Travaux Pratiques TP N°3

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LMD : Génie Energétique

Matière : Outils Numériques

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Equation de la chaleur

Détermination de la temperature $T(x, t)$ à travers l'épaisseur d'une plaque dont les extrémités sont maintenues à des températures constantes.

$$\frac{\partial}{\partial t} T(x,t) = \frac{\partial^2}{\partial x^2} T(x,t)$$

Conditions aux limites et initiale:

$$T(0,t) = 0,$$
$$T(1,t) = 0,$$
$$T(x,0) = 1$$

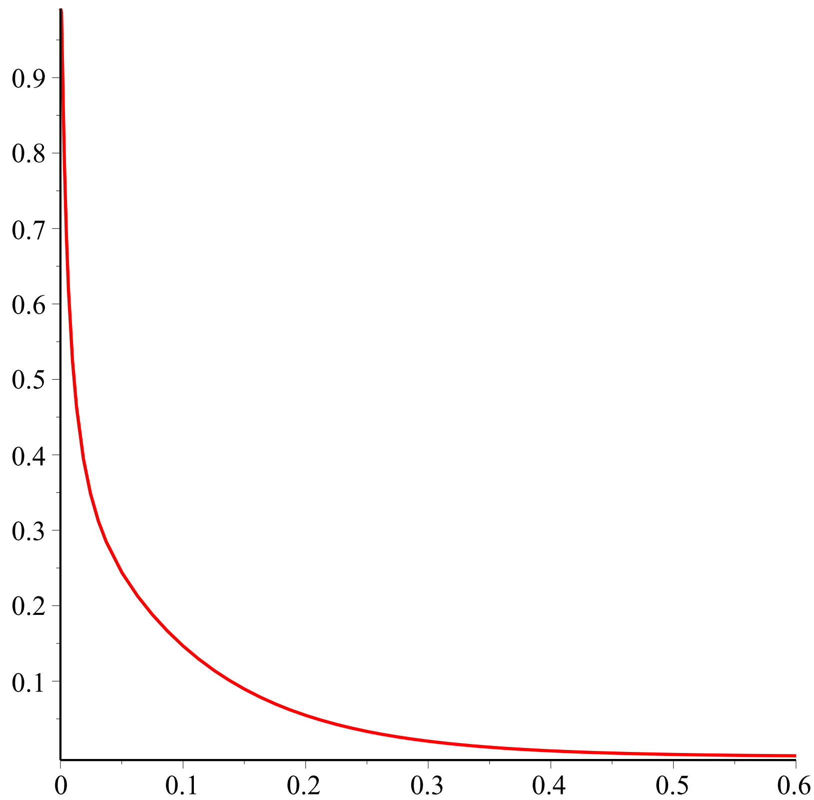
Solution analytique

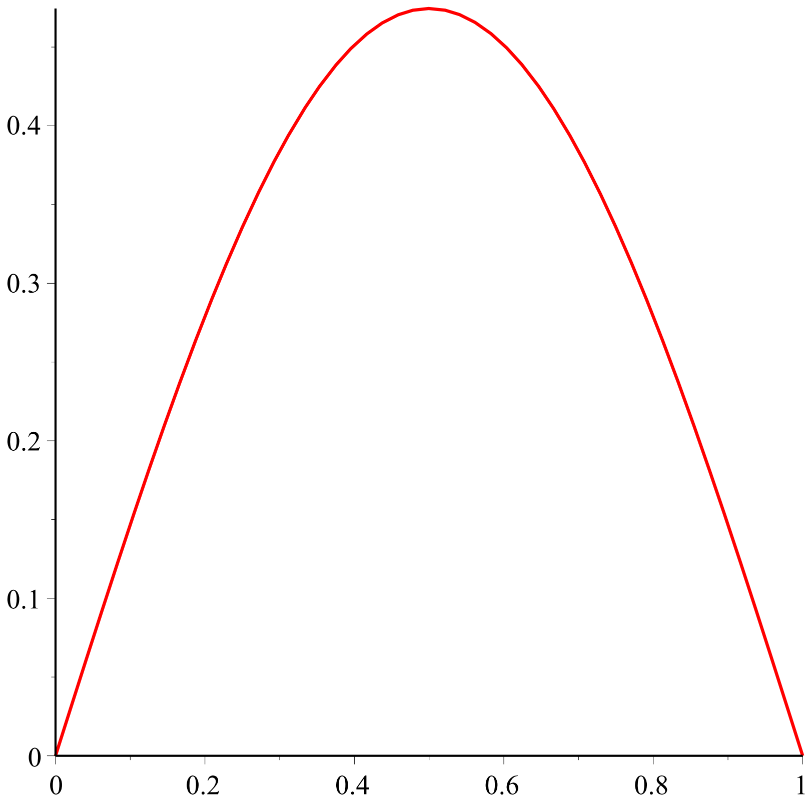
```
> Restart:
```

$$> T(X,\tau) := \sum_{k=0}^{100} \frac{4 \sin((2k+1)\pi X)\, e^{-(2k+1)^2 \pi^2 \tau}}{(2k+1)\pi};$$

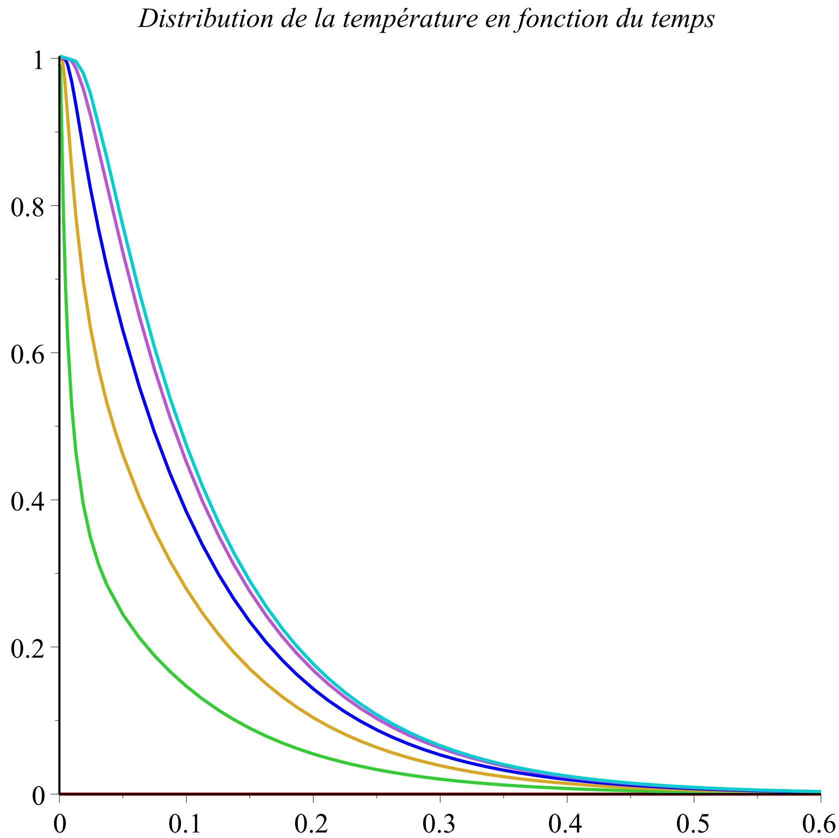
$$(X,\tau) \rightarrow \sum_{k=0}^{100} \frac{4 \sin((2k+1)\pi X)\, e^{-(2k+1)^2 \pi^2 \tau}}{(2k+1)\pi} \qquad \textbf{(1.1.1)}$$

```
> plot(T(0.1, τ), τ = 0 ..0.6); plot(T(X, 0.1), X = 0 ..1);
```

