Equation de Poisson 2D

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Détermination de la temperature $T(x, y)$ à travers la surface d'une plaque rectangulaire (a x b) dont les 4 extrémités sont soumises à des (C.L.) de Dirichlet et contenant une source de chaleur au centre.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = -\frac{Q}{\lambda}$$

Conditions aux limites (C.L):

$$T(x, 0) = 0,$$
$$T(x, b) = 0,$$
$$T(0, y) = 0,$$
$$T(a, y) = 0.$$

Solution discrétisée par la formulation à 5 points:

```
> Restart:
> a := 5: b := 15: ndx := 5: ndy := 15:
> β := 1.:
> Δx := a/ndx; λ := 0.4: Q := 40:
```

$$\Delta x := 1 \quad \textbf{(1.1)}$$

```
> i_max := ndx + 1; j_max := ndy + 1;
```

$$i_{max} := 6$$
$$j_{max} := 16 \quad \textbf{(1.2)}$$

Nombre d'équations:

$> N := (i_{max} - 2) \cdot (j_{max} - 2)$

$$N := 56 \quad \textbf{(1.3)}$$

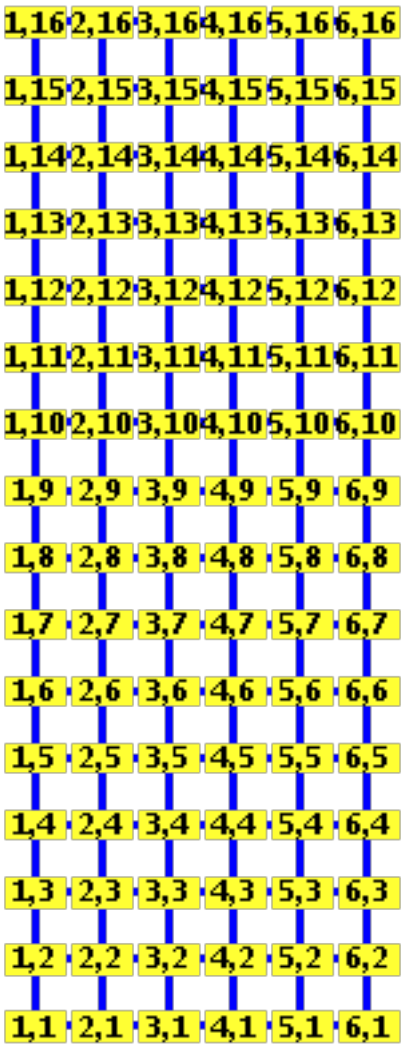
Maillage:

$> with(GraphTheory) : with(SpecialGraphs) :$

$> G := GridGraph(i_{max}, j_{max})$

$$G := Graph\ 1:\ an\ undirected\ unweighted\ graph\ with\ 96\ vertices\ and\ 170\ edge(s) \quad \textbf{(1.4)}$$

$> DrawGraph(G)$



Conditions aux Limites:

$>$ **for** i **from** 1 **to** i_{max} **do** $T[i, 1] := 0$ **end do**:

$>$ **for** i **from** 1 **to** i_{max} **do** $T[i, j_{max}] := 0$ **end do**:

$>$ **for** j **from** 1 **to** j_{max} **do** $T[1, j] := 0$ **end do**:

$>$ **for** j **from** 1 **to** j_{max} **do** $T[i_{max}, j] := 0$ **end do**:

$> k := 1 :$

Résolution pour les noeuds internes

$>$ **for** i **from** 2 **to** $i_{max} - 1$ **do**

for j **from** 2 **to** $j_{max} - 1$ **do**

$$Eq[k] := T[i+1, j] + T[i-1, j] + \beta^2 \cdot (T[i, j+1] + T[i, j-1]) - 2 \cdot \left(1 + \beta^2\right) \cdot T[i, j] + \Delta x^2 \cdot \frac{Q}{\lambda} = 0;$$

```
        Temps[k] := T[i, j];
    k := k + 1
  end do;
end do;
```

Ecriture du système d'équations:

