Equation de Poisson 2D

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Matière : Méthodes Numériques Appliquées I

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les 4 extrémités sont soumises à des (C.L.) de Dirichlet et contenant une source de chaleur au centre.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = -\frac{Q}{\lambda}$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 0,$$
$$T(0, y) = 0,$$
$$T(a, y) = 0.$$

Solution discrétisée par la formulation à 5 points:

```
> Restart :
> a := 5 : b := 15 : ndx := 10 : ndy := 30 :
> β := 1. :
> Δx := a/ndx ; λ := 0.4 : Q := 40 :
```

$$\Delta x := \frac{1}{2} \quad \textbf{(1.1)}$$

```
> i_max := ndx + 1; j_max := ndy + 1;
```

$$i_{max} := 11$$

(1.2)

$$j_{max} := 31 \quad \textbf{(1.2)}$$

Nombre d'équations:

```
> N := (i_max - 2)·(j_max - 2)
```

$$N := 261 \quad \textbf{(1.3)}$$

Maillage:

```
> with(GraphTheory) : with(SpecialGraphs) :
> G := GridGraph(i_max, j_max)
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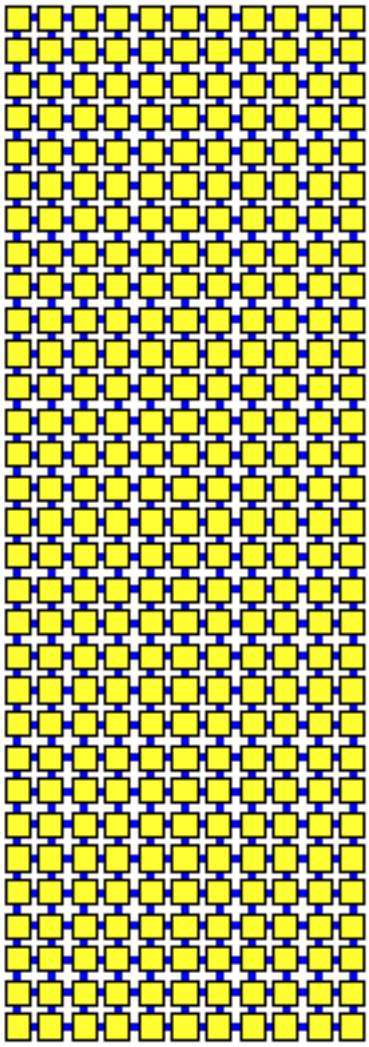
$$G := \text{Graph 1: an undirected unweighted graph with 341 vertices and 640 edge(s)} \quad \textbf{(1.4)}$$

```
> DrawGraph(G)
```



Conditions aux Limites:

```
> for i from 1 to i_max do T[i, 1] := 0 end do:
> for i from 1 to i_max do T[i, j_max] := 0 end do:
> for j from 1 to j_max do T[1, j] := 0 end do:
> for j from 1 to j_max do T[i_max, j] := 0 end do:
> k := 1 :
```

Résolution pour les noeuds internes