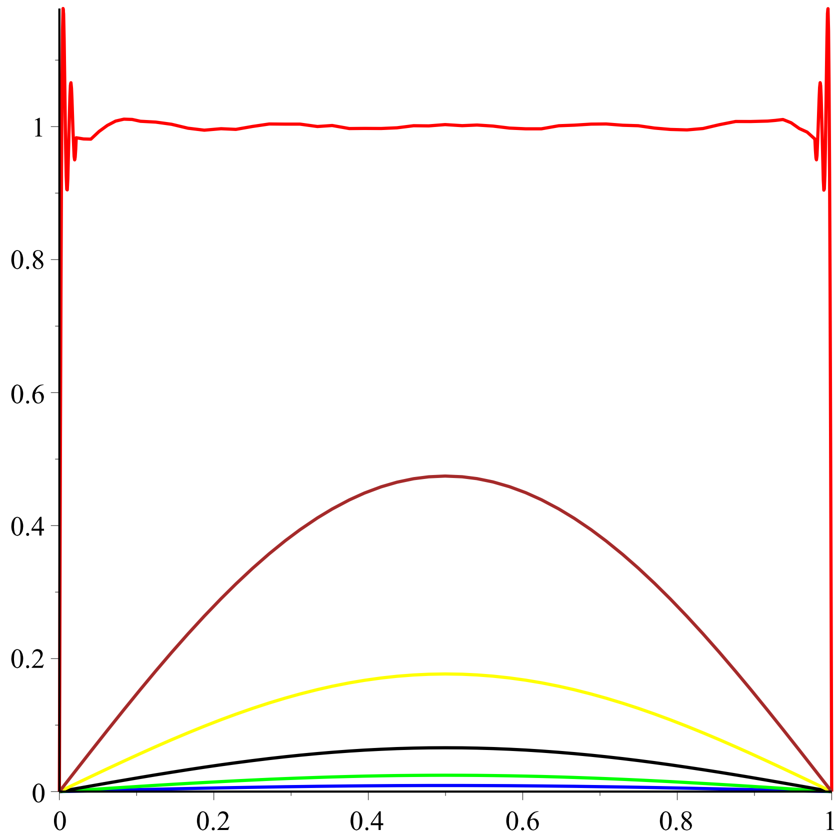

0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6



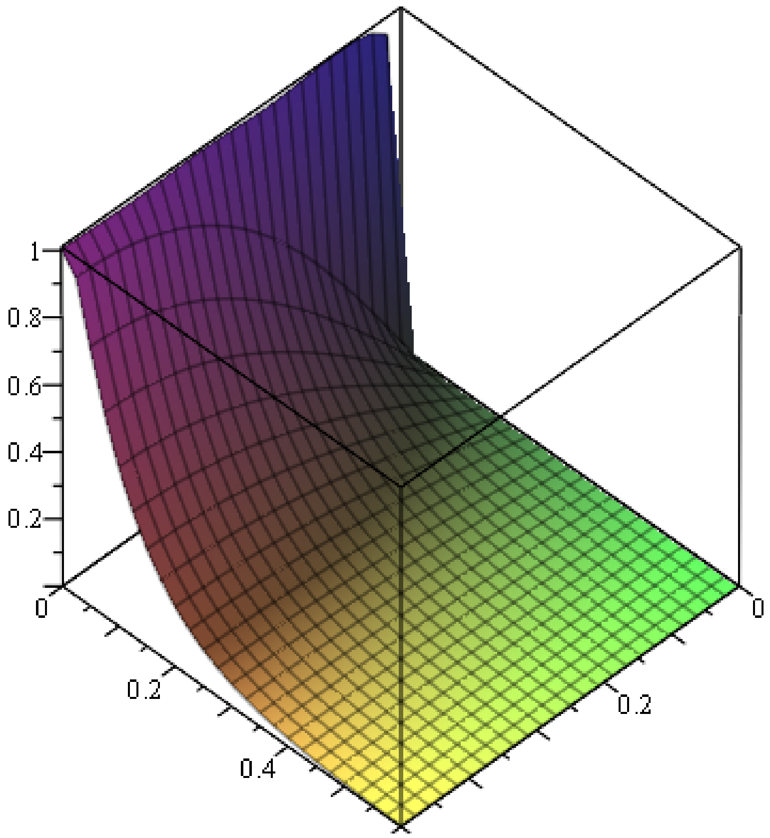
> plot({ T(0.1, τ), T(0.2, τ), T(0.3, τ), T(0.4, τ), T(0.5, τ), T(1, τ) }, τ = 0 ..0.6, title = Distribution de la température en fonction du temps, labels = [`τ`, `T`]);



> *plot*({ *T*(X, 0), *T*(X, 0.1), *T*(X, 0.2), *T*(X, 0.3), *T*(X, 0.4), *T*(X, 0.5) }, X = 0 ..1, colour = [red, blue, green, black, yellow, brown], labels = [\`*X*\`, \`*T*\`]);



> $plot3d(T(X, \tau), X = 0 ..0.5, \tau = 0 ..0.6, axes = boxed);$



> ?

> ?

Solution numérique

```
> Restart:with(plots):
```

$$> Eq := \frac{\partial}{\partial t} T(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} T(x,t)\right);$$

$$\frac{\partial}{\partial t} T(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} T(x,t)\right) \quad (1.2.1)$$

```
> CL:={T(x,0) = 1, T(0,t) = 0, T(1,t) = 0};
```

