Equation de Convection-Diffusion 1D

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EXAMEN

Détermination de la distribution de x6(x) transportée par convection-diffusion à travers un domaine 1D dont les extrémités sont soumises aux (C.L.):
Schéma Hybride pour le terme convectif.

$$\frac{d}{dx}\left(\rho \cdot u \cdot \phi(x)\right) - \frac{d}{dx}\left(\Gamma \cdot \frac{d}{dx}\phi(x)\right) = 0$$

Conditions aux limites (C.L):

$$\phi(0) = \phi_0 = 1,$$
$$\phi(L) = \phi_L = 0,$$

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> Restart : Digits := 4 :
> L := 1.0; F := 2.5; d := 0.5; ndx := 5;
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$$L := 1.0$$
$$F := 2.5$$
$$d := 0.5$$
$$ndx := 5 \qquad \textbf{(1.1)}$$

```
> δx := L/ndx; i_max := ndx;
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$$\delta x := 0.2000 \qquad \textbf{(1.2)}$$

$$i_{max} := 5 \quad \textbf{(1.2)}$$

Nombre d'équations:

$> Ne := i_{\max}$

$$Ne := 5 \quad \textbf{(1.3)}$$

Conditions aux Limites:

$> \phi[0] := 1.0;$

$\phi[i_{\max}+1] := 0;$

$$\phi_0 := 1.0$$
$$\phi_6 := 0 \quad \textbf{(1.4)}$$

Noeuds internes:

$>$ **for** i **from** 2 **to** $i_{\max}-1$ **do**

$Sp[i] := 0;$

$Su[i] := 0;$

$a_W[i] := F;$

$a_E[i] := 0;$

$a_P[i] := a_W[i] + a_E[i] - Sp[i];$

end do;

$$Sp_2 := 0$$
$$Su_2 := 0$$
$$a_{W_2} := 2.5$$
$$a_{E_2} := 0$$
$$a_{P_2} := 2.5$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 2.5$$
$$a_{E_3} := 0$$
$$a_{P_3} := 2.5$$
$$Sp_4 := 0$$
$$Su_4 := 0$$
$$a_{W_4} := 2.5$$
$$a_{E_4} := 0$$
$$a_{P_4} := 2.5 \quad \textbf{(1.5)}$$

Noeud gauche:

$> Sp[1] := -(2\cdot d + F);$

$Su[1] := (2\cdot d + F)\cdot\phi[0];$

$a_W[1] := 0;$

$a_E[1] := 0;$

$a_P[1] := a_W[1] + a_E[1] - Sp[1];$

$$Sp_1 := -3.5$$
$$Su_1 := 3.50$$
$$a_{W_1} := 0$$

$$a_{E_1} := 0$$

$$a_{P_1} := 3.5 \quad \textbf{(1.6)}$$

Noeud droit:

> $Sp[i_{max}] := -2 \cdot d;$
> $Su[i_{max}] := 2 \cdot d \cdot \phi[i_{max}+1];$