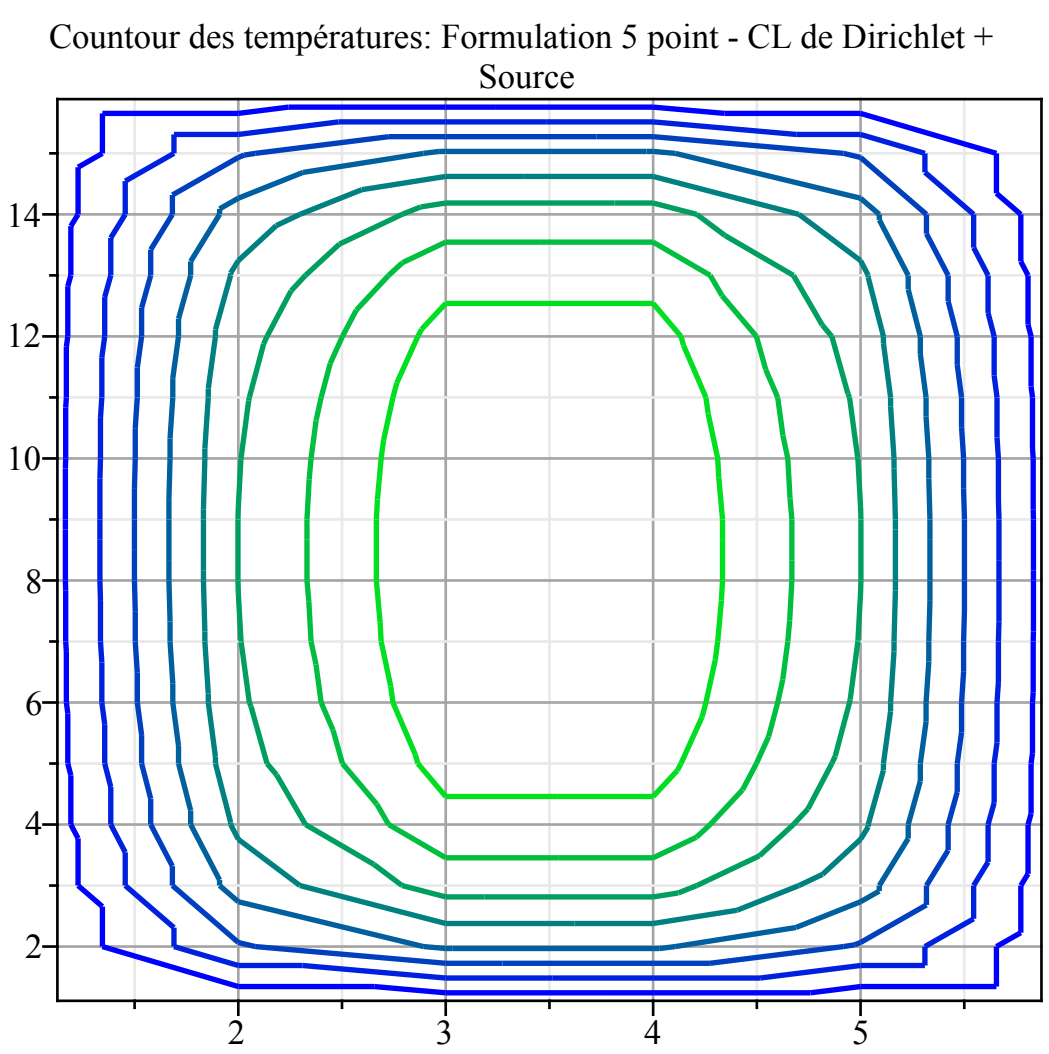
```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(Temps[i], i = 1 ..N)] :
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_{2,2} = 94.49042146, T_{2,3} = 143.3026820, T_{2,4} = 169.2337165, T_{2,5} = 183.1340996, T_{2,6} = 190.5287356, T_{2,7} = 194.2911877, T_{2,8} = 195.8697318, T_{2,9} = 195.8697318, T_{2,10} = 194.2911877, T_{2,11} = 190.5287356, T_{2,12} = 183.1340996, T_{2,13} = 169.2337165, T_{2,14} = 143.3026820, T_{2,15} = 94.49042146, T_{3,2} = 134.6590038, T_{3,3} = 209.4865900, T_{3,4} = 250.4980843, T_{3,5} = 272.7739464, T_{3,6} = 284.6896552, T_{3,7} = 290.7662835, T_{3,8} = 293.3180077, T_{3,9} = 293.3180077, T_{3,10} = 290.7662835, T_{3,11} = 284.6896552, T_{3,12} = 272.7739464, T_{3,13} = 250.4980843, T_{3,14} = 209.4865900, T_{3,15} = 134.6590038, T_{4,2} = 134.6590038, T_{4,3} = 209.4865900, T_{4,4} = 250.4980843, T_{4,5} = 272.7739464, T_{4,6} = 284.6896552, T_{4,7} = 290.7662835, T_{4,8} = 293.3180077, T_{4,9} = 293.3180077, T_{4,10} = 290.7662835, T_{4,11} = 284.6896552, T_{4,12} = 272.7739464, T_{4,13} = 250.4980843, T_{4,14} = 209.4865900, T_{4,15} = 134.6590038, T_{5,2} = 94.49042146, T_{5,3} = 143.3026820, T_{5,4} = 169.2337165, T_{5,5} = 183.1340996, T_{5,6} = 190.5287356, T_{5,7} = 194.2911877, T_{5,8} = 