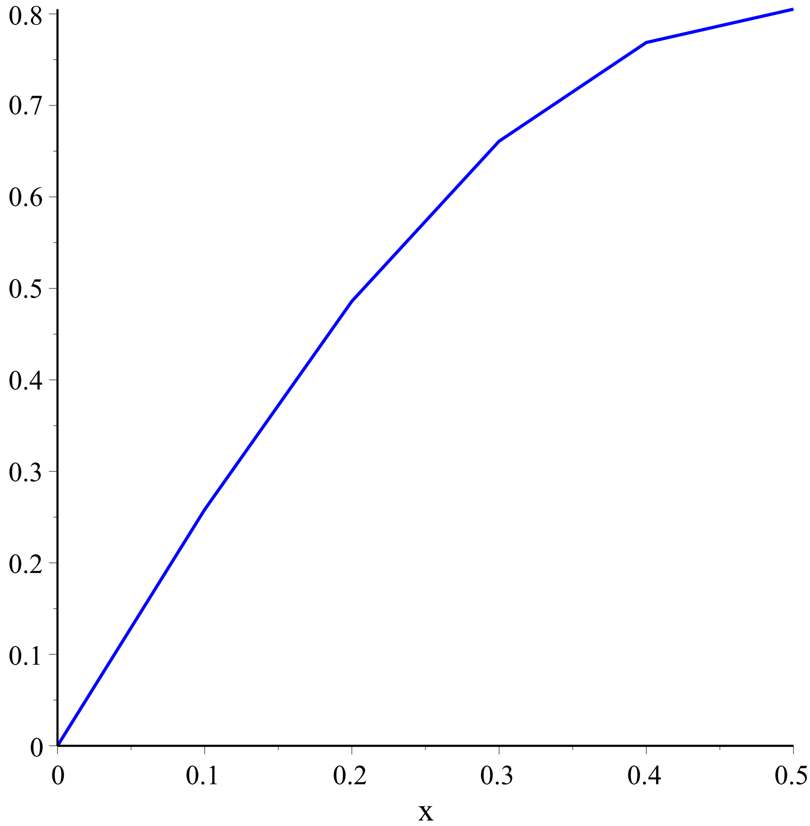
$$\{T(0,t)=0, T(1,t)=0, T(x,0)=1\} \quad (1.2.2)$$

Résolution et création d'un module de solutions comprenant les focntions: plot, plot3d, value, ...etc.

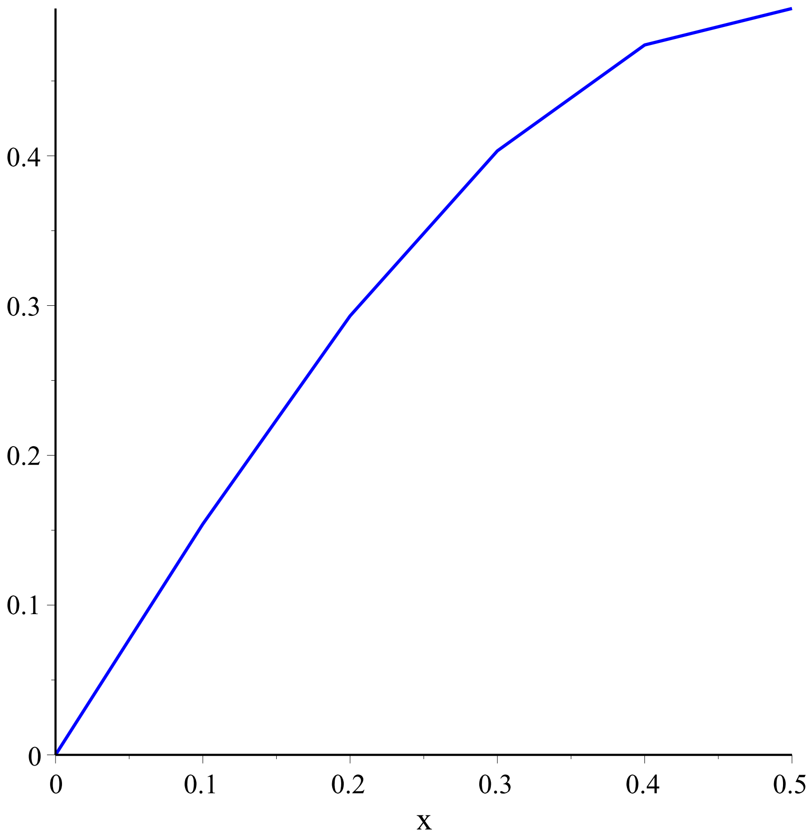
```
> Sol := pdsolve( Eq, CL, numeric, timestep=0.01, spacestep=
  0.1);
```

module( ) **export** *plot, plot3d, animate, value, settings*; ... **end module** **(1.2.3)**

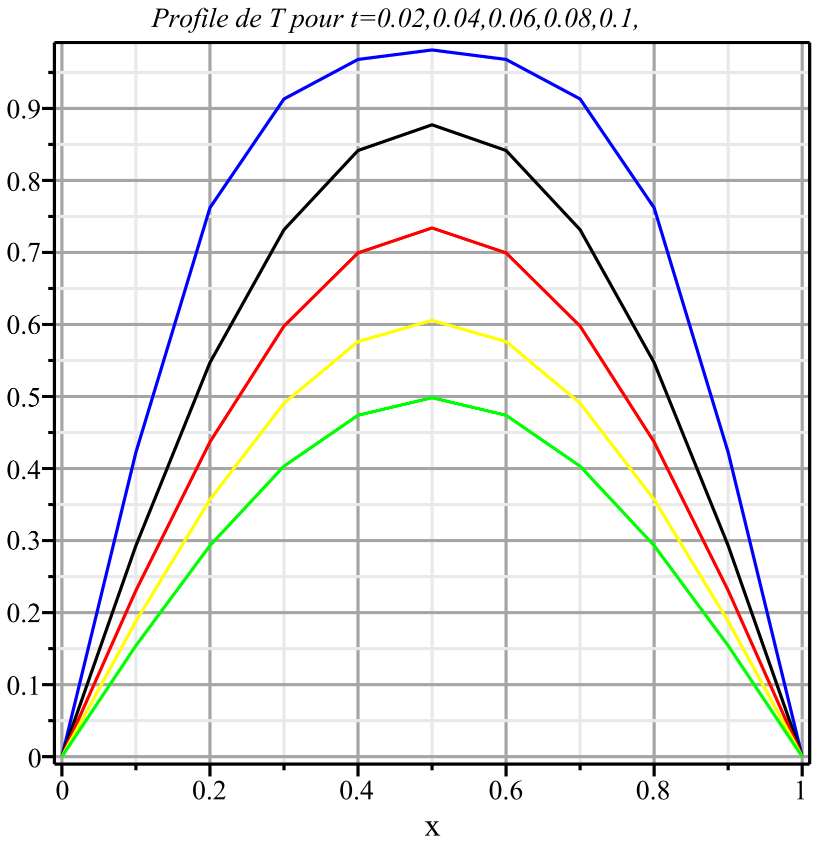
$> Sol :- plot(T(x,t), x = 0..0.5, t = 0.05, color = blue);$



