RÉPUBLIQUE ALGÉRIENNE DÉMOCRATIQUE ET POPULAIRE

MINISTÈRE DE L'ENSEIGNEMENT SUPÉRIEUR
ET DE LA RECHERCHE SCIENTIFIQUE

UNIVERSITÉ DE BATNA

FACULTE DE TECHNOLOGIE
DEPARTEMENT DE MÉCANIQUE

MEMOIRE

PRESENTE POUR OBTENIR LE DIPLOME

LICENCE

Spécialité : MECANIQUE

Option : ENERGETIQUE

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PARTIE THEORIQUE

les schéma	*les formules*	*erreur de troncature*	*stbilité*
schéma explicite	$T[i, n+1] := \lambda \cdot T[i-1, n] + (1-2\cdot\lambda)\cdot T[i, n] + \lambda \cdot T[i+1, n]$	**$o(\Delta t+(\Delta x)^2)$**	stable L^2 et L^∞ condition cfl $2\ \Delta t \leq (\Delta x)^2$
schéma implicite	$T[i, n] = -\lambda \cdot T[i-1, n+1] + (1+2\cdot\lambda)\cdot T[i, n+1] - \lambda \cdot T[i+1, n+1]$	**$o(\Delta t+(\Delta x)^2)$**	stable L^2 et L^∞
Crank - nicholson $(avec\ \theta = 1/2)$	$-\frac{\lambda}{2}\cdot T[i-1, n+1] + (1+\lambda)\cdot T[i, n+1] - \frac{\lambda}{2}\cdot T[i+1, n+1] = \frac{\lambda}{2}\cdot T[i-1, n] + (1-\lambda)\cdot T[i, n] + \frac{\lambda}{2}\cdot T[i+1, n];$	**$o((\Delta t)^2+(\Delta x)^2)$**	stable L^2
θ *schéma* $(avec \neq 1/2)$	$(-\lambda\cdot\theta)\cdot T[i-1, n+1] + (1+2\cdot\lambda\cdot\theta)\cdot T[i, n+1] - (\lambda\cdot\theta)\cdot T[i+1, n+1] = \lambda\cdot(1-\theta)\cdot T[i-1, n] + (1-2\cdot\lambda\cdot(1-\theta))\cdot T[i, n] + \lambda\cdot(1-\theta)\ T[i+1, n];$	**$o(\Delta t+(\Delta x)^2)$**	stable L^2 si condition CFL $2\ (1-2\theta)\ \Delta t \leq (\Delta x)^2$

Remarque: Le θ schéma est un schéma :
- Explicite pour θ = 0.
- Complètement Implicite pour θ = 1.
- Implicite (Crank-Nicholson) pour θ = 1/2.

SCHÉMA EXPLICITE

DONNEES

restart;

$$\Delta x := \frac{2.0}{10} :$$

$$\Delta t := \frac{5}{1000} :$$

$$\lambda := \frac{\Delta t}{\Delta x^2} :$$

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

nmax := 15 :

imax := 15 :

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**

$T[1, n] := \alpha$

end do:

for n **from** 0 **to** nmax **do**

$T[imax, n] := \beta$

end do:

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**

$T[i, 0] := \phi$

end do:

PROGRAMME PRINCIPAL

for n **from** 0 **to** nmax **do**

for i **from** 2 **to** imax − 1 **do**

$T[i, n+1] := \lambda \cdot T[i-1, n] + (1 - 2 \cdot \lambda) \cdot T[i, n] + \lambda \cdot T[i+1, n]$

end do;

end do;

SOULUTION

for n **from** 1 **to** nmax **do**

```
liste[n] := [α, seq(T[i, n], i = 2 .. imax − 1), β];
end do
```

[0, 0.8750000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 0.8750000000, 0]

[0, 0.7812500000, 0.9843750000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 0.9843750000, 0.7812500000, 0]

[0, 0.7089843750, 0.9609375000, 0.9980468750, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 0.9980468750, 0.9609375000, 0.7089843750, 0]

[0, 0.6518554687, 0.9340820313, 0.9936523437, 0.9997558594, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 0.9997558594, 0.9936523437, 0.9340820313, 0.6518554687, 0]

[0, 0.6056518554, 0.9062500001, 0.9869689941, 0.9990234376, 0.9999694824, 1.000000000, 1.000000000, 1.000000000, 0.9999694824, 0.9990234376, 0.9869689941, 0.9062500001, 0.6056518554, 0]

[0, 0.5675201416, 0.8787651063, 0.9783859253, 0.9976348878, 0.9998550415, 0.9999961853, 1.000000000, 0.9999961853, 0.9998550415, 0.9976348878, 0.9783859253, 0.8787651063, 0.5675201416, 0]

[0, 0.5354857445, 0.8523120881, 0.9683394433, 0.9955062867, 0.9995951653, 0.9999790192, 0.9999990464, 0.9999790192, 0.9995951653, 0.9955062867, 0.9683394433, 0.8523120881, 0.5354857445, 0]

