Equation de Diffusion 2D

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EXAMEN

Détermination de la distribution de temperature $T(x, y)$ à travers une plaque de largeur L, de hauteur H, d'éapisseur e et de conductivité thermique k soumise aux (C. L.) soit de Dirichler soit de Neumann..

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x},\mathrm{y})\right)+\frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x},\mathrm{y})\right)=0$$

$$\begin{aligned} T(0, y) = T_w \quad & ou\ bien \quad q(0, y) = q_w, \\ T(L, y) = T_e \quad & ou\ bien \quad q(L, y) = q_e, \\ T(x, 0) = T_s \quad & ou\ bien \quad q(x, 0) = q_s, \\ T(x, H) = T_n \quad & ou\ bien \quad q(x, H) = q_n, \end{aligned}$$

Solution

```
> Restart: Digits := 4:
```

Données:

```
> L := 0.3; H := 0.4; e := 0.01; k := 1000; δx := 0.1; δy := 0.1;
```

$$L := 0.3$$
$$H := 0.4$$
$$e := 0.01$$
$$k := 1000$$
$$\delta x := 0.1$$
$$\delta y := 0.1$$

Calcul du nombre de divisions:

```
> ndx := L/δx ; ndy := H/δy;
```

$$ndx := 3.000$$
$$ndy := 4.000$$

Calcul des surfaces:

```
> Aw := δy·e;
  Ae := δy·e;
  As := δx·e;
  An := δx·e;
```

$$Aw := 0.001$$
$$Ae := 0.001$$
$$As := 0.001$$
$$An := 0.001$$

```
> i_max := round(ndx); j_max := round(ndy);
```

$$i_{max} := 3$$
$$j_{max} := 4$$

Nombre d'équations:

```
> Ne := i_max· j_max
```

$$Ne := 12$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to i_max do
      x[i] := δx/2 + (i-1)·δx;
  end do;
  x[ i_max + 1 ] := L;
```

$$x_0 := 0$$
$$x_1 := 0.05000$$
$$x_2 := 0.1500$$
$$x_3 := 0.2500$$
$$x_4 := 0.3$$

Ordonnées des noeuds:

```
> y[0] := 0;
```

```
for j from 1 to j_max do
    y[j] := δy/2 + (j - 1)·δy;
end do;
y[j_max + 1] := H;
```

$$y_0 := 0$$
$$y_1 := 0.05000$$
$$y_2 := 0.1500$$
$$y_3 := 0.2500$$
$$y_4 := 0.3500$$
$$y_5 := 0.4$$

Conditions aux Limites:

```
> T_w := 0; T_e := 0; T_s := 0; T_n := 100;
```

$$T_w := 0$$
$$T_{0.01} := 0$$
$$T_s := 0$$
$$T_n := 100$$

```
> q_w := 5·10^5; q_e := 0; q_s := 0; q_n := 0;
```

$$q_w := 500000$$
$$q_{0.01} := 0$$
$$q_s := 0$$
$$q_n := 0$$

Noeuds internes:

```
> for j from 2 to j_max - 1 do
    for i from 2 to i_max - 1 do
      Su[i, j] := 0;
      a_W[i, j] := k·Aw/δx;
      a_E[i, j] := k·Ae/δx;
      a_S[i, j] := k·As/δy;
      a_N[i, j] := k·An/δy;
      a_P[i, j] := k·Aw/δx + k·Ae/δx + k·As/δy + k·An/δy;
      Sp[i, j] := a_W[i, j] + a_E[i, j] + a_S[i, j] + a_N[i, j] - a_P[i, j];
    end do;
```

end do;

Noeuds Ouest:

> **for** j **from** 2 **to** $j_{\max}-1$ **do**

$$Su[1,j] := q_w \cdot Aw + \frac{2\cdot k\cdot Aw}{\delta x}\cdot T_w;$$

$$a_W[1,j] := 0;$$

$$a_E[1,j] := \frac{k\cdot Ae}{\delta x};$$

$$a_S[1,j] := \frac{k\cdot As}{\delta y};$$

$$a_N[1,j] := \frac{k\cdot An}{\delta y};$$

$$a_P[1,j] := \frac{2\cdot k\cdot Aw}{\delta x} + \frac{k\cdot Ae}{\delta x} + \frac{k\cdot As}{\delta y} + \frac{k\cdot An}{\delta y};$$

if $T_w = 0$ **then** $a_P[1,j] := \frac{k\cdot Ae}{\delta x} + \frac{k\cdot As}{\delta y} + \frac{k\cdot An}{\delta y}$ **end if**;

$$Sp[1,j] := a_W[1,j] + a_E[1,j] + a_S[1,j] + a_N[1,j] - a_P[1,j];$$

end do;

$$Su_{1,2} := 500.0$$
$$a_{W_{1,2}} := 0$$
$$a_{E_{1,2}} := 10.00$$
$$a_{S_{1,2}} := 10.00$$
$$a_{N_{1,2}} := 10.00$$
$$a_{P_{1,2}} := 50.00$$
$$Sp_{1,2} := 0.$$
$$Su_{1,3} := 500.0$$
$$a_{W_{1,3}} := 0$$
$$a_{E_{1,3}} := 10.00$$
$$a_{S_{1,3}} := 10.00$$
$$a_{N_{1,3}} := 10.00$$
$$a_{P_{1,3}} := 50.00$$
$$Sp_{1,3} := 0.$$

Noeuds Est:

> **for** j **from** 2 **to** $j_{\max}-1$ **do**

$Su[\ i_{max}, j] := q_e \cdot Ae + \frac{2 \cdot k \cdot Ae}{\delta x} \cdot T_e;$

$a_W[\ i_{max}, j] := \frac{k \cdot Aw}{\delta x};$

$a_E[\ i_{max}, j] := 0;$

$a_S[\ i_{max}, j] := \frac{k \cdot As}{\delta y};$

$a_N[\ i_{max}, j] := \frac{k \cdot An}{\delta y};$

$a_P[\ i_{max}, j] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

if $T_e = 0$ **then** $a_P[\ i_{max}, j] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**;

$Sp[\ i_{max}, j] := a_W[\ i_{max}, j] + a_E[\ i_{max}, j] + a_S[\ i_{max}, j] + a_N[\ i_{max}, j] - a_P[\ i_{max}, j];$

end do;

$$Su_{3,\ 2} := 0.$$
$$a_{W_{3,\ 2}} := 10.00$$
$$a_{E_{3,\ 2}} := 0$$
$$a_{S_{3,\ 2}} := 10.00$$
$$a_{N_{3,\ 2}} := 10.00$$
$$a_{P_{3,\ 2}} := 50.00$$
$$Sp_{3,\ 2} := 0.$$
$$Su_{3,\ 3} := 0.$$
$$a_{W_{3,\ 3}} := 10.00$$
$$a_{E_{3,\ 3}} := 0$$
$$a_{S_{3,\ 3}} := 10.00$$
$$a_{N_{3,\ 3}} := 10.00$$
$$a_{P_{3,\ 3}} := 50.00$$
$$Sp_{3,\ 3} := 0.$$

Noeuds Sud:

> **for** i **from** 2 **to** $i_{max} - 1$ **do**

$Su[\ i,\ 1\ ] := q_s \cdot As + \frac{2 \cdot k \cdot As}{\delta y} \cdot T_s;$

$a_W[\ i,\ 1\ ] := \frac{k \cdot Aw}{\delta x};$

$a_E[\ i,\ 1\ ] := \frac{k \cdot Ae}{\delta x};$

$a_S[i, 1] := 0;$

$a_N[i, 1] := \frac{k \cdot An}{\delta y};$

$a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

if $T_s = 0$ **then** $a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if;**

$Sp[i, 1] := a_W[i, 1] + a_E[i, 1] + a_S[i, 1] + a_N[i, 1] - a_P[i, 1];$

end do;

$$Su_{2,1} := 0.$$
$$a_{W_{2,1}} := 10.00$$
$$a_{E_{2,1}} := 10.00$$
$$a_{S_{2,1}} := 0$$
$$a_{N_{2,1}} := 10.00$$
$$a_{P_{2,1}} := 50.00$$
$$Sp_{2,1} := 0.$$

Noeuds Nord:

> **for** i **from** 2 **to** $i_{\max} - 1$ **do**

$Su[i, j_{\max}] := q_n \cdot An + \frac{2 \cdot k \cdot An}{\delta y} \cdot T_n;$

$a_W[i, j_{\max}] := \frac{k \cdot Aw}{\delta x};$

$a_E[i, j_{\max}] := \frac{k \cdot Ae}{\delta x};$

$a_S[i, j_{\max}] := \frac{k \cdot As}{\delta y};$

$a_N[i, j_{\max}] := 0;$

$a_P[i, j_{\max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y};$

if $T_n = 0$ **then** $a_P[i, j_{\max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if;**

$Sp[i, j_{\max}] := a_W[i, j_{\max}] + a_E[i, j_{\max}] + a_S[i, j_{\max}] + a_N[i, j_{\max}] - a_P[i, j_{\max}];$

end do;

$$Su_{2,4} := 2000.$$
$$a_{W_{2,4}} := 10.00$$
$$a_{E_{2,4}} := 10.00$$
$$a_{S_{2,4}} := 10.00$$

$$a_{N_{2,4}} := 0$$
$$a_{P_{2,4}} := 50.00$$
$$Sp_{2,4} := -20.00$$

Noeud (1,1):

> $Su[1,1] := Su[1,2] + Su[2,1];$

$a_W[1,1] := 0;$

$a_E[1,1] := \frac{k \cdot Ae}{\delta x};$

$a_S[1,1] := 0;$

$a_N[1,1] := \frac{k \cdot An}{\delta y};$

if $(T_w = 0$ **and** $T_s = 0)$ **then** $a_P[1,1] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if;**

if $(T_w = 0$ **and** $T_s \neq 0)$ **then** $a_P[1,1] := \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if;**

if $(T_w \neq 0$ **and** $T_s = 0)$ **then** $a_P[1,1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if;**

if $(T_w \neq 0$ **and** $T_s \neq 0)$ **then** $a_P[1,1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if;

$Sp[1,1] := a_W[1,1] + a_E[1,1] + a_S[1,1] + a_N[1,1] - a_P[1,1];$

$$Su_{1,1} := 500.0$$
$$a_{W_{1,1}} := 0$$
$$a_{E_{1,1}} := 10.00$$
$$a_{S_{1,1}} := 0$$
$$a_{N_{1,1}} := 10.00$$
$$a_{P_{1,1}} := 20.00$$
$$Sp_{1,1} := 0.$$

Noeud (imax,1):

> $Su[i_{max},1] := Su[i_{max},2] + Su[2,1];$

$a_W[i_{max},1] := \frac{k \cdot Aw}{\delta x};$

$a_E[i_{max},1] := 0;$

$a_S[i_{max},1] := 0;$

$a_N[i_{max},1] := \frac{k \cdot An}{\delta y};$

if $(T_e = 0$ **and** $T_s = 0)$ **then** $a_P[i_{max},1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if;**

if $(T_e = 0$ and $T_s \neq 0)$ then $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ end if;

if $(T_e \neq 0$ and $T_s = 0)$ then $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ end if;

if $(T_e \neq 0$ and $T_s \neq 0)$ then $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if;

$Sp[i_{max}, 1] := a_W[i_{max}, 1] + a_E[i_{max}, 1] + a_S[i_{max}, 1] + a_N[i_{max}, 1] - a_P[i_{max}, 1]$;

$$Su_{3,1} := 0.$$
$$a_{W_{3,1}} := 10.00$$
$$a_{E_{3,1}} := 0$$
$$a_{S_{3,1}} := 0$$
$$a_{N_{3,1}} := 10.00$$
$$a_{P_{3,1}} := 20.00$$
$$Sp_{3,1} := 0.$$

Noeud (1,jmax):

> $Su[1, j_{max}] := Su[1, 2] + Su[2, j_{max}]$;

$a_W[1, j_{max}] := 0$;

$a_E[1, j_{max}] := \frac{k \cdot Ae}{\delta x}$;

$a_S[1, j_{max}] := \frac{k \cdot As}{\delta y}$;

$a_N[1, j_{max}] := 0$;

if $(T_w = 0$ and $T_n = 0)$ then $a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ end if;

if $(T_w = 0$ and $T_n \neq 0)$ then $a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ end if;

if $(T_w \neq 0$ and $T_n = 0)$ then $a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ end if;

if $(T_w \neq 0$ and $T_n \neq 0)$ then $a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ end if;

$Sp[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - a_P[1, j_{max}]$;

$$Su_{1,4} := 2500.$$
$$a_{W_{1,4}} := 0$$

$$a_{E_{1,4}} := 10.00$$
$$a_{S_{1,4}} := 10.00$$
$$a_{N_{1,4}} := 0$$
$$a_{P_{1,4}} := 40.00$$
$$Sp_{1,4} := -20.00$$

Noeud (imax,jmax):

> $Su[i_{max}, j_{max}] := Su[i_{max}, 2] + Su[2, j_{max}];$

$a_W[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x};$

$a_E[i_{max}, j_{max}] := 0;$

$a_S[i_{max}, j_{max}] := \frac{k \cdot As}{\delta y};$

$a_N[i_{max}, j_{max}] := 0;$

if $(T_e = 0$ **and** $T_n = 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if;**

if $(T_e = 0$ **and** $T_n \neq 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if;**

if $(T_e \neq 0$ **and** $T_n = 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if;**

if $(T_e \neq 0$ **and** $T_n \neq 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if;**

$Sp[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}] - a_P[i_{max}, j_{max}];$

$$Su_{3,4} := 2000.$$
$$a_{W_{3,4}} := 10.00$$
$$a_{E_{3,4}} := 0$$
$$a_{S_{3,4}} := 10.00$$
$$a_{N_{3,4}} := 0$$
$$a_{P_{3,4}} := 40.00$$
$$Sp_{3,4} := -20.00$$

Equations:

> $k := 1:$

Résolution pour les noeuds internes:

> **for** j **from** 1 **to** j_{max} **do**
for i **from** 1 **to** i_{max} **do**
$Eq[k] := a_P[i, j] \cdot T[i, j] = a_W[i, j] \cdot T[i-1, j] + a_E[i, j] \cdot T[i+1, j]$