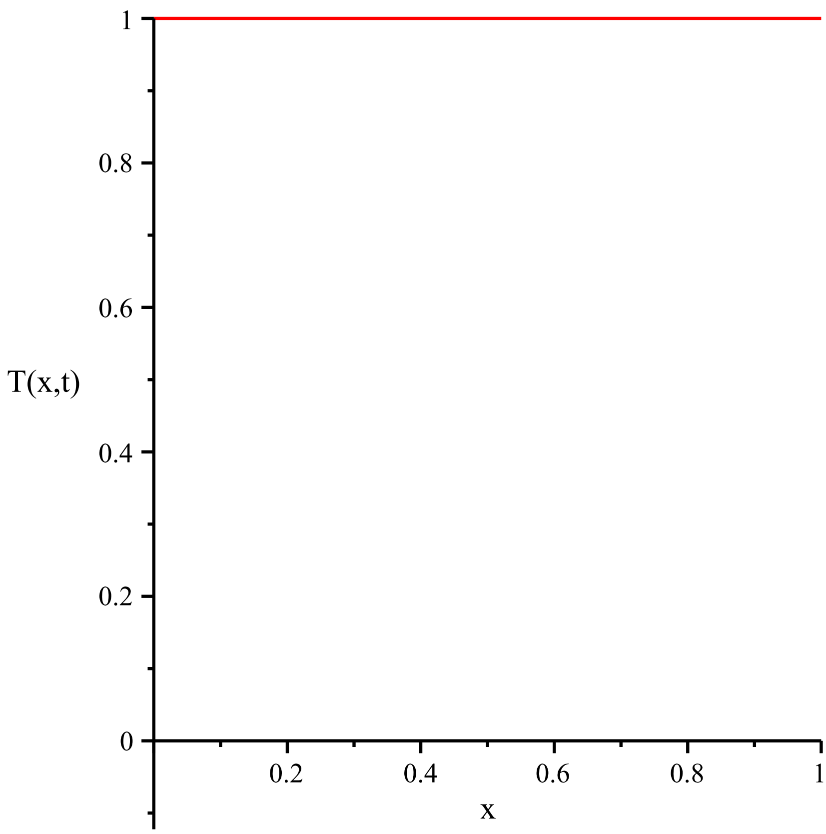
> $Sol := plot(T(x, t), x = 0..0.5, t = 0.1, color = blue);$



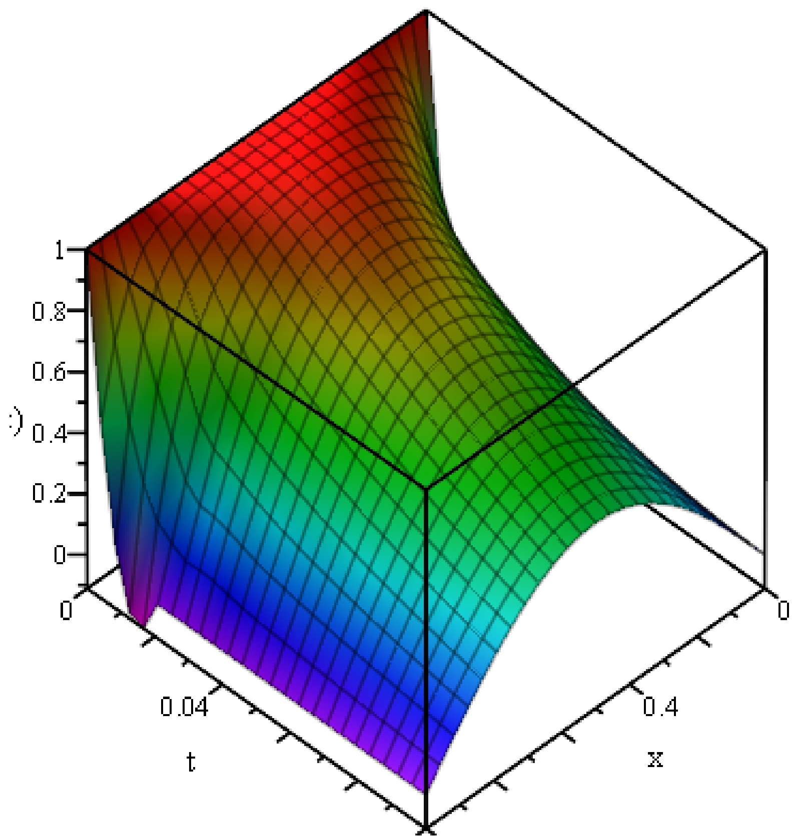

```
> g1 := Sol :- plot(t = 0.02, colour = blue) :
  g2 := Sol :- plot(t = 0.04, colour = black) :
  g3 := Sol :- plot(t = 0.06, colour = red) :
  g4 := Sol :- plot(t = 0.08, colour = yellow) :
  g5 := Sol :- plot(t = 0.1, colour = green) :
  plots[display]({g1, g2, g3, g4, g5}, gridlines = true, axes = boxed, title
       = `Profile de T pour t=0.02,0.04,0.06,0.08,0.1,`);
```




```
Sol :- animate( T(x,t) ,t=0..0.7, frames=50, labels=["x", "T
(x,t)"], labelfont=[TIMES,ROMAN,14]);
```




```
> Sol :- plot3d(T(x,t), t=0..0.1, shading=zhue, axes=boxed,
  labels=["x","t","T(x,t)"], labelfont=[TIMES,ROMAN,16]);
```



Pour avoir les valeur numériques, on crée une procédure utilisant la commande **value** à partir du module de la solution.

```
> ValTemp := Sol :- value();
```

? **(1.2.4)**

La veleur de la temperature en $x = 0.1$, $t = 0.01$ *est* :

```
> ValTemp( 0.1, 0.01 );
```

$[x = 0.1, t = 0.01, T(x, t) = 0.7320441988895027254]$ **(1.2.5)**

> ?

> ?

Equation de Laplace

Détermination de la temperature $T(x, y)$ à travers une plaque rectangulaire dont les extrémités sont maintenues à des températures constantes.

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$$

Conditions aux limites:

$$T(0, y) = 0,$$

$$T(\mathrm{a}, \mathrm{y}) = 0,$$
$$T(x, 0) = 0,$$
$$T(x, \mathrm{b}) = \mathrm{T}_0,$$

Solution analytique

> *restart* :

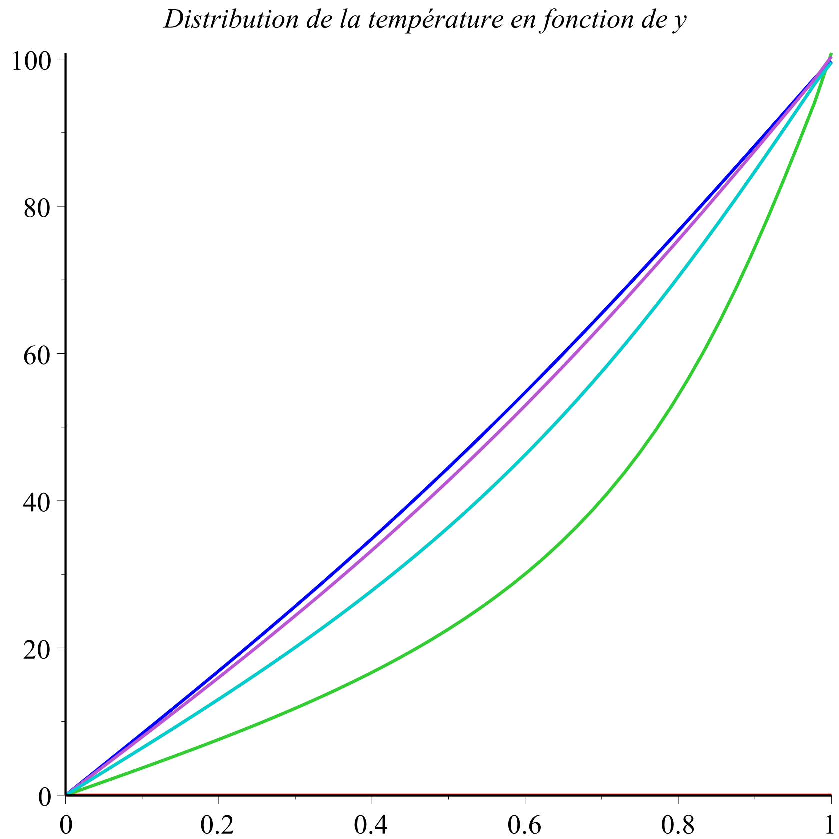
> $T(x, y) := \dfrac{4 \cdot T0}{\pi} \displaystyle\sum_{k=1}^{100} \dfrac{\sin\left(\dfrac{(2 \cdot k - 1)\, \pi\, x}{a}\right) \sinh\left(\dfrac{(2 \cdot k - 1)\, \pi\, y}{a}\right)}{(2 \cdot k - 1) \cdot \sinh\left(\dfrac{(2 \cdot k - 1)\, \pi\, b}{a}\right)};$