[0, 0.5081533194, 0.8272122146, 0.9572318793, 0.9926215411, 0.9991320372, 0.9999335409, 0.9999940396, 0.9999335409, 0.9991320372, 0.9926215411, 0.9572318793, 0.8272122146, 0.5081533194, 0]

[0, 0.4845165164, 0.8035823108, 0.9454031289, 0.9890116453, 0.9984184131, 0.9998409153, 0.9999789149, 0.9998409153, 0.9984184131, 0.9890116453, 0.9454031289, 0.8035823108, 0.4845165164, 0]

[0, 0.4638351761, 0.7814266887, 0.9331265912, 0.9847364267, 0.9974203799, 0.9996803525, 0.9999444150, 0.9996803525, 0.9974203799, 0.9847364267, 0.9331265912, 0.7814266888, 0.4638351761, 0]

[0, 0.4455547182, 0.7606902374, 0.9206153328, 0.9798706914, 0.9961173823, 0.9994308638, 0.9998783994, 0.9994308638, 0.9961173823, 0.9798706914, 0.9206153328, 0.7606902375, 0.4455547182, 0]

[0, 0.4292523183, 0.7412889344, 0.9080316157, 0.9744946080, 0.9945007311, 0.9990726205, 0.9997665156, 0.9990726205, 0.9945007311, 0.9744946080, 0.9080316157, 0.7412889345, 0.4292523183, 0]

[0, 0.4146003555, 0.7231271926, 0.8954966546, 0.9686874994, 0.9925714519, 0.9985878712, 0.9995930419, 0.9985878712, 0.9925714519, 0.9686874994, 0.8954966546, 0.7231271927, 0.4146003555, 0]

[0, 0.4013411657, 0.7061075206, 0.8830993275, 0.9625241379, 0.9903380102, 0.9979614651, 0.9993417492, 0.9979614651, 0.9903380102, 0.9625241379, 0.8830993275, 0.7061075207, 0.4013411657, 0]

[0, 0.3892693144, 0.6901357020, 0.8709034529, 0.9560727706, 0.9878142079, 0.9971810687, 0.9989966781, 0.9971810687, 0.9878142079, 0.9560727706, 0.8709034529, 0.6901357021, 0.3892693144, 0] **(2.5.1)**

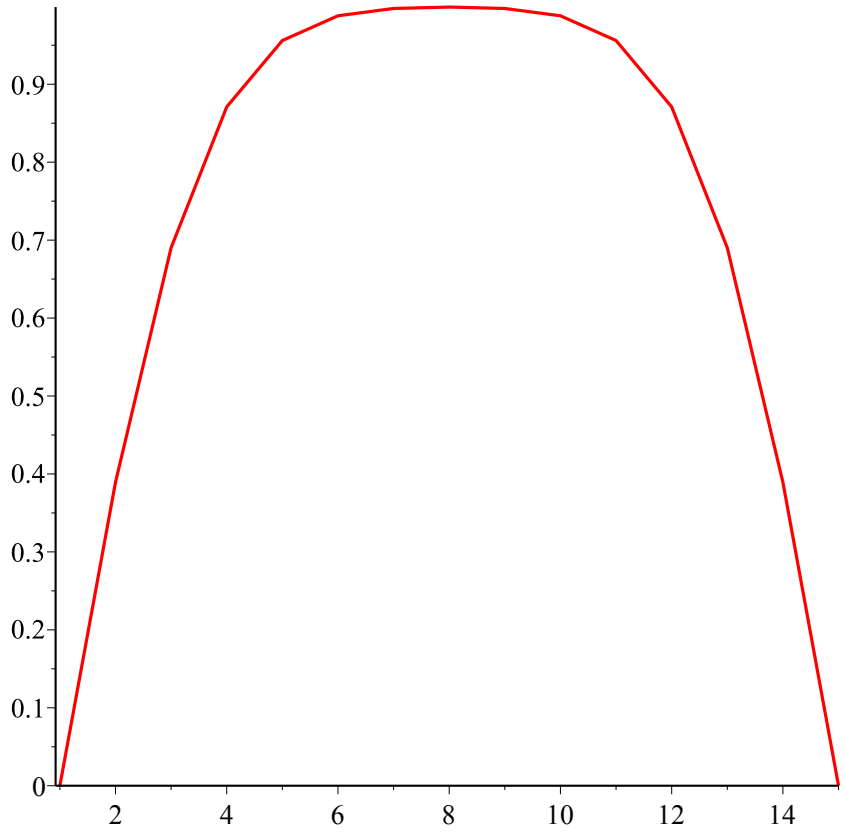
$lliste := seq(T[i, 15], i = 1..15)$

0, 0.3892693144, 0.6901357020, 0.8709034529, 0.9560727706, 0.9878142079, 0.9971810687, 0.9989966781, 0.9971810687, 0.9878142079, 0.9560727706, 0.8709034529, 0.6901357021, 0.3892693144, 0 **(2.5.2)**