```
> for i from 2 to i_max - 1 do
    for j from 2 to j_max - 1 do
        Eq[k] := T[i + 1, j] + T[i - 1, j] + β^2·(T[i, j + 1] + T[i, j - 1]) - 2·(1 + β^2)
```

$$\cdot T[i, j] + \Delta x^2 \cdot \frac{Q}{\lambda} = 0;$$

$Temps[k] := T[i, j];$

$k := k + 1$

end do;

end do;

Ecriture du système d'équations:

> $Eqs := \{seq(Eq[i], i = 1..N)\}:$

> $Tmps := [seq(Temps[i], i = 1..N)]:$

> $SolT := solve(Eqs, Tmps);$

(1.1.1)

$$SolT := [[T_{2,2} = 34.70952036, T_{2,3} = 57.18321068, T_{2,4} = 72.60750739, T_{2,5} = 83.51766328, T_{2,6} = 91.35522217, T_{2,7} = 97.02955407, T_{2,8} = 101.1518697, T_{2,9} = 104.1480089, T_{2,10} = 106.3203045, T_{2,11} = 107.8850966, T_{2,12} = 108.9970792, T_{2,13} = 109.7657281, T_{2,14} = 110.2665430, T_{2,15} = 110.5486533, T_{2,16} = 110.6397152, T_{2,17} = 110.5486533, T_{2,18} = 110.2665430, T_{2,19} = 109.7657281, T_{2,20} = 108.9970792, T_{2,21} = 107.8850966, T_{2,22} = 106.3203045, T_{2,23} = 104.1480089, T_{2,24} = 101.1518697, T_{2,25} = 97.02955407, T_{2,26} = 91.35522217, T_{2,27} = 83.51766328, T_{2,28} = 72.60750739, T_{2,29} = 57.18321068, T_{2,30} = 34.70952036, T_{3,2} = 56.65487077, T_{3,3} = 96.41581497, T_{3,4} = 124.7291556, T_{3,5} = 145.1079235, T_{3,6} = 159.8736713, T_{3,7} = 170.6111244, T_{3,8} = 178.4299158, T_{3,9} = 184.1198613, T_{3,10} = 188.2481126, T_{3,11} = 191.2230028, T_{3,12} = 193.3374919, T_{3,13} = 194.7992901, T_{3,14} = 195.7517906, T_{3,15} = 196.2883549, T_{3,16} = 196.4615541, T_{3,17} = 196.2883549, T_{3,18} = 195.7517906, T_{3,19} = 194.7992901, T_{3,20} = 193.3374919, T_{3,21} = 191.2230028, T_{3,22} = 188.2481126, T_{3,23} = 184.1198613, T_{3,24} = 178.4299158, T_{3,25} = 170.6111244, T_{3,26} = 