$$(x, y) \rightarrow \frac{4\, T0 \left(\displaystyle\sum_{k=1}^{100} \frac{\sin\left(\frac{(2\, k - 1)\, \pi\, x}{a}\right) \sinh\left(\frac{(2\, k - 1)\, \pi\, y}{a}\right)}{(2\, k - 1)\, \sinh\left(\frac{(2\, k - 1)\, \pi\, b}{a}\right)} \right)}{\pi} \quad \textbf{(2.1.1)}$$

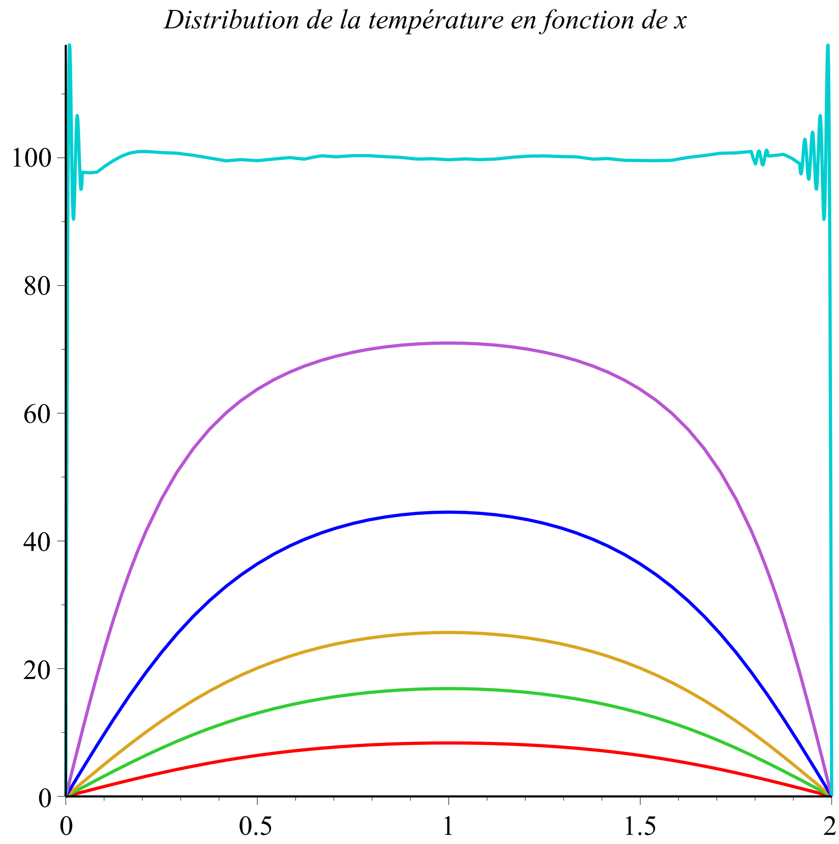
> $a := 2;\ b := 1;\ T0 := 100;$

2 **(2.1.2)**
1
100

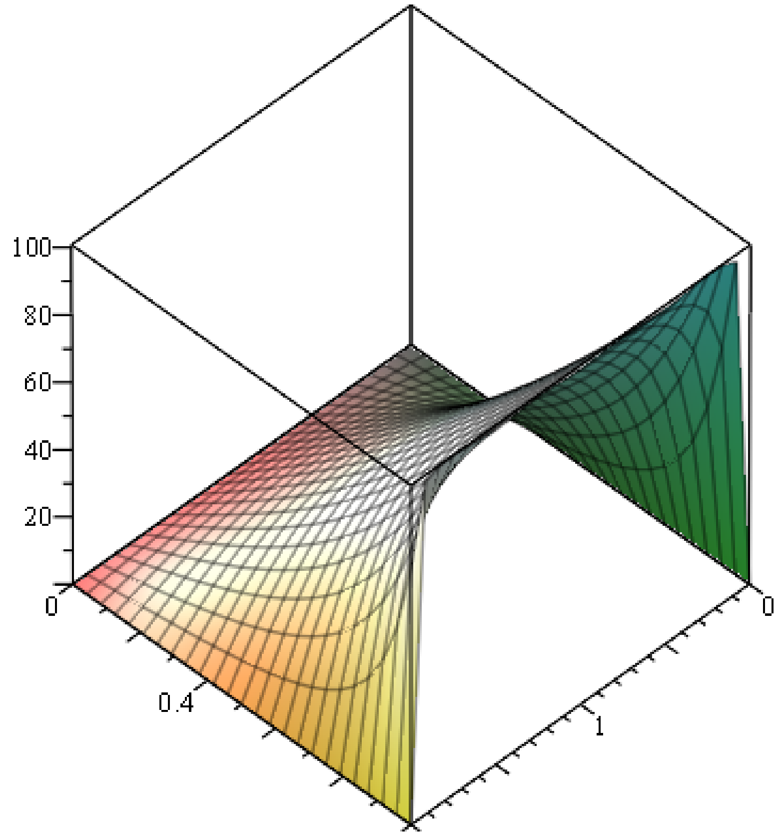
> *plot*({ $T(0.25, \mathrm{y})$, $T(0.5, \mathrm{y})$, $T(1.0, \mathrm{y})$, $T(1.25, \mathrm{y})$, $T(1.5, \mathrm{y})$, $T(2, \mathrm{y})$ }, y = 0 ..1, title = Distribution de la température en fonction de y, *labels* = [\`y\`, \`T\`]);



> plot({ T(x, 0.1), T(x, 0.2), T(x, 0.3), T(x, 0.5), T(x, 0.75), T(x, 1.0) }, x = 0 ..2, title = Distribution de la température en fonction de x, labels = [`x`, `T`])



> plot3d(T(x, y), x = 0 ..2, y = 0 ..1, axes = boxed);



> ?