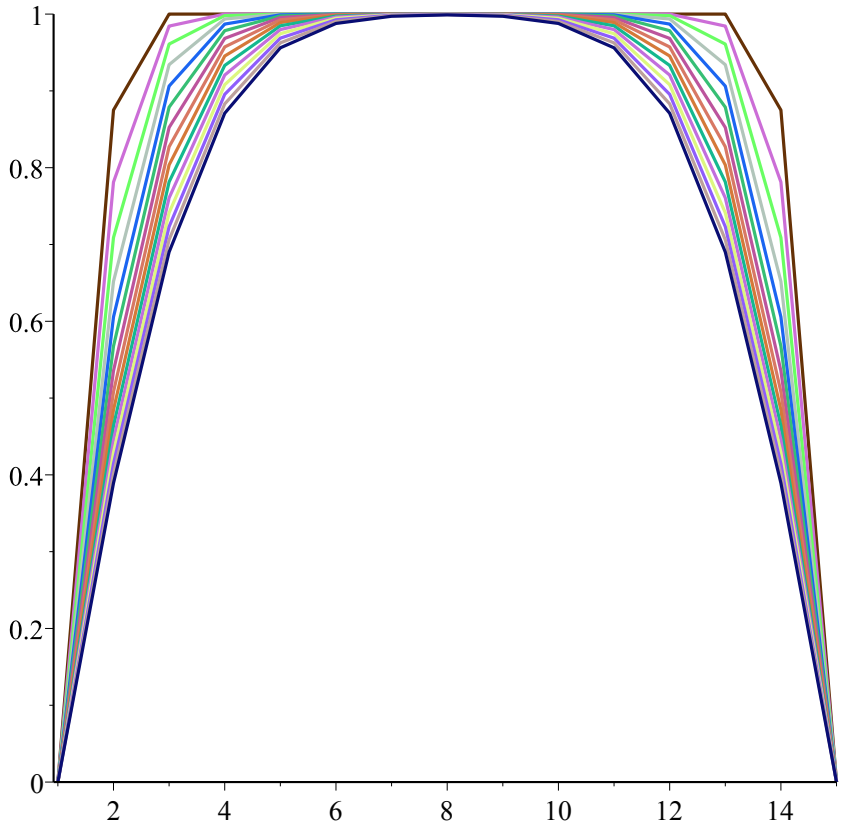
GRAPHES

$with(plots)$:

$listplot(liste[\text{nmax}], color = red)$



$$multiple\left(listplot\ ,\ seq\left(\left[liste[n],\ color = COLOR\left(RGB,\ \frac{rand(\ )}{10^{12}},\ \frac{rand(\ )}{10^{12}},\ \frac{rand(\ )}{10^{12}} \right) \right],\ n = 1..nmax \right) \right)$$



$$writedata(chaleur,\ liste[nmax])$$

SCHÉMA IMPLICITE

DONNEES

restart :

$imax := 15$:

$nmax := 15$:

$k := 1$:

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

$\Delta t := 0.005$:

$$\Delta x := \frac{2.0}{10} :$$

$$\lambda := \frac{\Delta t}{\Delta x^2} :$$

$$k := 1 :$$

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**
$T[1, n] := \alpha$
end :
for n **from** 0 **to** nmax **do**
$T[imax, n] := \beta$
end :

(3.2.1)

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**
$T[i, 0] := \phi$
end :

PROGRAMME PRINCIPAL ET SOLUTIONS

for n **from** 0 **to** $nmax - 1$ **do**
for i **from** 2 **to** $imax - 1$ **do**

$$eq[k] := -\lambda \cdot T[i-1, n+1] + (1 + 2\cdot\lambda)\cdot T[i, n+1] - \lambda \cdot T[i+1, n+1] = T[i, n];$$

$k := k + 1;$
end do;
$sys := \{seq(eq[k], k = 1 .. imax - 2)\}$:
$var := [seq(T[i, n+1], i = 2 .. imax - 1)]$:
$sols := solve(sys, var)$;
$liste := map(op, sols);$
$k := 1;$
for i **from** 2 **to** $imax - 1$ **do**

$$eq[k] := -\lambda \cdot T[i-1, n+2] + (1 + 2\cdot\lambda)\cdot T[i, n+2] - \lambda \cdot T[i+1, n+2] = rhs(sols_{1,k});$$

$k := k + 1;$
end do;
end do:
$nliste := [seq(rhs(liste[k]), k = 1 .. imax - 2)]$

[0.4057575809, 0.7038407305, 0.8736606315, 0.9527859933, 0.9841576770, 0.9948692402, 0.9973248804, 0.9948692402, 0.9841576770, 0.9527859933, 0.8736606315, 0.7038407305, 0.4057575809] **(3.4.1)**