> $a_W[i_{max}] := F;$
> $a_E[i_{max}] := 0;$
> $a_P[i_{max}] := a_W[i_{max}] + a_E[i_{max}] - Sp[i_{max}];$

$$Sp_5 := -1.0$$

$$Su_5 := 0.$$

$$a_{W_5} := 2.5$$

$$a_{E_5} := 0$$

$$a_{P_5} := 3.5 \quad \textbf{(1.7)}$$

Equations:

> $k := 1$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

> **for** i **from** 1 **to** i_{max} **do**
> $\quad Eq[\mathrm{k}] := a_P[\mathrm{i}] \cdot \phi[\mathrm{i}] = a_W[\mathrm{i}] \cdot \phi[\mathrm{i}-1] + a_E[\mathrm{i}] \cdot \phi[\mathrm{i}+1] + Su[\mathrm{i}];$
> $\quad k := k+1;$
> **end do**;

$$Eq_1 := 3.5\ \phi_1 = 3.50$$

$$k := 2$$

$$Eq_2 := 2.5\ \phi_2 = 2.5\ \phi_1$$

$$k := 3$$

$$Eq_3 := 2.5\ \phi_3 = 2.5\ \phi_2$$

$$k := 4$$

$$Eq_4 := 2.5\ \phi_4 = 2.5\ \phi_3$$

$$k := 5$$

$$Eq_5 := 3.5\ \phi_5 = 2.5\ \phi_4$$

$$k := 6 \quad \textbf{(1.1.2)}$$

Ecriture du système d'équations:

> **for** k **from** 1 **to** Ne **do** $Eq[\mathrm{k}]$ **end do**;