Solution numérique

```
> Restart:with(plots):
```

$$> Eq := \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} T(x,y)\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} T(x,y)\right) = 0;$$

$$Eq := \frac{\partial^2}{\partial x^2} T(x,y) + \frac{\partial^2}{\partial y^2} T(x,y) = 0 \quad (2.2.1)$$

```
> CL:={T(0,y) = 0, T(2,y) = 0, T(x,0) = 0, T(x,1) = 100};
```

$$CL := \{T(0,y) = 0, T(2,y) = 0, T(x,0) = 0, T(x,1) = 100\} \quad (2.2.2)$$

Résolution et création d'un module de solutions comprenant les focntions: plot, plot3d, value, ...etc.

```
> Sol := pdsolve(Eq, CL, numeric, spacestep = 0.1);
Error, (in pdsolve/numeric) unable to handle elliptic PDEs
```

> ?

> ?

Equation d'onde

Détermination du déplacement u(x, t) d'une corde dont les extrémités sont maintenues.

$$\frac{\partial^2}{\partial t^2} u(x, t) = c^2 \cdot \frac{\partial^2}{\partial x^2} u(x, t)$$

Conditions aux limites et initiale:

$$u(0, t) = 0,$$
$$u(L, t) = 0,$$
$$u(x, 0) = 2 \cdot \sin\left(\frac{\pi}{L} x\right),$$
$$\frac{\partial}{\partial t}(u(x, 0)) = -\sin\left(\frac{2 \cdot \pi}{L} x\right),$$

Solution analytique

```
> restart;
```

$$> u(x, t) := 2 \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{c \pi t}{L}\right) - \frac{L}{2 \pi c} \sin\left(\frac{2 \pi x}{L}\right) \sin\left(\frac{2 c \pi t}{L}\right);$$

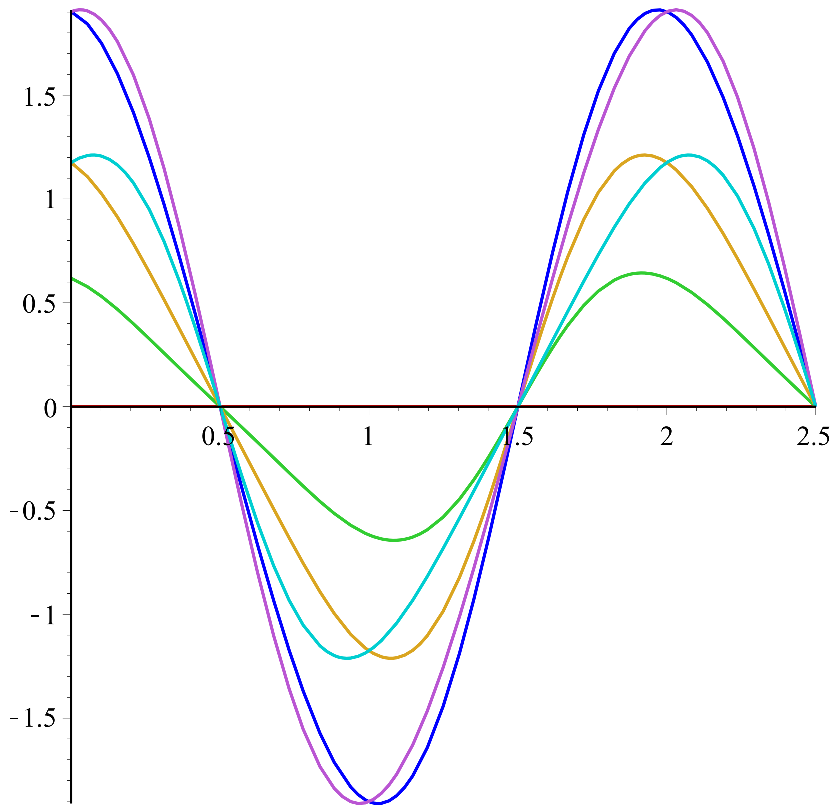
$$(x, t) \rightarrow 2 \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{c \pi t}{L}\right) - \frac{1}{2} \frac{L \sin\left(\frac{2 \pi x}{L}\right) \sin\left(\frac{2 c \pi t}{L}\right)}{c \pi} \quad \textbf{(3.1.1)}$$

$$> L := 1; c := 1;$$

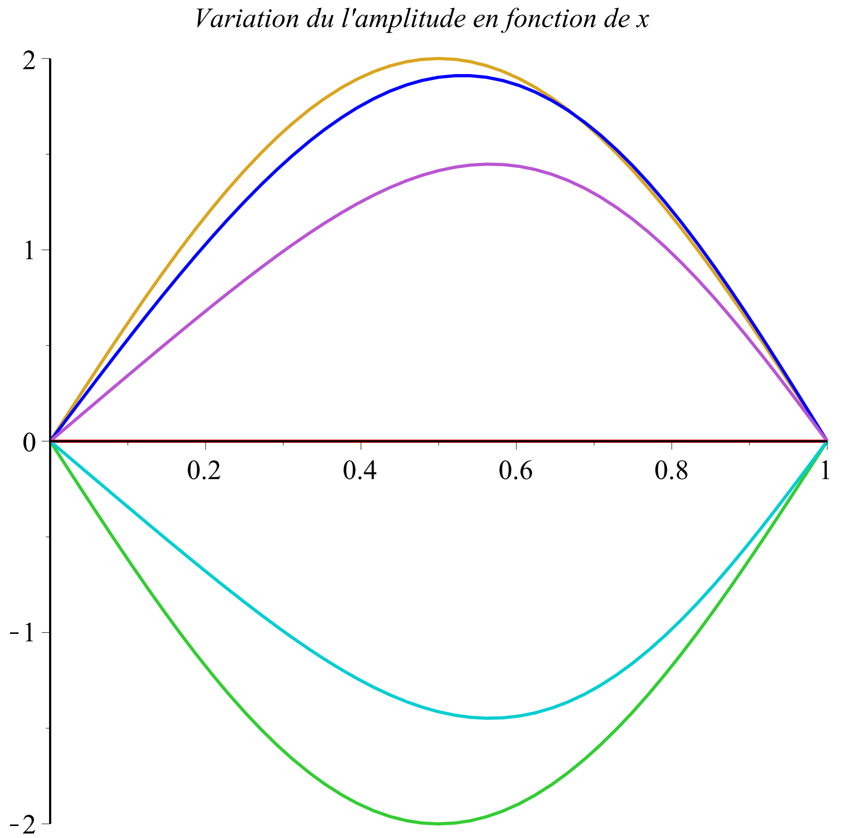
$$1$$
$$1 \quad \textbf{(3.1.2)}$$

> plot({u(0.1, t), u(0.2, t), u(0.4, t), u(0.6, t), u(0.8, t), u(1, t) }, t=0 ..2.5, title = `Variation de l'amplitude en fonction du temps`, labels = [`t`, `u`]);