$nliste := [\alpha, seq(rhs(liste[k]), k = 1 .. imax - 2), \beta]$

[0, 0.4057575809, 0.7038407305, 0.8736606315, 0.9527859933, 0.9841576770, 0.9948692402, 0.9973248804, 0.9948692402, 0.9841576770, 0.9527859933, **(3.4.2)**

0.8736606315, 0.7038407305, 0.4057575809, 0]

writedata(*chaleurr*, *nliste*)

Graphe pour nmax

with(*plots*) : *listplot*(*nliste*, *color* = green)

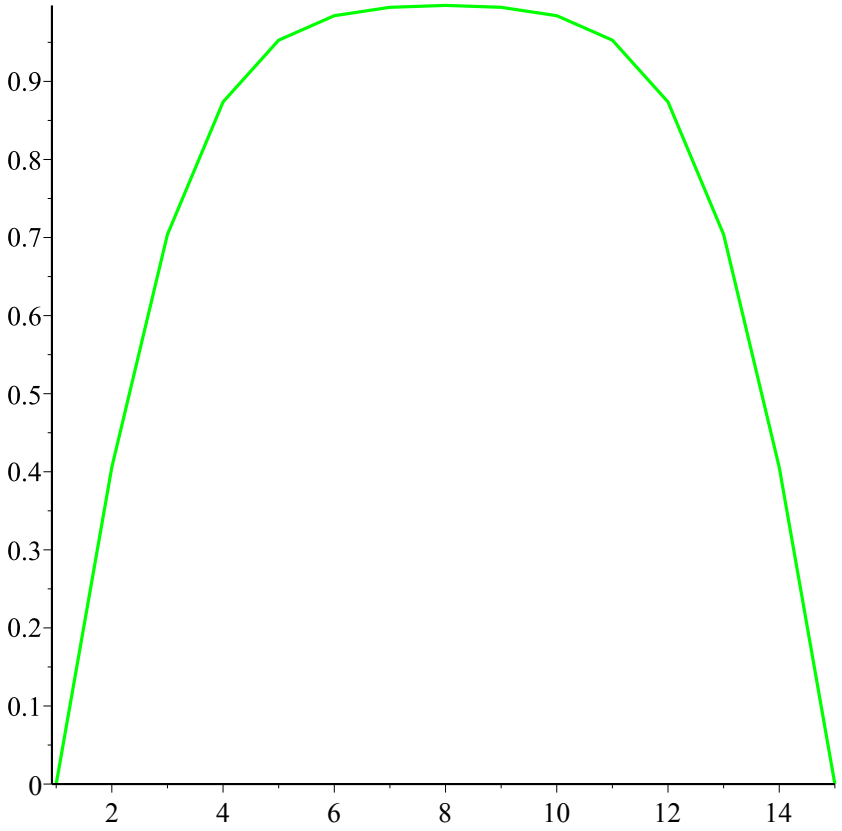


SCHÉMA DE CRANK-NICHOLSON

DONNEES

```
restart :
 imax := 15 :
 nmax := 15 :
  k := 1 :
```

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

$\Delta t := 0.005$:

$\Delta x := \frac{2.0}{10}$:

$\lambda := \frac{\Delta t}{\Delta x^2}$:

$k := 1$:

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**

$T[1, n] := \alpha$

end :

for n **from** 0 **to** nmax **do**

$T[imax, n] := \beta$

end :

(4.2.1)

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**

$T[i, 0] := \phi$

end :

PROGRAMME PRINCIPAL ET SOLUTIONS

for n **from** 0 **to** $nmax - 1$ **do**

for i **from** 2 **to** $imax - 1$ **do**

$$eq[k] := -\frac{\lambda}{2} \cdot T[i-1, n+1] + (1+\lambda) \cdot T[i, n+1] - \frac{\lambda}{2} \cdot T[i+1, n+1] = \frac{\lambda}{2} \cdot T[i-1, n] + (1-\lambda) \cdot T[i, n] + \frac{\lambda}{2} \cdot T[i+1, n];$$

$k := k + 1;$

end do;

$sys := \{seq(eq[k], k = 1 .. imax - 2)\}$:

$var := [seq(T[i, n+1], i = 2 .. imax - 1)]$:

$sols := solve(sys, var);$

$liste := map(op, sols);$

$k := 1;$

for i **from** 2 **to** $imax - 1$ **do**

$$eq[k] := -\frac{\lambda}{2} \cdot T[i-1, n+2] + (1+\lambda) \cdot T[i, n+2] - \frac{\lambda}{2} \cdot T[i+1, n+2] = rhs\left(sols_{1,k}\right);$$

$k := k + 1;$

end do;

end do:

(4.4.1)