159.8736713, T_{3,27} = 145.1079235, T_{3,28} = 124.7291556, T_{3,29} = 96.41581497, T_{3,30} = 56.65487077, T_{4,2} = 70.49414776, T_{4,3} = 122.0960228, T_{4,4} = 159.7853765, T_{4,5} = 187.3112040, T_{4,6} = 207.4204152, T_{4,7} = 222.1113565, T_{4,8} = 232.8368077, T_{4,9} = 240.6534078, T_{4,10} = 246.3292819, T_{4,11} = 250.4213101, T_{4,12} = 253.3305956, T_{4,13} = 255.3421499, T_{4,14} = 256.6529744, T_{4,15} = 257.3914216, T_{4,16} = 257.6297916, T_{4,17} = 257.3914216, T_{4,18} = 256.6529744, T_{4,19} = 255.3421499, T_{4,20} = 253.3305956, T_{4,21} = 250.4213101, T_{4,22} = 246.3292819, T_{4,23} = 240.6534078, T_{4,24} = 232.8368077, T_{4,25} = 222.1113565, T_{4,26} = 207.4204152, T_{4,27} = 187.3112040, T_{4,28} = 159.7853765, T_{4,29} = 122.0960228, T_{4,30} = 70.49414776, T_{5,2} = 78.22569744, T_{5,3} = 136.6887521, T_{5,4} = 180.0051235, T_{5,5} = 211.9311008, T_{5,6} = 235.3854289, T_{5,7} = 252.5770787, T_{5,8} = 265.1525509, T_{5,9} = 274.3276804, T_{5,10} = 280.9942971, T_{5,11} = 285.8023602, T_{5,12} = 289.2214304, T_{5,13} = 291.5857394, T_{5,14} = 293.1265355, T_{5,15} = 293.9945656, T_{5,16} = 294.2747689, T_{5,17} = 293.9945656, T_{5,18} = 293.1265355, T_{5,19} = 291.5857394, T_{5,20} = 289.2214304, T_{5,21}$$

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$= 78.22569744,\ T_{6,\,2} = 80.71988989,\ T_{6,\,3} = 141.4281647,\ T_{6,\,4} = 186.6152646,$
$T_{6,\,5} = 220.0226467,\ T_{6,\,6} = 244.6131208,\ T_{6,\,7} = 262.6589787,\ T_{6,\,8}$
$= 275.8686365,\ T_{6,\,9} = 285.5104658,\ T_{6,\,10} = 292.5178658,\ T_{6,\,11} = 297.5724032,$
$T_{6,\,12} = 301.1670265,\ T_{6,\,13} = 303.6528419,\ T_{6,\,14} = 305.2728624,\ T_{6,\,15}$