```
        + a_S[i, j]·T[i, j − 1] + a_N[i, j]·T[i, j + 1] + Su[i, j];
          Var[k] := T[i, j];
          k := k + 1;
    end do;
    end do;
```

Ecriture du systme d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$20.00\,T_{1,1} = 10.00\,T_{1,2} + 10.00\,T_{2,1} + 500.0$$
$$30.00\,T_{2,1} = 10.00\,T_{2,2} + 10.00\,T_{1,1} + 10.00\,T_{3,1}$$
$$20.00\,T_{3,1} = 10.00\,T_{3,2} + 10.00\,T_{2,1}$$
$$30.00\,T_{1,2} = 10.00\,T_{1,1} + 10.00\,T_{1,3} + 10.00\,T_{2,2} + 500.0$$
$$40.00\,T_{2,2} = 10.00\,T_{2,1} + 10.00\,T_{2,3} + 10.00\,T_{1,2} + 10.00\,T_{3,2}$$
$$30.00\,T_{3,2} = 10.00\,T_{3,1} + 10.00\,T_{3,3} + 10.00\,T_{2,2}$$
$$30.00\,T_{1,3} = 10.00\,T_{1,2} + 10.00\,T_{1,4} + 10.00\,T_{2,3} + 500.0$$
$$40.00\,T_{2,3} = 10.00\,T_{2,2} + 10.00\,T_{2,4} + 10.00\,T_{1,3} + 10.00\,T_{3,3}$$
$$30.00\,T_{3,3} = 10.00\,T_{3,2} + 10.00\,T_{3,4} + 10.00\,T_{2,3}$$
$$40.00\,T_{1,4} = 10.00\,T_{1,3} + 10.00\,T_{2,4} + 2500.$$
$$50.00\,T_{2,4} = 10.00\,T_{2,3} + 10.00\,T_{1,4} + 10.00\,T_{3,4} + 2000.$$
$$40.00\,T_{3,4} = 10.00\,T_{3,3} + 10.00\,T_{2,4} + 2000.$$