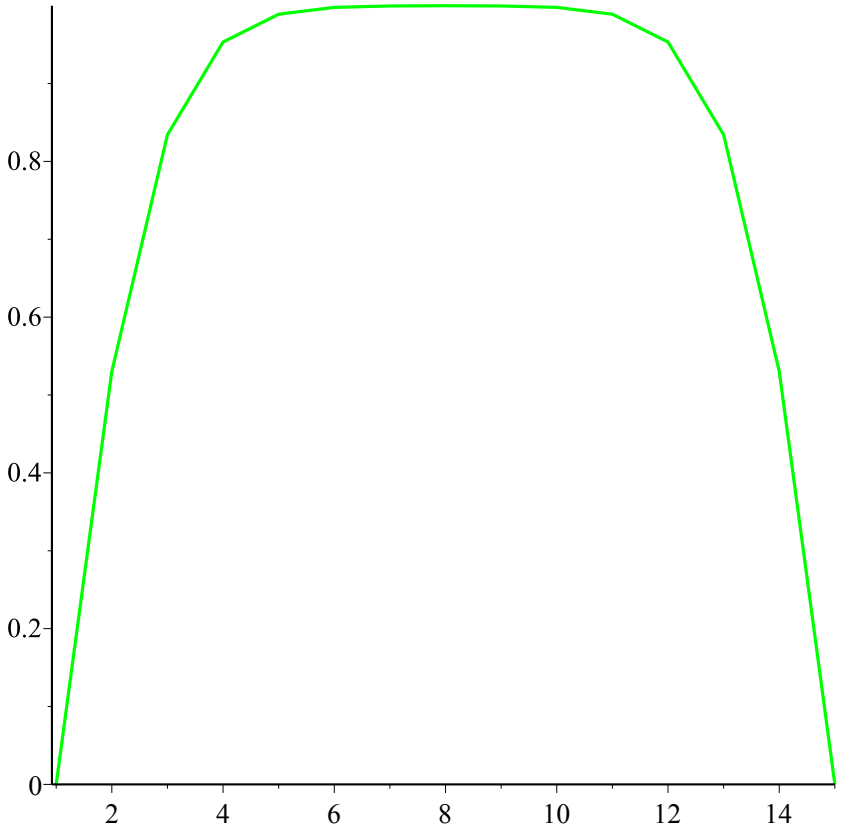
$lliste := [\alpha, seq(rhs(liste[k]), k = 1 ..imax - 2\ ), \beta]$

[0, 0.5302317500, 0.8341721264, 0.9532787133, 0.9889769662, 0.9977425588, 0.9995785494, 0.9998639857, 0.9995785494, 0.9977425588, 0.9889769662, 0.9532787133, 0.8341721264, 0.5302317500, 0] **(4.4.2)**

$writedata(chaleurrr, lliste)$

Graphe pour nmax

$with(plots) : listplot(lliste, color = green)$



LE θ SCHÉMAT (POUR θ=0)

DONNEES

```
restart :
imax := 15 :
nmax := 15 :
```

$k := 1$:

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

$\Delta t := 0.005$:

$\Delta x := \frac{2.0}{10}$:

$\lambda := \frac{\Delta t}{\Delta x^2}$:

$k := 1$:

$\theta := 0$:

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**

$T[1, n] := \alpha$

end :

for n **from** 0 **to** nmax **do**

$T[imax, n] := \beta$

end :

(5.2.1)

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**

$T[i, 0] := \phi$

end :

PROGRAMME PRINCIPAL

if $\theta = 0$ **then**

for n **from** 0 **to** $nmax - 1$ **do**

for i **from** 2 **to** $imax - 1$ **do**

$T[i, n+1] := \lambda \cdot T[i-1, n] + (1 - 2 \cdot \lambda) \cdot T[i, n] + \lambda \cdot T[i+1, n]$

end do;

end do;

else

for n **from** 0 **to** $nmax - 1$ **do**

for i **from** 2 **to** $imax - 1$ **do**

$eq[k] := (-\lambda \cdot \theta) \cdot T[i-1, n+1] + (1 + 2 \cdot \lambda \cdot \theta) \cdot T[i, n+1] - (\lambda \cdot \theta) \cdot T[i+1, n+1] = \lambda \cdot (1 - \theta) \cdot T[i-1, n] + (1 - 2 \cdot \lambda \cdot (1 - \theta)) \cdot T[i, n] + \lambda \cdot (1 - \theta)\ T[i+1, n]$;

$k := k + 1$;

end do;

$sys := \{seq(eq[k], k = 1 .. imax - 2)\}$;

$var := [seq(T[i, n+1], i = 2 .. imax - 1)]$;

$sols := solve(sys, var)$;