$= 306.1855366,\ T_{6,\,16} = 306.4801528,\ T_{6,\,17} = 306.1855366,\ T_{6,\,18}$
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$$= 72.60750739,\ T_{10,\,29} = 57.18321068,\ T_{10,\,30} = 34.70952036]]$$

> $LT := [seq(T_{1,j}, j = 1..j_{max}), seq(rhs(SolT_{1,i}), i = 1..N)]:$

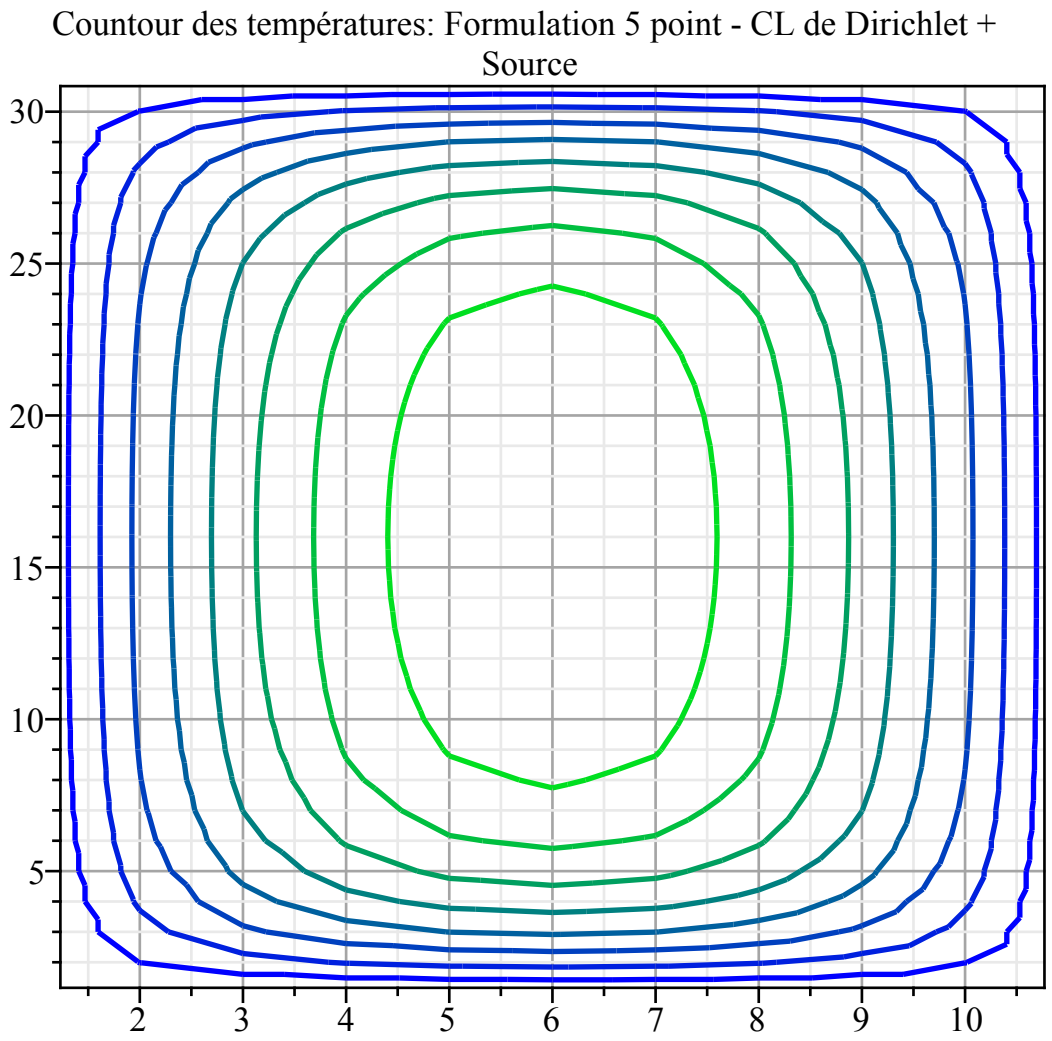
> $with(plots):$

> **for** i **from** 1 **to** $i_{max} - 2$ **do** $Ns[i] := i \cdot \frac{N}{i_{max} - 2}$ **end do**:

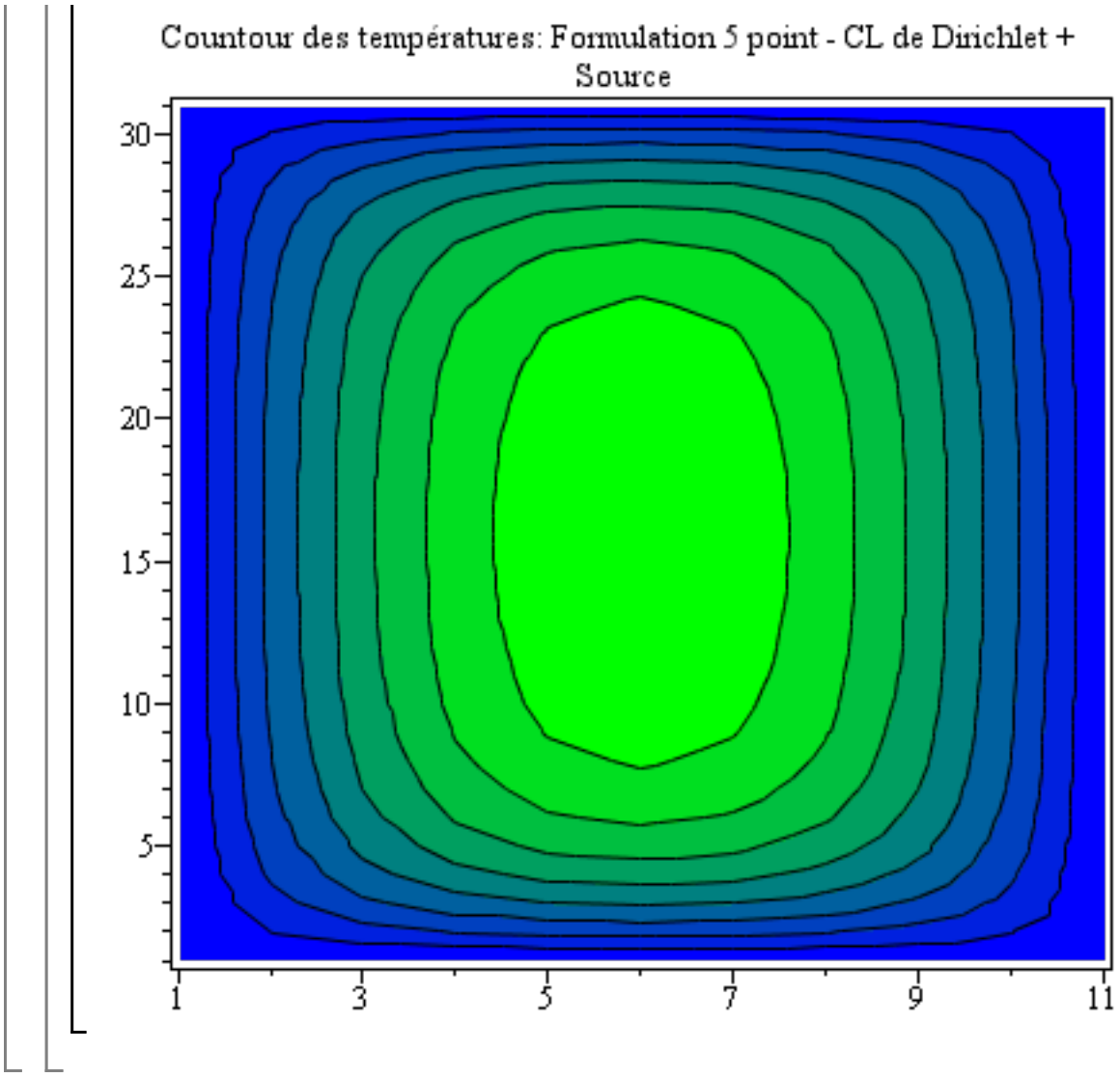
> $GTemps := [[seq(T_{1,j}, j = 1..j_{max})], [T_{2,1}, seq(rhs(SolT_{1,i}), i = 1..Ns_1), T_{2,j_{max}}],$
$[T_{3,1}, seq(rhs(SolT_{1,i}), i = Ns_1 + 1..Ns_2), T_{3,j_{max}}], [T_{4,1}, seq(rhs(SolT_{1,i}), i$
$= Ns_2 + 1..Ns_3), T_{4,j_{max}}], [T_{5,1}, seq(rhs(SolT_{1,i}), i = Ns_3 + 1..Ns_4), T_{5,j_{max}}],$
$[T_{6,1}, seq(rhs(SolT_{1,i}), i = Ns_4 + 1..Ns_5), T_{6,j_{max}}], [T_{7,1}, seq(rhs(SolT_{1,i}), i$
$= Ns_5 + 1..Ns_6), T_{7,j_{max}}], [T_{8,1}, seq(rhs(SolT_{1,i}), i = Ns_6 + 1..Ns_7), T_{8,j_{max}}],$
$[T_{9,1}, seq(rhs(SolT_{1,i}), i = Ns_7 + 1..Ns_8), T_{9,j_{max}}], [T_{10,1}, seq(rhs(SolT_{1,i}), i$
$= Ns_8 + 1..Ns_9), T_{10,j_{max}}], [seq(T_{i_{max},j}, j = 1..j_{max})]]:$

Tracé des isothermes:

> $listcontplot(GTemps, title$
$= "Countour des températures: Formulation 5 point - CL de Dirichlet + Source",$
$axes = boxed, gridlines = true, thickness = 2, coloring = [blue, green])$




```
> listcontplot(GTemps, title
    = "Countour des températures: Formulation 5 point - CL de Dirichlet + Source",
    axes = boxed, gridlines = false, thickness = 1, coloring = [blue, green], filledregions
    = true)
```


Countour des températures: Formulation 5 point - CL de Dirichlet + Source
30
25
20
15
10
5
1
3
5
7
9
11