195.8697318, T_{5,9} = 195.8697318, T_{5,10} = 194.2911877, T_{5,11} = 190.5287356, T_{5,12} = 183.1340996, T_{5,13} = 169.2337165, T_{5,14} = 143.3026820, T_{5,15} = 94.49042146]] \quad \textbf{(1.1.1)}$$

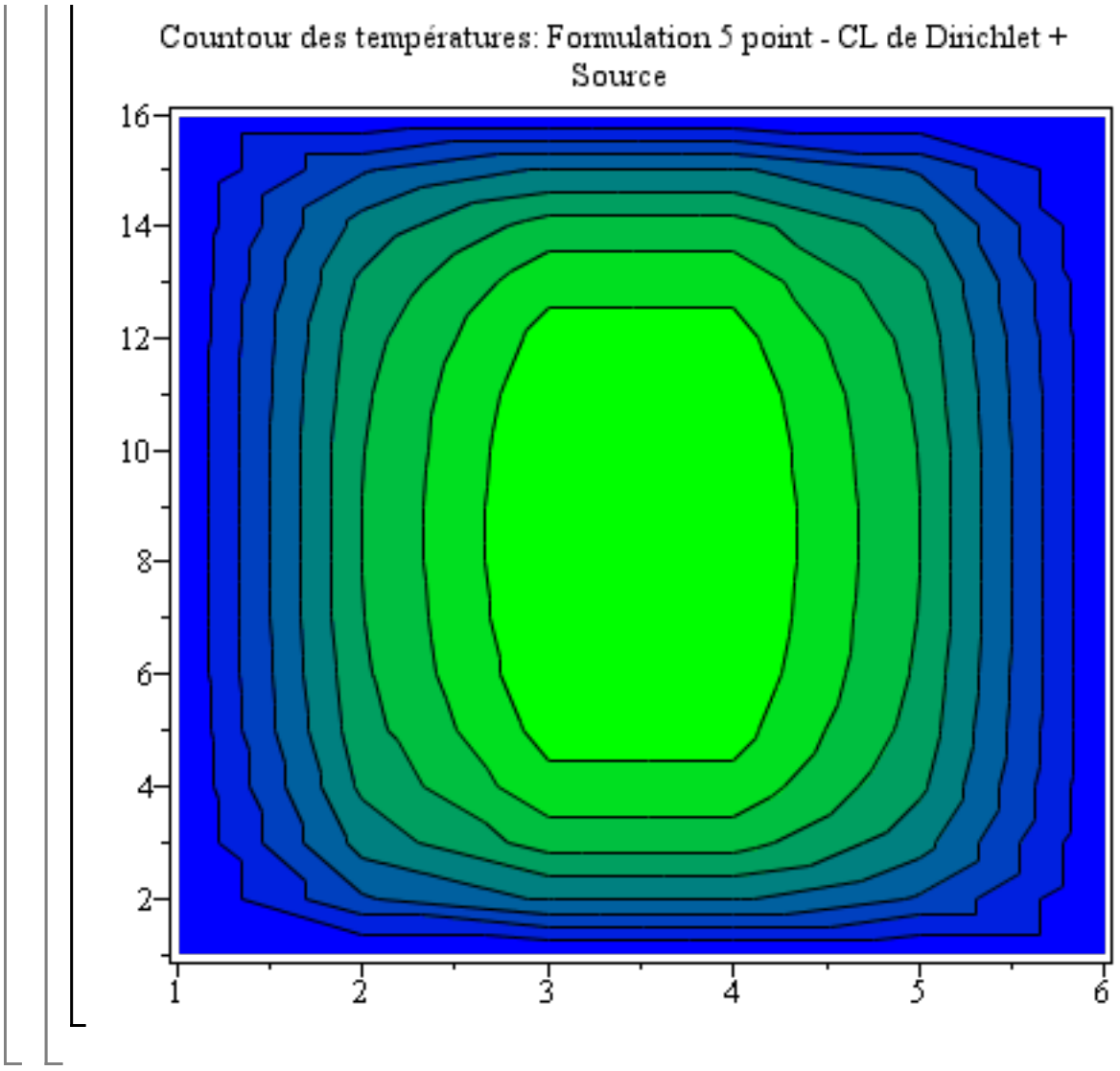
```
> LT := [seq(T[1,j], j = 1 ..jmax), seq(rhs(SolT[1,i]), i = 1 ..N)] :
> with(plots) :
> for i from 1 to imax - 2 do Ns[i] := i·N/(imax - 2) end do:
> GTemps := [[seq(T[1,j], j = 1 ..jmax)], [T[2,1], seq(rhs(SolT[1,i]), i = 1 ..Ns[1]), T[2,jmax]],
    [T[3,1], seq(rhs(SolT[1,i]), i = Ns[1] + 1 ..Ns[2]), T[3,jmax]], [T[4,1], seq(rhs(SolT[1,i]), i
    = Ns[2] + 1 ..Ns[3]), T[4,jmax]], [T[5,1], seq(rhs(SolT[1,i]), i = Ns[3] + 1 ..Ns[4]), T[5,jmax]],
    [seq(T[imax,j], j = 1 ..jmax)]] :
```

Tracé des isothermes:

```
> listcontplot(GTemps, title
    = "Countour des températures: Formulation 5 point - CL de Dirichlet + Source",
    axes = boxed, gridlines = true, thickness = 2, coloring = [blue, green])
```




```
> listcontplot(GTemps, title
    = "Countour des températures: Formulation 5 point - CL de Dirichlet + Source",
    axes = boxed, gridlines = false, thickness = 1, coloring = [blue, green], filledregions
    = true)
```


Countour des températures: Formulation 5 point - CL de Dirichlet + Source
16
14
12
10
8
6
4
2
1
2
3
4
5
6