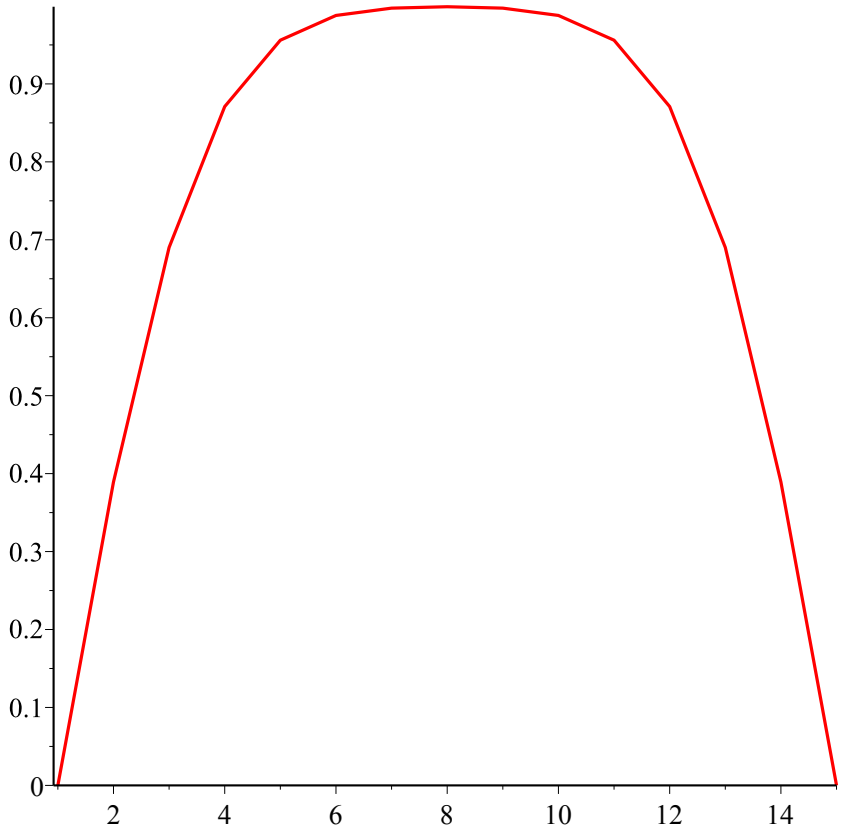
$liste := map(op, sols)$;

$k := 1$;

```
 for i from 2 to imax − 1 do
eq[k] := (-λ·θ)·T[i − 1, n + 2] + (1 + 2·λ·θ)·T[i, n + 2] − (λ·θ)·T[i + 1, n + 2]
      = rhs(sols1, k);
 k := k + 1;
 end do;
 end do;
 end if;
```

SOLUTION ET GRAPHES

```
 if θ = 0 then
for n from 1 to nmax do
 liste[n] := [α, seq(T[i, n], i = 2 .. imax − 1), β]
 end do;
 end if
 with(plots) :
 listplot(liste[nmax], color = red)
```




```
if θ ≠ 0 then
 nliste := [seq(rhs(liste[k]), k = 1 ..imax − 2 )]
```

end if ; *with*(*plots*) : *listplot*(*nliste*, *color* = green) ; Error, (in plots:-pointplot) points are not in the correct format

LE θ SCHÉMAT (POUR θ=1)

DONNEES

restart :

$imax := 15$:

$nmax := 15$:

$k := 1$:

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

$\Delta t := 0.005$:

$\Delta x := \frac{2.0}{10}$:

$\lambda := \frac{\Delta t}{\Delta x^2}$:

$k := 1$:

$\theta := 1$:

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**

$T[1, n] := \alpha$

end :

for n **from** 0 **to** nmax **do**

$T[imax, n] := \beta$

end :

(6.2.1)

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**

$T[i, 0] := \phi$

end :

