do
    liste[n] := [seq([(i-1)·Δx, u[i, n]], i = 1..i_max)]
  end do:
```

Tracés des courbes:

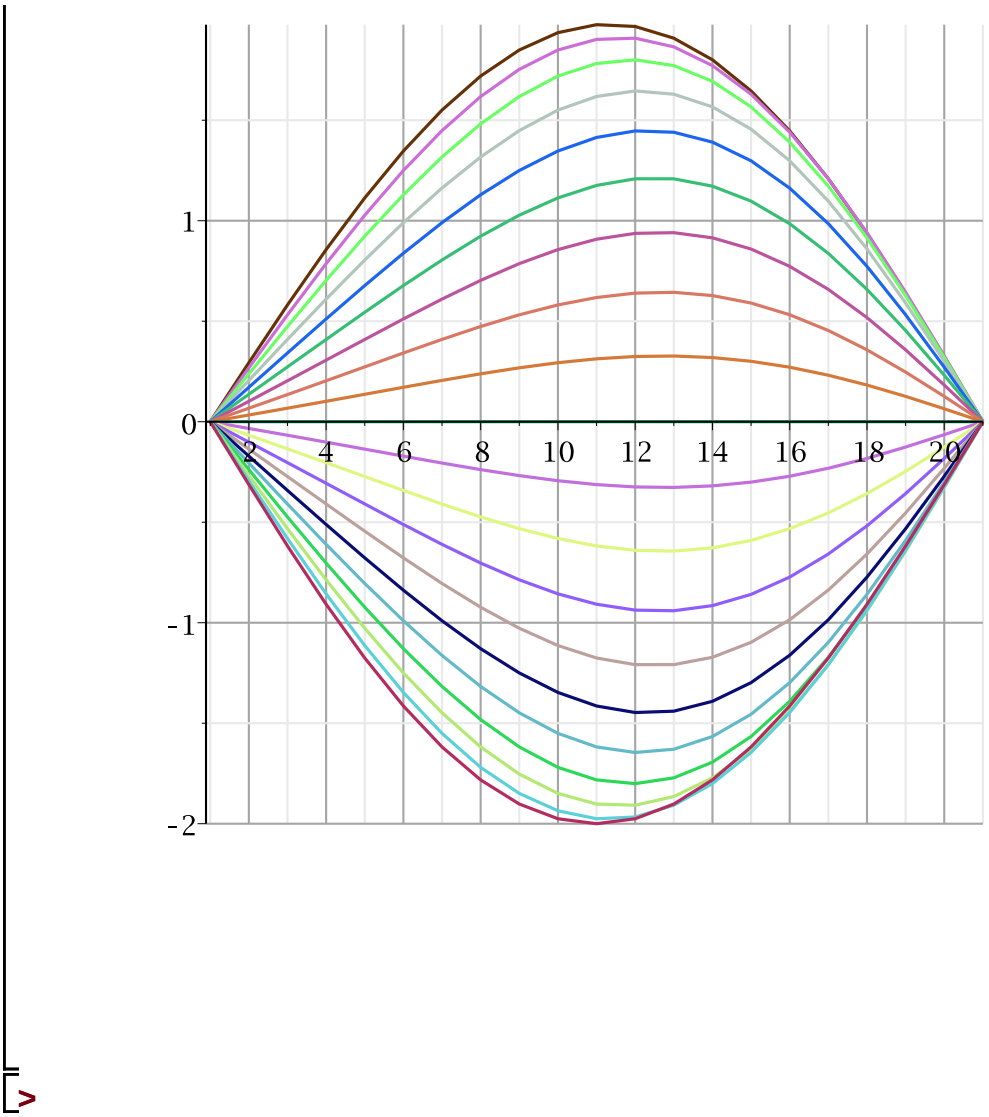
```
> multiple(listplot, [liste[1], color = red, legend = ["n=1"]], [liste[3], color = blue,
    legend = ["n=3"]], [liste[5], color = black, legend = ["n=5"]], [liste[7], color
    = green, legend = ["n=7"]], [liste[9], color = yellow, legend = ["n=9"]],
    [liste[11], color = brown, legend = ["n=11"]], [liste[13], color = red, legend
    = ["n=13"]], [liste[15], color = blue, legend = ["n=15"]], gridlines = true);
```




```
> for n from 1 to n_max do
    liste[n] := [α, seq(u[i, n], i = 2 .. i_max - 1), β]
  end do:
> listplot(liste[n_max]);
```



> $multiple\left(listplot,\ seq\left(\left[liste[i],\ color = COLOR\left(RGB,\ \frac{rand(\,)}{10^{12}},\ \frac{rand(\,)}{10^{12}},\ \frac{rand(\,)}{10^{12}} \right) \right],\ i = 1..n_{\max} \right),\ gridlines = true \right);$


1
0
-1
-2
2
4
6
8
10
12
14
16
18
20