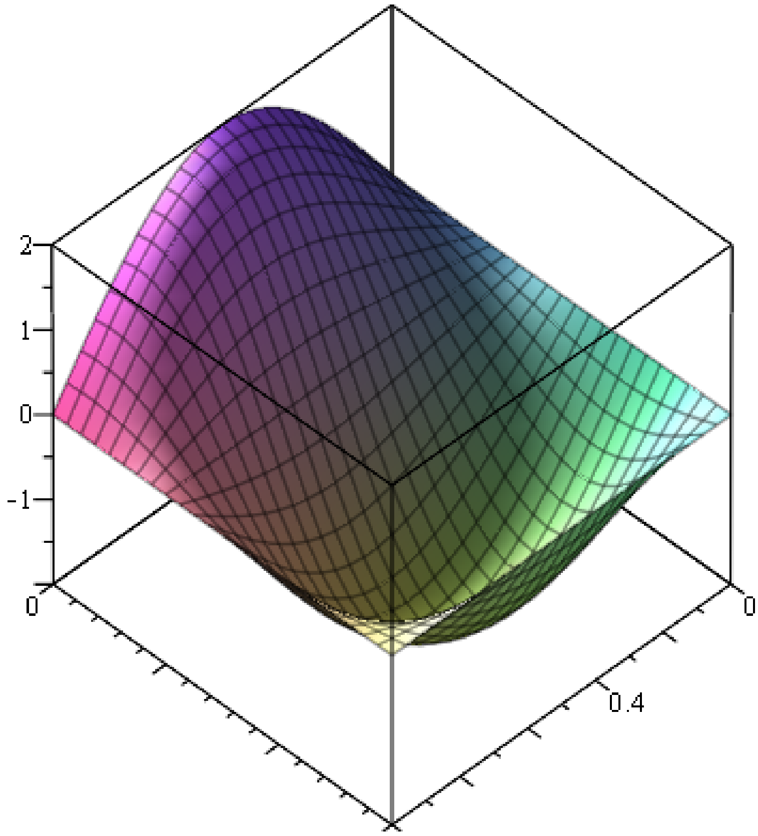
Variation de l'amplitude en fonction du temps



> *plot*({*u*(x, 0.1), *u*(x, 0.25), *u*(x, 0.75), *u*(x, 1.0), *u*(x, 1.5), *u*(x, 2.0)}, x = 0 ..1, title = `Variation du l'amplitude en fonction de x`, *labels* = [`*x*`, `*u*`]);



> $plot3d(u(x, t), x = 0..1, t = 0..1.5, axes = boxed);$



> ?

Solution numérique

> *restart*

> $PDE := \frac{\partial^2}{\partial t^2} u(x,t) = \left(\frac{\partial^2}{\partial x^2} u(x,t)\right)$

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} u(x,t)\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} u(x,t)\right) \quad \textbf{(3.2.1)}$$

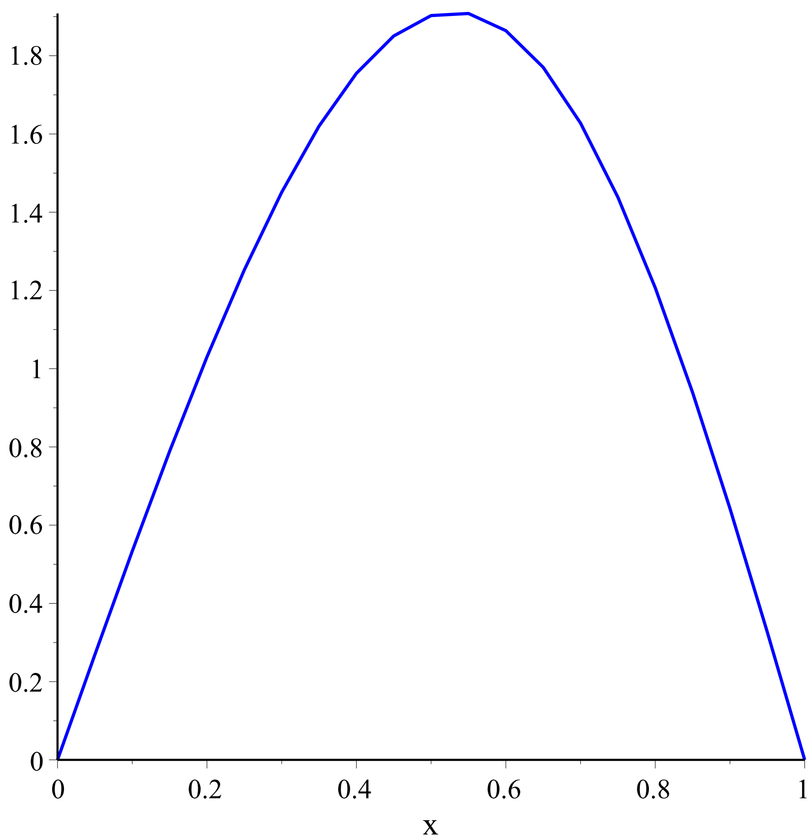
> $IBC := \{u(0,t)=0, u(1,t)=0, u(x,0)=2\cdot\sin(\pi\cdot x), D_2(u)(x,0)=-\sin(2\cdot\pi\cdot x)\}$

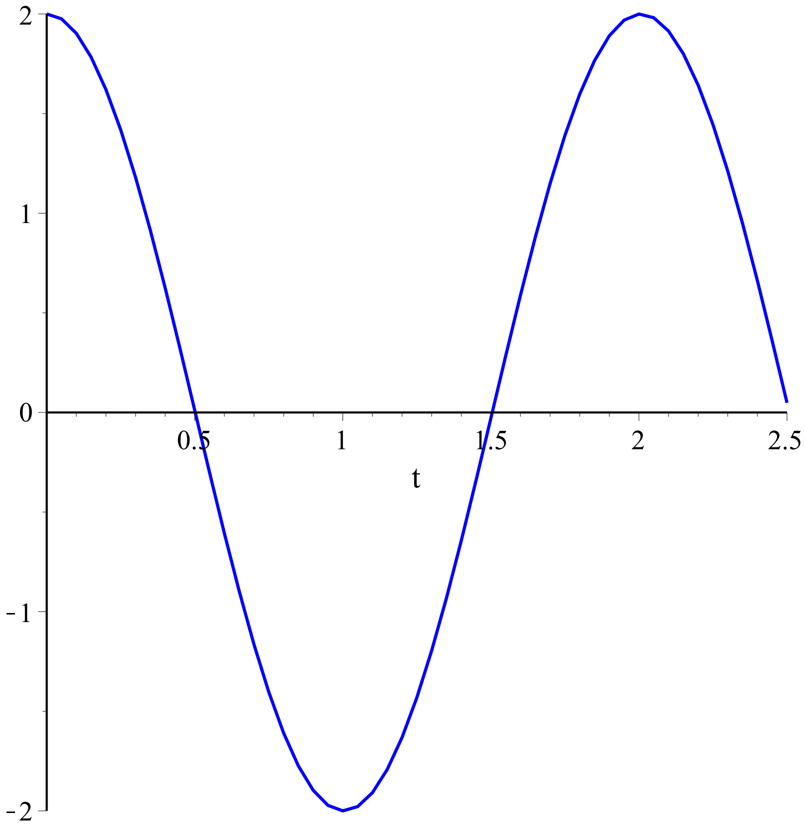
$$\{u(0,t)=0, u(1,t)=0, u(x,0)=2\sin(\pi x), D_2(u)(x,0)=-\sin(2\pi x)\} \quad \textbf{(3.2.2)}$$

