Equation de Convection-Diffusion 1D

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Dr. Laïd MESSAOUDI

Département de Mécanique

Université de Batna

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EXAMEN

Détermination de la distribution de $x6(x)$ transportée par convection-diffusion à travers un domaine 1D dont les extrémités sont soumises aux (C.L.):
Schéma Upwind pour le terme convectif.

$$\frac{d}{dx}\left(\rho \cdot u \cdot \phi(x)\right) - \frac{d}{dx}\left(\Gamma \cdot \frac{d}{dx}\phi(x)\right) = 0$$

Conditions aux limites (C.L):

$$\phi(0) = \phi_0 = 1,$$
$$\frac{d}{dx}\phi(L) = \phi'_L = 0,$$

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> Restart : Digits := 4 :
> F := 0.2; d := 0.4; ndx := 10;
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$$F := 0.2$$
$$d := 0.4$$
$$ndx := 10 \quad \textbf{(1.1)}$$

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> i_max := ndx;
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$$i_{max} := 10 \quad \textbf{(1.2)}$$

Nombre d'équations:

> $Ne := i_{max}$

$$Ne := 10 \quad (1.3)$$

Conditions aux Limites:

> $\phi[0] := 1.0;$
$d\phi[i_{max} + 1] := 0;$

$$\phi_0 := 1.0$$
$$d\phi_{11} := 0 \quad (1.4)$$

Noeuds internes:

> **for** i **from** 2 **to** $i_{max} - 1$ **do**
$Sp[i] := 0;$
$Su[i] := 0;$
$a_W[i] := d + F;$
$a_E[i] := d;$
$a_P[i] := a_W[i] + a_E[i] - Sp[i];$
end do;

$$Sp_2 := 0$$
$$Su_2 := 0$$
$$a_{W_2} := 0.6$$
$$a_{E_2} := 0.4$$
$$a_{P_2} := 1.0$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 0.6$$
$$a_{E_3} := 0.4$$
$$a_{P_3} := 1.0$$
$$Sp_4 := 0$$
$$Su_4 := 0$$
$$a_{W_4} := 0.6$$
$$a_{E_4} := 0.4$$
$$a_{P_4} := 1.0$$
$$Sp_5 := 0$$
$$Su_5 := 0$$
$$a_{W_5} := 0.6$$
$$a_{E_5} := 0.4$$
$$a_{P_5} := 1.0$$
$$Sp_6 := 0$$
$$Su_6 := 0$$
$$a_{W_6} := 0.6$$
$$a_{E_6} := 0.4$$
$$a_{P_6} := 1.0$$

$$Sp_7 := 0$$
$$Su_7 := 0$$
$$a_{W_7} := 0.6$$
$$a_{E_7} := 0.4$$
$$a_{P_7} := 1.0$$
$$Sp_8 := 0$$
$$Su_8 := 0$$
$$a_{W_8} := 0.6$$
$$a_{E_8} := 0.4$$
$$a_{P_8} := 1.0$$
$$Sp_9 := 0$$
$$Su_9 := 0$$
$$a_{W_9} := 0.6$$
$$a_{E_9} := 0.4$$
$$a_{P_9} := 1.0 \quad (1.5)$$

Noeud gauche:

$> Sp[1] := -(2 \cdot d + F);$

$Su[1] := (2 \cdot d + F) \cdot \phi[0];$

$a_W[1] := 0;$

$a_E[1] := d;$

$a_P[1] := a_W[1] + a_E[1] - Sp[1];$

$$Sp_1 := -1.0$$
$$Su_1 := 1.00$$
$$a_{W_1} := 0$$
$$a_{E_1} := 0.4$$
$$a_{P_1} := 1.4 \quad (1.6)$$

Noeud droit:

$> Sp[i_{max}] := 0;$

$Su[i_{max}] := 0;$

$a_W[i_{max}] := d + F;$

$a_E[i_{max}] := 0;$

$a_P[i_{max}] := a_W[i_{max}] + a_E[i_{max}] - Sp[i_{max}];$

$$Sp_{10} := 0$$
$$Su_{10} := 0$$
$$a_{W_{10}} := 0.6$$
$$a_{E_{10}} := 0$$
$$a_{P_{10}} := 0.6 \quad (1.7)$$

Equations:

> $k := 1$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

> **for** i **from** 1 **to** i_{max} **do**
>
> $Eq[k] := a_P[i] \cdot \phi[i] = a_W[i] \cdot \phi[i-1] + a_E[i] \cdot \phi[i+1] + Su[i];$
>
> $k := k + 1;$
>
> **end do**;

$$Eq_1 := 1.4\ \phi_1 = 1.00 + 0.4\ \phi_2$$
$$k := 2$$
$$Eq_2 := 1.0\ \phi_2 = 0.6\ \phi_1 + 0.4\ \phi_3$$
$$k := 3$$
$$Eq_3 := 1.0\ \phi_3 = 0.6\ \phi_2 + 0.4\ \phi_4$$
$$k := 4$$
$$Eq_4 := 1.0\ \phi_4 = 0.6\ \phi_3 + 0.4\ \phi_5$$
$$k := 5$$
$$Eq_5 := 1.0\ \phi_5 = 0.6\ \phi_4 + 0.4\ \phi_6$$
$$k := 6$$
$$Eq_6 := 1.0\ \phi_6 = 0.6\ \phi_5 + 0.4\ \phi_7$$
$$k := 7$$
$$Eq_7 := 1.0\ \phi_7 = 0.6\ \phi_6 + 0.4\ \phi_8$$
$$k := 8$$
$$Eq_8 := 1.0\ \phi_8 = 0.6\ \phi_7 + 0.4\ \phi_9$$
$$k := 9$$
$$Eq_9 := 1.0\ \phi_9 = 0.6\ \phi_8 + 0.4\ \phi_{10}$$
$$k := 10$$
$$Eq_{10} := 0.6\ \phi_{10} = 0.6\ \phi_9$$
$$k := 11 \quad \textbf{(1.1.2)}$$

Ecriture du système d'équations:

> **for** k **from** 1 **to** Ne **do** $Eq[k]$ **end do**;

$$1.4\ \phi_1 = 1.00 + 0.4\ \phi_2$$
$$1.0\ \phi_2 = 0.6\ \phi_1 + 0.4\ \phi_3$$
$$1.0\ \phi_3 = 0.6\ \phi_2 + 0.4\ \phi_4$$
$$1.0\ \phi_4 = 0.6\ \phi_3 + 0.4\ \phi_5$$
$$1.0\ \phi_5 = 0.6\ \phi_4 + 0.4\ \phi_6$$
$$1.0\ \phi_6 = 0.6\ \phi_5 + 0.4\ \phi_7$$
$$1.0\ \phi_7 = 0.6\ \phi_6 + 0.4\ \phi_8$$
$$1.0\ \phi_8 = 0.6\ \phi_7 + 0.4\ \phi_9$$
$$1.0\ \phi_9 = 0.6\ \phi_8 + 0.4\ \phi_{10}$$
$$0.6\ \phi_{10} = 0.6\ \phi_9 \quad \textbf{(1.1.3)}$$

> $Eqs := \{seq(Eq[k], k = 1..Ne)\};$

$$Eqs := \{1.4\ \phi_1 = 1.00 + 0.4\ \phi_2, 1.0\ \phi_2 = 0.6\ \phi_1 + 0.4\ \phi_3, 1.0\ \phi_3 = 0.6\ \phi_2 + 0.4\ \phi_4, 1.0\ \phi_4 \quad \textbf{(1.1.4)}$$
$$= 0.6\ \phi_3 + 0.4\ \phi_5, 1.0\ \phi_5 = 0.6\ \phi_4 + 0.4\ \phi_6, 1.0\ \phi_6 = 0.6\ \phi_5 + 0.4\ \phi_7, 1.0\ \phi_7 = 0.6\ \phi_6$$

$+ 0.4\ \phi_8,\ 1.0\ \phi_8 = 0.6\ \phi_7 + 0.4\ \phi_9,\ 1.0\ \phi_9 = 0.6\ \phi_8 + 0.4\ \phi_{10},\ 0.6\ \phi_{10} = 0.6\ \phi_9\}$

> *Tmps* := [*seq*(ϕ[*i*], *i* = 1 ..*Ne*)];

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}] \quad \textbf{(1.1.5)}$$

> *SolT* := *solve*(*Eqs*, *Tmps*);

$$SolT := [[\phi_1 = 1., \phi_2 = 1., \phi_3 = 1., \phi_4 = 1., \phi_5 = 1., \phi_6 = 1., \phi_7 = 1., \phi_8 = 1., \phi_9 = 1., \phi_{10} = 1.]] \quad \textbf{(1.1.6)}$$

> *with*(*LinearAlgebra*) :

Forme matricielle:

> *A*, *b* := *GenerateMatrix*(*Eqs*, *Tmps*)

$$A, b := \begin{bmatrix} 1.4 & -0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6 & 0.6 \\ -0.6 & 1.0 & -0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.6 & 1.0 & -0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.6 & 1.0 & -0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6 & 1.0 & -0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.6 & 1.0 & -0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.6 & 1.0 & -0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.6 & 1.0 & -0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6 & 1.0 & -0.4 \end{bmatrix}, \begin{bmatrix} 1.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(1.1.7)}$$