```
> Eqs := [seq(Eq[k], k = 1 ..Ne)]:
> Vars := [seq(Var[k], k = 1 ..Ne)]:
> SolT := solve(Eqs, Vars);
```

$$SolT := [[T_{1,1} = 260.0,\ T_{2,1} = 227.8,\ T_{3,1} = 212.2,\ T_{1,2} = 242.3,\ T_{2,2} = 211.2,\ T_{3,2} = 196.5,\ T_{1,3} = 205.6,\ T_{2,3} = 178.2,\ T_{3,3} = 166.2,\ T_{1,4} = 146.3,\ T_{2,4} = 129.7,\ T_{3,4} = 124.0]]$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Vars)
```

$$A, b := \begin{bmatrix} 12\ x\ 12\ Matrix \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}, \begin{bmatrix} 1\ ..\ 12\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
> seq(b[i], i = 1 ..Ne)
```

500.0, 0, 0, 500.0, 0, 0, 500.0, 0, 0, 2500., 2000., 2000.

```
> seq(A[i, i], i = 1 ..Ne)
```

20.00, 30.00, 20.00, 30.00, 40.00, 30.00, 30.00, 40.00, 30.00, 40.00, 50.00, 40.00

Récapitulation:

```
> seq(seq(a_W[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

0, 10.00, 10.00, 0, 10.00, 10.00, 0, 10.00, 10.00, 0, 10.00, 10.00

```
> seq(seq(a_E[i, j], i = 1 ..i_max), j = 1 ..j_max)
```

10.00, 10.00, 0, 10.00, 10.00, 0, 10.00, 10.00, 0, 10.00, 10.00, 0

> $seq(seq(a_S[i,j], i=1..i_{max}), j=1..j_{max})$

0, 0, 0, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00

> $seq(seq(a_N[i,j], i=1..i_{max}), j=1..j_{max})$

10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 10.00, 0, 0, 0

> $seq(seq(Su[i,j], i=1..i_{max}), j=1..j_{max})$

500.0, 0., 0., 500.0, 0, 0., 500.0, 0, 0., 2500., 2000., 2000.

> $seq(seq(a_P[i,j], i=1..i_{max}), j=1..j_{max})$

20.00, 30.00, 20.00, 30.00, 40.00, 30.00, 30.00, 40.00, 30.00, 40.00, 50.00, 40.00

> $seq(seq(Sp[i,j], i=1..i_{max}), j=1..j_{max})$

0., 0., 0., 0., 0., 0., 0., 0., 0., -20.00, -20.00, -20.00

>