$$3.5\ \phi_1 = 3.50$$

$$2.5\ \phi_2 = 2.5\ \phi_1$$

$$2.5\ \phi_3 = 2.5\ \phi_2$$

$$2.5\ \phi_4 = 2.5\ \phi_3$$

$$3.5\ \phi_5 = 2.5\ \phi_4 \quad \textbf{(1.1.3)}$$

> $Eqs := \{seq(Eq[k], k = 1..Ne)\};$

$$Eqs := \{3.5\ \phi_1 = 3.50, 2.5\ \phi_2 = 2.5\ \phi_1, 2.5\ \phi_3 = 2.5\ \phi_2, 2.5\ \phi_4 = 2.5\ \phi_3, 3.5\ \phi_5 = 2.5\ \phi_4\} \quad \textbf{(1.1.4)}$$

> $Tmps := [seq(\phi[i], i=1..Ne)];$

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5] \quad \textbf{(1.1.5)}$$

> $SolT := solve(Eqs, Tmps);$

$$SolT := [[\phi_1 = 1., \phi_2 = 1., \phi_3 = 1., \phi_4 = 1., \phi_5 = 0.7143]] \quad \textbf{(1.1.6)}$$

> $with(LinearAlgebra):$

Forme matricielle:

> $A, b := GenerateMatrix(Eqs, Tmps)$

$$A, b := \begin{bmatrix} 3.5 & 0 & 0 & 0 & 0 \\ -2.5 & 2.5 & 0 & 0 & 0 \\ 0 & -2.5 & 2.5 & 0 & 0 \\ 0 & 0 & -2.5 & 2.5 & 0 \\ 0 & 0 & 0 & -2.5 & 3.5 \end{bmatrix}, \begin{bmatrix} 3.50 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(1.1.7)}$$

