Equation de Diffusion 2D

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EXAMEN

Détermination de la distribution de temperature $T(x, y)$ à travers une plaque de largeur L, de hauteur H, d'éapisseur e et de conductivité thermique k soumise aux (C. L.) soit de Dirichler soit de Neumann..

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x},\mathrm{y})\right)+\frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x},\mathrm{y})\right)=0$$

$$\begin{aligned} T(0, y) = T_w \quad & ou\ bien \quad q(0, y) = q_w, \\ T(L, y) = T_e \quad & ou\ bien \quad q(L, y) = q_e, \\ T(x, 0) = T_s \quad & ou\ bien \quad q(x, 0) = q_s, \\ T(x, H) = T_n \quad & ou\ bien \quad q(x, H) = q_n, \end{aligned}$$

Solution

```
> Restart: Digits := 4:
```

Données:

```
> L := 0.3; H := 0.4; e := 0.01; k := 1000; δx := 0.1; δy := 0.1;
```

$$L := 0.3$$
$$H := 0.4$$
$$e := 0.01$$
$$k := 1000$$
$$\delta x := 0.1$$
$$\delta y := 0.1$$

Calcul du nombre de divisions:

```
> ndx := trunc(L/δx); ndy := trunc(H/δy);
```

$$ndx := 3$$
$$ndy := 4$$

Calcul des surfaces:

```
> Aw := δy·e;
  Ae := δy·e;
  As := δx·e;
  An := δx·e;
```

$$Aw := 0.001$$
$$Ae := 0.001$$
$$As := 0.001$$
$$An := 0.001$$

```
> i_max := round(ndx); j_max := round(ndy);
```

$$i_{max} := 3$$
$$j_{max} := 4$$

Nombre d'équations:

```
> Ne := i_max · j_max
```

$$Ne := 12$$

Conditions aux Limites:

```
> T_w := 0: T_e := 0: T_s := 0: T_n := 100:

> q_w := 5·10^5: q_e := 0: q_s := 0: q_n := 0:
```

Noeuds internes:

```
> for j from 2 to j_max − 1 do
    for i from 2 to i_max − 1 do
      Su[i, j] := 0;
      a_W[i, j] := k·Aw/δx;
      a_E[i, j] := k·Ae/δx;
```

$$a_S[i,j] := \frac{k \cdot As}{\delta y};$$

$$a_N[i,j] := \frac{k \cdot An}{\delta y};$$

$$a_P[i,j] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$$

$$Sp[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - a_P[i,j];$$

end do:

end do:

Noeuds Ouest:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$$Su[1,j] := q_w \cdot Aw + \frac{2 \cdot k \cdot Aw}{\delta x} \cdot T_w;$$

$$a_W[1,j] := 0;$$

$$a_E[1,j] := \frac{k \cdot Ae}{\delta x};$$

$$a_S[1,j] := \frac{k \cdot As}{\delta y};$$

$$a_N[1,j] := \frac{k \cdot An}{\delta y};$$

$$a_P[1,j] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$$

if $T_w = 0$ **then** $a_P[1,j] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**;

$$Sp[1,j] := a_W[1,j] + a_E[1,j] + a_S[1,j] + a_N[1,j] - a_P[1,j];$$

end do:

Noeuds Est:

> **for** j **from** 2 **to** $j_{\max} - 1$ **do**

$$Su[i_{\max},j] := q_e \cdot Ae + \frac{2 \cdot k \cdot Ae}{\delta x} \cdot T_e;$$

$$a_W[i_{\max},j] := \frac{k \cdot Aw}{\delta x};$$

$$a_E[i_{\max},j] := 0;$$

$$a_S[i_{\max},j] := \frac{k \cdot As}{\delta y};$$

$$a_N[i_{\max},j] := \frac{k \cdot An}{\delta y};$$

$$a_P[i_{\max},j] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$$

if $T_e = 0$ then $a_P[i_{max}, j] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ end if;

$Sp[i_{max}, j] := a_W[i_{max}, j] + a_E[i_{max}, j] + a_S[i_{max}, j] + a_N[i_{max}, j] - a_P[i_{max}, j];$

end do:

Noeuds Sud:

> for i from 2 to $i_{max} - 1$ do

$Su[i, 1] := q_s \cdot As + \frac{2 \cdot k \cdot As}{\delta y} \cdot T_s;$

$a_W[i, 1] := \frac{k \cdot Aw}{\delta x};$

$a_E[i, 1] := \frac{k \cdot Ae}{\delta x};$

$a_S[i, 1] := 0;$

$a_N[i, 1] := \frac{k \cdot An}{\delta y};$

$a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y};$

if $T_s = 0$ then $a_P[i, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ end if;