PROGRAMME PRINCIPAL

if θ = 0 **then**

for *n* **from** 0 **to** *nmax* − 1 **do**

for *i* **from** 2 **to** *imax* − 1 **do**

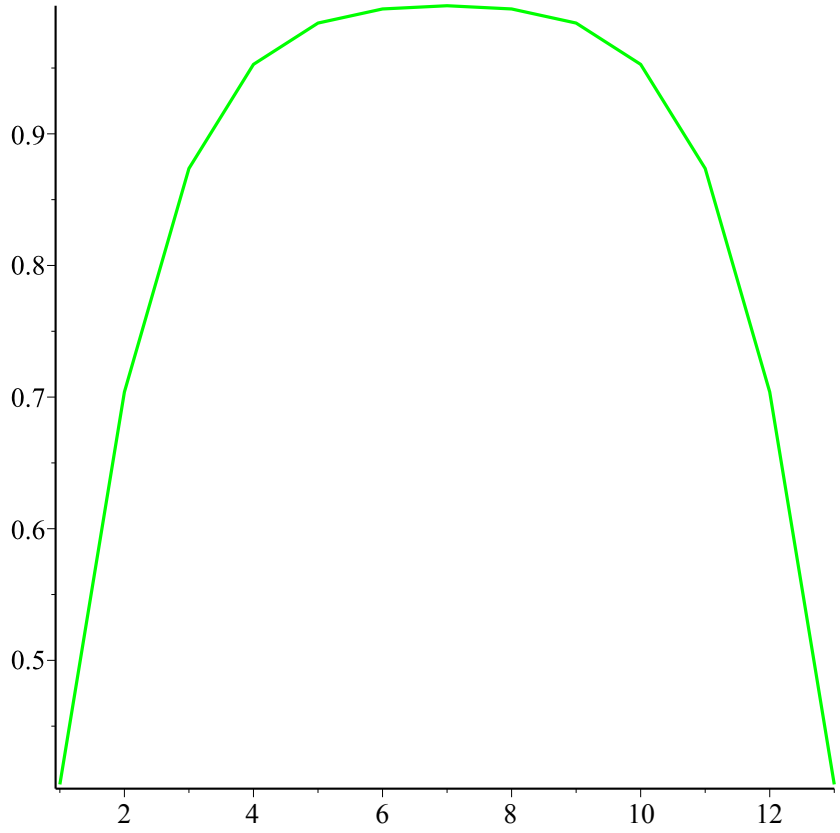
$T[i, n+1] := \lambda \cdot T[i-1, n] + (1 - 2 \cdot \lambda) \cdot T[i, n] + \lambda \cdot T[i+1, n]$

```
 end do;
 end do;
  else
for n from 0 to nmax − 1 do
 for i from 2 to imax − 1 do
eq[k] := (-λ·θ)·T[i − 1, n + 1] + (1 + 2·λ·θ)·T[i, n + 1] − (λ·θ)·T[i + 1, n + 1] = λ·(1
      − θ)· T[i − 1, n] + (1 − 2·λ·(1 − θ))·T[i, n] + λ·(1 − θ) T[i + 1, n];
 k := k + 1;
 end do;
sys := {seq(eq[k], k = 1 ..imax − 2)};
var := [seq(T[i, n + 1], i = 2 ..imax − 1)];
sols := solve(sys, var) ;
 liste := map(op, sols);
 k := 1;
 for i from 2 to imax − 1 do
eq[k] := (-λ·θ)·T[i − 1, n + 2] + (1 + 2·λ·θ)·T[i, n + 2] − (λ·θ)·T[i + 1, n + 2]
      = rhs(sols1, k);
 k := k + 1;
 end do;
 end do;
 end if;
```

SOLUTION ET GRAPHES

```
 if θ = 0 then
for n from 1 to nmax do
 liste[n] := [α, seq(T[i, n], i = 2 .. imax − 1), β]
 end do;
 end if
 with(plots) :
 listplot(liste[nmax], color = red)Error, invalid subscript selector

if θ ≠ 0 then
 nliste := [seq(rhs(liste[k]), k = 1 ..imax − 2 )]
  end if ; with(plots) : listplot(nliste, color = green) ;
[0.4057575809, 0.7038407305, 0.8736606315, 0.9527859933, 0.9841576770,
    0.9948692402, 0.9973248804, 0.9948692402, 0.9841576770, 0.9527859933,
    0.8736606315, 0.7038407305, 0.4057575809]
```



LE θ SCHÉMAT (POUR θ=1/2)

DONNEES

restart :

$imax := 15$:

$nmax := 15$:

$k := 1$:

$\alpha := 0$:

$\beta := 0$:

$\phi := 1$:

$\Delta t := 0.005$:

$\Delta x := \frac{2.0}{10}$:

$$\lambda := \frac{\Delta t}{\Delta x^2} :$$

$k := 1 :$

$\theta := \frac{1}{2} :$

CONDITIONS AUX LIMITES

for n **from** 0 **to** nmax **do**
$T[1, n] := \alpha$
end :
for n **from** 0 **to** nmax **do**
$T[imax, n] := \beta$
end :

(7.2.1)

CODITION INITIALE

for i **from** 2 **to** imax − 1 **do**
$T[i, 0] := \phi$
end :

PROGRAMME PRINCIPAL

if $\theta = 0$ **then**
for n **from** 0 **to** $nmax - 1$ **do**
for i **from** 2 **to** $imax - 1$ **do**
$T[i, n+1] := \lambda \cdot T[i-1, n] + (1 - 2 \cdot \lambda) \cdot T[i, n] + \lambda \cdot T[i+1, n]$
end do;
end do;
else
for n **from** 0 **to** $nmax - 1$ **do**
for i **from** 2 **to** $imax - 1$ **do**
$eq[k] := (-\lambda \cdot \theta) \cdot T[i-1, n+1] + (1 + 2 \cdot \lambda \cdot \theta) \cdot T[i, n+1] - (\lambda \cdot \theta) \cdot T[i+1, n+1] = \lambda \cdot (1 - \theta) \cdot T[i-1, n] + (1 - 2 \cdot \lambda \cdot (1 - \theta)) \cdot T[i, n] + \lambda \cdot (1 - \theta)\ T[i+1, n];$
$k := k + 1;$
end do;
$sys := \{seq(eq[k], k = 1 .. imax - 2)\};$
$var := [seq(T[i, n+1], i = 2 .. imax - 1)];$
$sols := solve(sys, var)$;
$liste := map(op, sols);$
$k := 1;$
for i **from** 2 **to** $imax - 1$ **do**
$eq[k] := (-\lambda \cdot \theta) \cdot T[i-1, n+2] + (1 + 2 \cdot \lambda \cdot \theta) \cdot T[i, n+2] - (\lambda \cdot \theta) \cdot T[i+1, n+2] = rhs(sols_{1, k});$
$k := k + 1;$
end do;
end do;