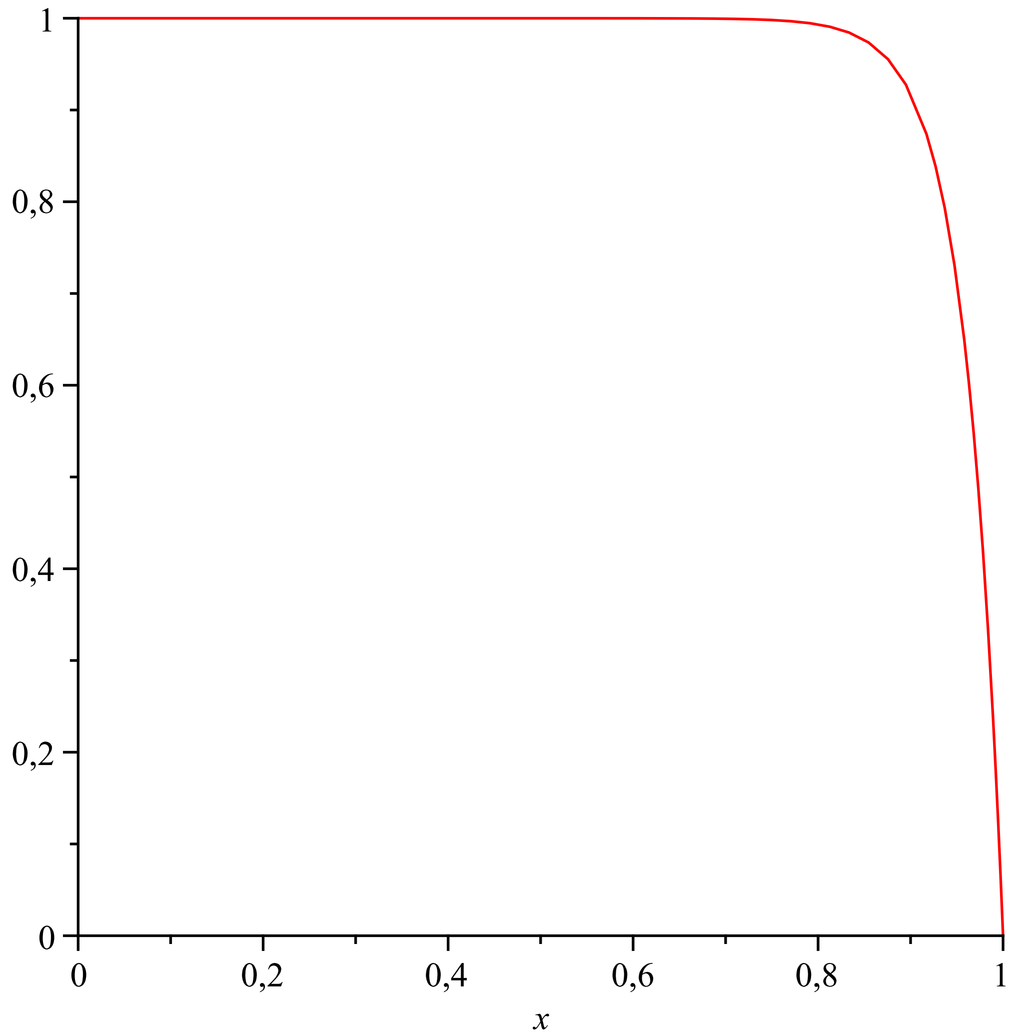
Solution exacte:

> $F(x) := 1 + \frac{1 - e^{(25 \cdot x)}}{7.20 \cdot 10^{10}};$

$$F := x \rightarrow 1 + \frac{1 - e^{25\,x}}{7.20 \cdot 10000000000} \quad \textbf{(1.1.8)}$$

> $with(plots):$

> $plot(F(x), x = 0..L);$




```
> for i from 1 to Ne do
      ϕ[i] := rhs(SolT[1, i])
  end do;
```

$$\phi_1 := 1.$$
$$\phi_2 := 1.$$
$$\phi_3 := 1.$$
$$\phi_4 := 1.$$
$$\phi_5 := 0.7143 \qquad \textbf{(1.1.9)}$$

```
> x[0] := 0;
  for i from 1 to i_max do
      x[i] := δx/2 + (i − 1)·δx;
  end do;
  x[i_max + 1] := L;
```

$$x_0 := 0$$
$$x_1 := 0.1000$$
$$x_2 := 0.3000$$
$$x_3 := 0.5000$$
$$x_4 := 0.7000$$

$$x_5 := 0.9000$$
$$x_6 := 1.0 \quad \textbf{(1.1.10)}$$

Abscisses des noeuds:

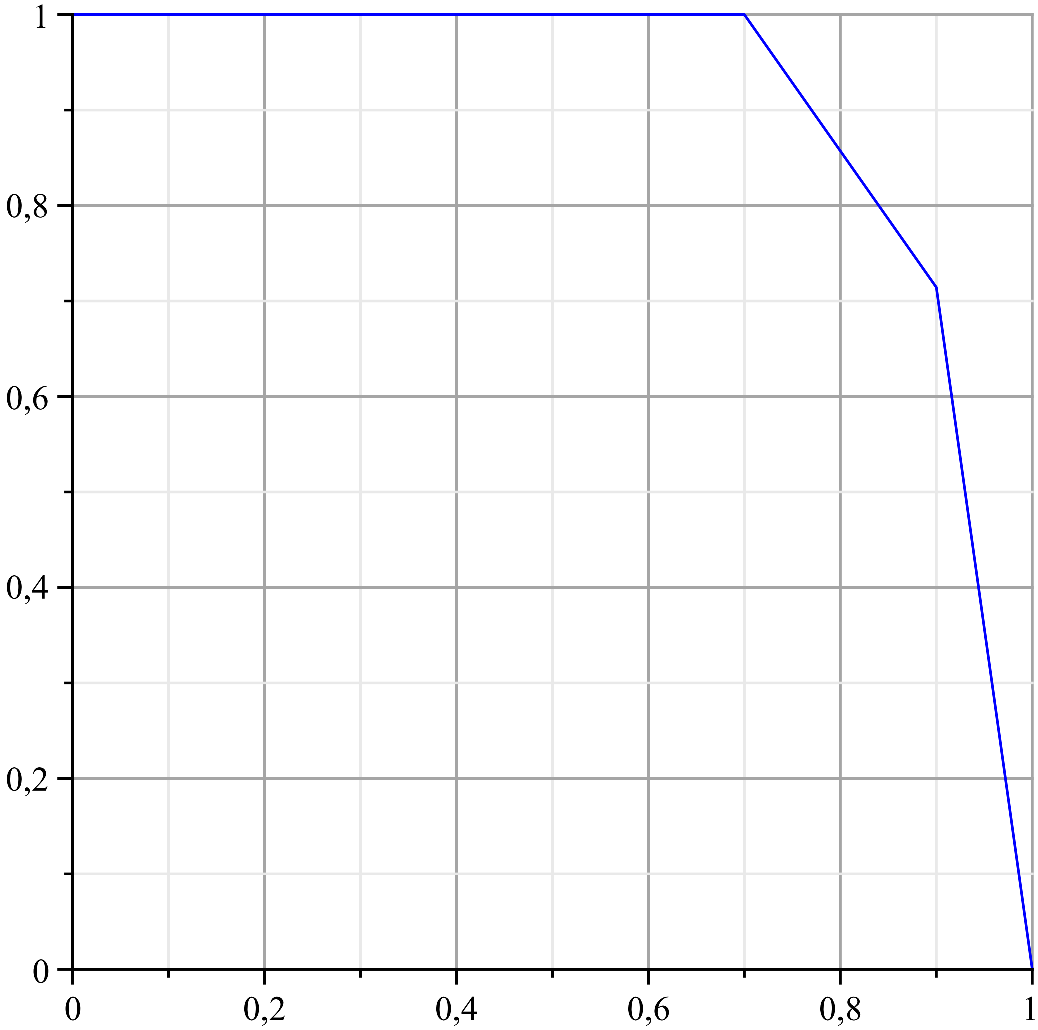
> $lpN := \left[\, seq\left(\, [x[i], \phi[i]], i = 0\,..\,i_{\max} + 1\right)\right]$

>

$lpN := [[0, 1.0], [0.1000, 1.], [0.3000, 1.], [0.5000, 1.], [0.7000, 1.], [0.9000, 0.7143], [1.0, 0]]$ **(1.1.11)**