$Sp[i, 1] := a_W[i, 1] + a_E[i, 1] + a_S[i, 1] + a_N[i, 1] - a_P[i, 1];$

end do:

Noeuds Nord:

> for i from 2 to $i_{max} - 1$ do

$Su[i, j_{max}] := q_n \cdot An + \frac{2 \cdot k \cdot An}{\delta y} \cdot T_n;$

$a_W[i, j_{max}] := \frac{k \cdot Aw}{\delta x};$

$a_E[i, j_{max}] := \frac{k \cdot Ae}{\delta x};$

$a_S[i, j_{max}] := \frac{k \cdot As}{\delta y};$

$a_N[i, j_{max}] := 0;$

$a_P[i, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y};$

if $T_n = 0$ then $a_P[i, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ end if;

$Sp[i, j_{max}] := a_W[i, j_{max}] + a_E[i, j_{max}] + a_S[i, j_{max}] + a_N[i, j_{max}] - a_P[i, j_{max}];$

end do:

Noeud (1,1):

> $Su[1, 1] := Su[1, 2] + Su[2, 1]$:

$a_W[1, 1] := 0$:

$a_E[1, 1] := \frac{k \cdot Ae}{\delta x}$:

$a_S[1, 1] := 0$:

$a_N[1, 1] := \frac{k \cdot An}{\delta y}$:

if $(T_w = 0$ **and** $T_s = 0)$ **then** $a_P[1, 1] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_w = 0$ **and** $T_s \neq 0)$ **then** $a_P[1, 1] := \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_w \neq 0$ **and** $T_s = 0)$ **then** $a_P[1, 1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_w \neq 0$ **and** $T_s \neq 0)$ **then** $a_P[1, 1] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if:

$Sp[1, 1] := a_W[1, 1] + a_E[1, 1] + a_S[1, 1] + a_N[1, 1] - a_P[1, 1]$:

Noeud (imax,1):

> $Su[i_{max}, 1] := Su[i_{max}, 2] + Su[2, 1]$:

$a_W[i_{max}, 1] := \frac{k \cdot Aw}{\delta x}$:

$a_E[i_{max}, 1] := 0$:

$a_S[i_{max}, 1] := 0$:

$a_N[i_{max}, 1] := \frac{k \cdot An}{\delta y}$:

if $(T_e = 0$ **and** $T_s = 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e = 0$ **and** $T_s \neq 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_s = 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot An}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_s \neq 0)$ **then** $a_P[i_{max}, 1] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{2 \cdot k \cdot As}{\delta y} + \frac{k \cdot An}{\delta y}$

end if:

$Sp[i_{max}, 1] := a_W[i_{max}, 1] + a_E[i_{max}, 1] + a_S[i_{max}, 1] + a_N[i_{max}, 1] - a_P[i_{max},$

$1]:$

Noeud (1,jmax):

> $Su[1, j_{max}] := Su[1, 2] + Su[2, j_{max}]:$

$a_W[1, j_{max}] := 0:$

$a_E[1, j_{max}] := \frac{k \cdot Ae}{\delta x}:$

$a_S[1, j_{max}] := \frac{k \cdot As}{\delta y}:$

$a_N[1, j_{max}] := 0:$

if $(T_w = 0$ **and** $T_n = 0)$ **then** $a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if**:

if $(T_w = 0$ **and** $T_n \neq 0)$ **then** $a_P[1, j_{max}] := \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if**:

if $(T_w \neq 0$ **and** $T_n = 0)$ **then** $a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if**:

if $(T_w \neq 0$ **and** $T_n \neq 0)$ **then** $a_P[1, j_{max}] := \frac{2 \cdot k \cdot Aw}{\delta x} + \frac{k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if**:

$Sp[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - a_P[1, j_{max}]:$

Noeud (imax,jmax):

> $Su[i_{max}, j_{max}] := Su[i_{max}, 2] + Su[2, j_{max}]:$

$a_W[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x}:$

$a_E[i_{max}, j_{max}] := 0:$

$a_S[i_{max}, j_{max}] := \frac{k \cdot As}{\delta y}:$

$a_N[i_{max}, j_{max}] := 0:$

if $(T_e = 0$ **and** $T_n = 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if**:

if $(T_e = 0$ **and** $T_n \neq 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_n = 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y}$ **end if**:

if $(T_e \neq 0$ **and** $T_n \neq 0)$ **then** $a_P[i_{max}, j_{max}] := \frac{k \cdot Aw}{\delta x} + \frac{2 \cdot k \cdot Ae}{\delta x} + \frac{k \cdot As}{\delta y} + \frac{2 \cdot k \cdot An}{\delta y}$ **end if**:

$Sp[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}]$

$- a_P[i_{max}, j_{max}]:$

Equations:

$> k := 1:$

Résolution pour les noeuds internes:

```
> for j from 1 to jmax do
  for i from 1 to imax do
        Eq[k] := aP[i, j]·T[i, j] = aW[i, j]·T[i−1, j] + aE[i, j]·T[i+1, j] + aS[i, j]
    ·T[i, j−1] + aN[i, j]·T[i, j+1] + Su[i, j];
      Var[k] := T[i, j];
       k := k+1;
  end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$20.00\,T_{1,1} = 10.00\,T_{1,2} + 10.00\,T_{2,1} + 500.0$$
$$30.00\,T_{2,1} = 10.00\,T_{2,2} + 10.00\,T_{1,1} + 10.00\,T_{3,1}$$
$$20.00\,T_{3,1} = 10.00\,T_{3,2} + 10.00\,T_{2,1}$$
$$30.00\,T_{1,2} = 10.00\,T_{1,1} + 10.00\,T_{1,3} + 10.00\,T_{2,2} + 500.0$$
$$40.00\,T_{2,2} = 10.00\,T_{2,1} + 10.00\,T_{2,3} + 10.00\,T_{1,2} + 10.00\,T_{3,2}$$
$$30.00\,T_{3,2} = 10.00\,T_{3,1} + 10.00\,T_{3,3} + 10.00\,T_{2,2}$$
$$30.00\,T_{1,3} = 10.00\,T_{1,2} + 10.00\,T_{1,4} + 10.00\,T_{2,3} + 500.0$$
$$40.00\,T_{2,3} = 10.00\,T_{2,2} + 10.00\,T_{2,4} + 10.00\,T_{1,3} + 10.00\,T_{3,3}$$
$$30.00\,T_{3,3} = 10.00\,T_{3,2} + 10.00\,T_{3,4} + 10.00\,T_{2,3}$$
$$40.00\,T_{1,4} = 10.00\,T_{1,3} + 10.00\,T_{2,4} + 2500.$$
$$50.00\,T_{2,4} = 10.00\,T_{2,3} + 10.00\,T_{1,4} + 10.00\,T_{3,4} + 2000.$$
$$40.00\,T_{3,4} = 10.00\,T_{3,3} + 10.00\,T_{2,4} + 2000.$$

Système d'équations:

$> Eqs := [seq(Eq[k], k = 1..Ne)]:$

Variables:

$> Vars := [seq(Var[k], k = 1..Ne)]:$

Résolution du système d'équations pour les variables:

$> SolT := solve(Eqs, Vars);$

$$SolT := [[T_{1,1} = 260.0, T_{2,1} = 227.8, T_{3,1} = 212.2, T_{1,2} = 242.3, T_{2,2} = 211.2, T_{3,2} = 196.5, T_{1,3} = 205.6, T_{2,3} = 178.2, T_{3,3} = 166.2, T_{1,4} = 146.3, T_{2,4} = 129.7, T_{3,4} = 124.0]]$$

$> with(LinearAlgebra):$

Forme matricielle:

$> A, b := GenerateMatrix(Eqs, Vars)$

$$A, b := \begin{bmatrix} 12 \text{ x } 12 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}, \begin{bmatrix} 1 \text{ .. } 12 \text{ Vector}_{column} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$$

Affichage du second membre:

> $seq(\text{round}(b[i]), i = 1..Ne)$

$$500, 0, 0, 500, 0, 0, 500, 0, 0, 2500, 2000, 2000$$

Affichage de la diagonale:

> $seq(\text{round}(A[i, i]), i = 1..Ne)$

$$20, 30, 20, 30, 40, 30, 30, 40, 30, 40, 50, 40$$

Récapitulation: Affichage de tous les coefficients:

> $seq(seq(\text{round}(a_W[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$0, 10, 10, 0, 10, 10, 0, 10, 10, 0, 10, 10$$

> $seq(seq(\text{round}(a_E[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$10, 10, 0, 10, 10, 0, 10, 10, 0, 10, 10, 0$$

> $seq(seq(\text{round}(a_S[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$0, 0, 0, 10, 10, 10, 10, 10, 10, 10, 10, 10$$

> $seq(seq(\text{round}(a_N[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$10, 10, 10, 10, 10, 10, 10, 10, 10, 0, 0, 0$$

> $seq(seq(\text{round}(Su[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$500, 0, 0, 500, 0, 0, 500, 0, 0, 2500, 2000, 2000$$

> $seq(seq(\text{round}(a_P[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$20, 30, 20, 30, 40, 30, 30, 40, 30, 40, 50, 40$$

> $seq(seq(\text{round}(Sp[i, j]), i = 1..i_{\max}), j = 1..j_{\max})$

$$0, 0, 0, 0, 0, 0, 0, 0, 0, -20, -20, -20$$

>