> *Sol* := *pdsolve*(*PDE*, *IBC*, *numeric*, *spacestep* = 0.05)

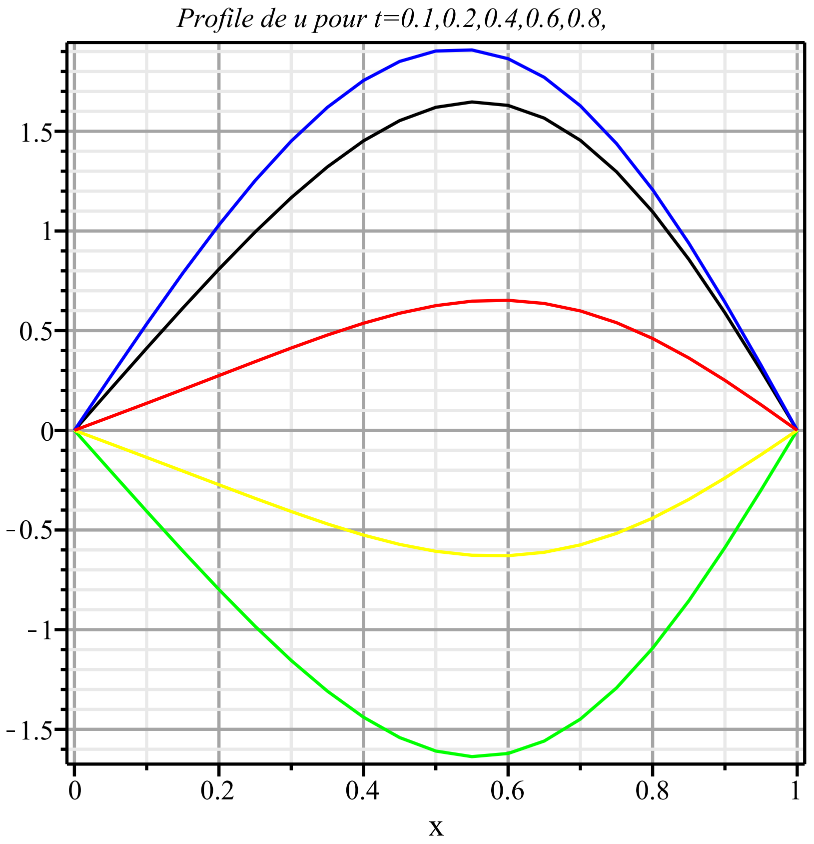
module( ) **export** *plot*, *plot3d*, *animate*, *value*, *settings*; ... **end module** **(3.2.3)**

> *Sol* :- *plot*(*u*(*x*, *t*) , *x* = 0 ..1, *t* = 0.1, *color* = *blue*); *Sol* :- *plot*(*u*(*x*, *t*) , *x* = 0.5, *t* = 0 ..2.5, *color* = *blue*)

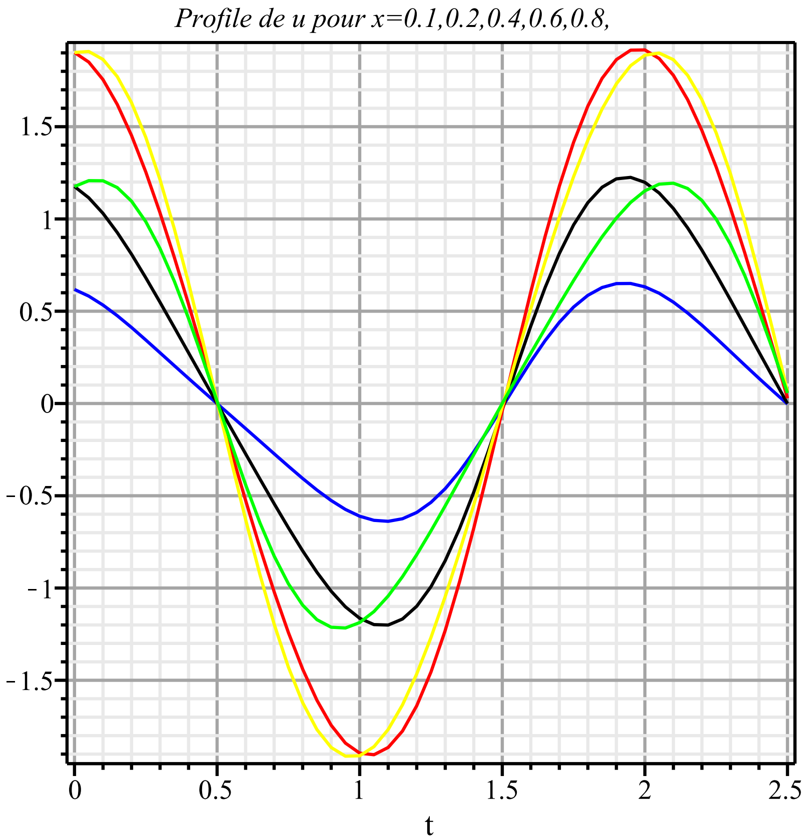

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x




```
> g1 := Sol :- plot(t = 0.1, colour = blue) :
  g2 := Sol :- plot(t = 0.2, colour = black) :
  g3 := Sol :- plot(t = 0.4, colour = red) :
  g4 := Sol :- plot(t = 0.6, colour = yellow) :
  g5 := Sol :- plot(t = 0.8, colour = green) :
  plots[display]({g1, g2, g3, g4, g5}, gridlines = true, axes = boxed, title
       = `Profile de u pour t=0.1,0.2,0.4,0.6,0.8,`);
```



$> g1 := Sol:\text{-}plot(x = 0.1, t = 0 ..2.5, colour = blue) :$
$g2 := Sol:\text{-}plot(x = 0.2, t = 0 ..2.5, colour = black) :$
$g3 := Sol:\text{-}plot(x = 0.4, t = 0 ..2.5, colour = red) :$
$g4 := Sol:\text{-}plot(x = 0.6, t = 0 ..2.5, colour = yellow) :$
$g5 := Sol:\text{-}plot(x = 0.8, t = 0 ..2.5, colour = green) :$
$plots[display](\{g1, g2, g3, g4, g5\}, gridlines = true, axes = boxed, title$
$= \text{`Profile de u pour x=0.1,0.2,0.4,0.6,0.8,`});$


Profile de u pour x=0.1,0.2,0.4,0.6,0.8,
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
t

> ?