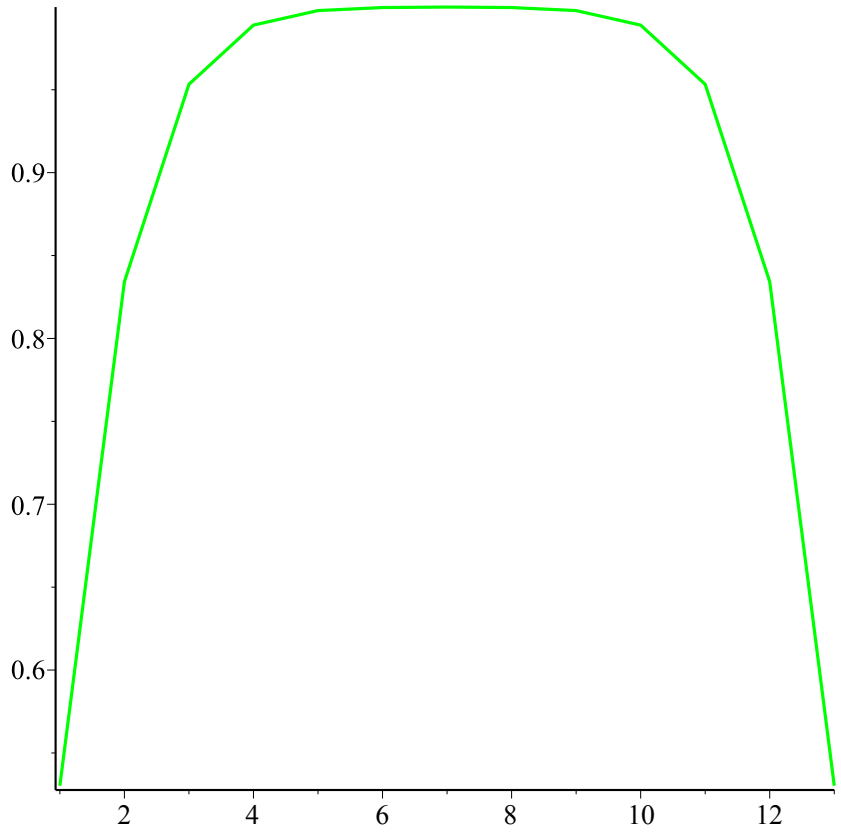
end if;

SOLUTION ET GRAPHES

if θ = 0 **then**
for n **from** 1 **to** nmax **do**
liste[n] := [α, seq(T[i, n], i = 2 .. imax − 1), β]
end do;
end if
with(*plots*) :
listplot(*liste*[*nmax*], *color* = *red*)Error, invalid subscript selector

if θ ≠ 0 **then**
nliste := [*seq*(*rhs*(*liste*[*k*]), *k* = 1 ..*imax* − 2)]
end if ; *with*(*plots*) : *listplot*(*nliste*, *color* = green) ;

[0.5302317500, 0.8341721264, 0.9532787133, 0.9889769662, 0.9977425588, 0.9995785494, 0.9998639857, 0.9995785494, 0.9977425588, 0.9889769662, 0.9532787133, 0.8341721264, 0.5302317500]



COMPARAISON

$soluexplicit := readdata(chaleur)$

$[0., 0.3892693144, 0.690135702, 0.8709034529, 0.9560727706, 0.9878142079, 0.9971810687, 0.9989966781, 0.9971810687, 0.9878142079, 0.9560727706, 0.8709034529, 0.6901357021, 0.3892693144, 0.]$ **(8.1)**

$soluimplicit := readdata(chaleurr)$

$[0., 0.4057575809, 0.7038407305, 0.8736606315, 0.9527859933, 0.984157677, 0.9948692402, 0.9973248804, 0.9948692402, 0.984157677, 0.9527859933, 0.8736606315, 0.7038407305, 0.4057575809, 0.]$ **(8.2)**

$solucrank := readdata(chaleurrr)$

$[0., 0.53023175, 0.8341721264, 0.9532787133, 0.9889769662, 0.9977425588, 0.9995785494, 0.9998639857, 0.9995785494, 0.9977425588, 0.9889769662, 0.9532787133, 0.8341721264, 0.53023175, 0.]$ **(8.3)**

$multiple(listplot, [soluexplicit, color = red], [soluimplicit, color = green], [solucrank])$