Courbe Numérique:

> $listplot(lpN, color = blue, gridlines = true)$

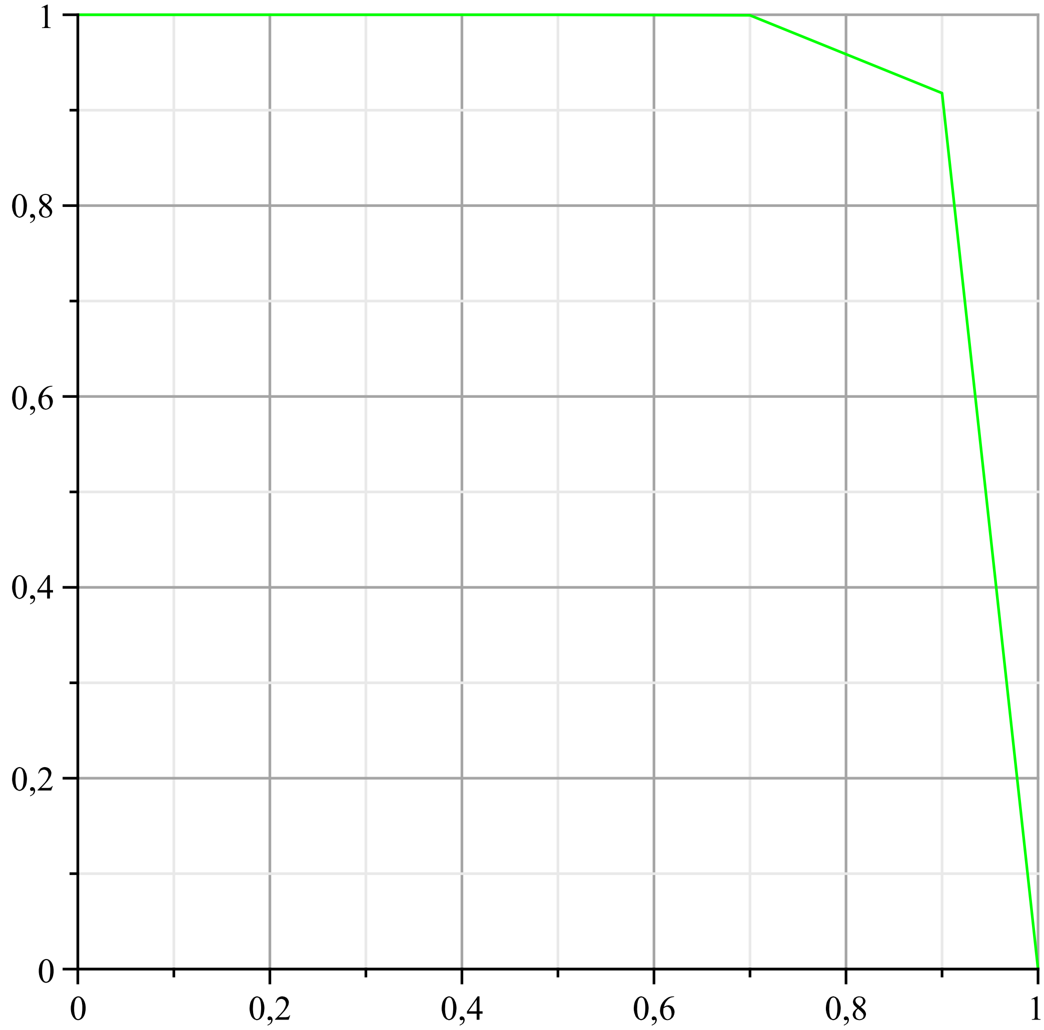


> $lpT := \left[seq\left(\, [x[\mathrm{i}], F(x[\mathrm{i}])\,], i = 0\,..\,i_{\max} + 1\right)\right]$

$lpT := [[0, 1.], [0.1000, 1.], [0.3000, 1.], [0.5000, 1.], [0.7000, 0.9994], [0.9000, 0.9179], [1.0, 0.]]$ **(1.1.12)**

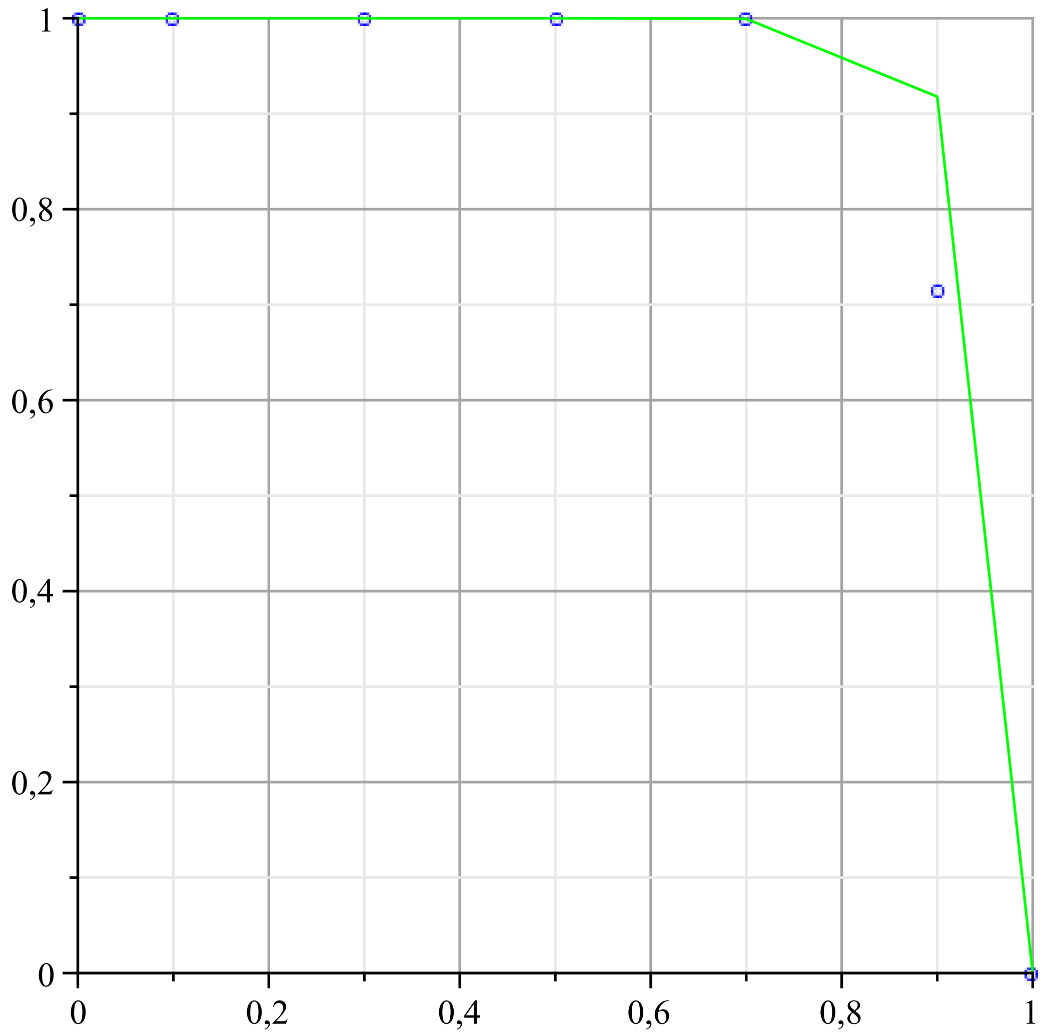
Courbe Théorique avec une liste de points:

> $listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

> *multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color = green, style = line], color = black, gridlines = true)*



Erreur relative:

> for i **from** 1 **to** Ne **do**

$x[i]$;

$\phi[i]$;

$F(x[i])$;

$$\frac{\phi[i]-F(x[i])}{F(x[i])}\cdot 100$$

end do

0.1000
1.
1.
0.
0.3000
1.
1.
0.
0.5000
1.
1.
0.
0.7000
1.
0.9994
0.06004

$$\begin{bmatrix} 0.9000 \\ 0.7143 \\ 0.9179 \\ -22.18 \end{bmatrix} \quad \textbf{(1.1.13)}$$