Equation de Diffusion 1D

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Master : Energétique

Matière : Méthodes Numériques Appliquées II

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2010/2011

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EXEMPLE 1

Détermination de la distribution de temperature $T(x)$ à travers une barre de section A, de conductivité thermique k et de longueur L dont les extrémités sont soumises à des (C.L.) de Dirichlet.

$$\frac{d}{dx}\left(k\frac{d}{dx}T(x)\right)=0$$

Conditions aux limites (C.L):

$$T(0)=T_A=100,$$
$$T(L)=T_B=500,$$

Solution

```
> Restart : Digits := 4 :
> L := 0.5; λ := 1000; S := 0.01; ndx := 5;
```

$$L := 0.5$$
$$\lambda := 1000$$
$$S := 0.01$$
$$ndx := 5 \qquad \textbf{(1.1)}$$

$$> \Delta x := \frac{L}{ndx};$$

$$\Delta x := 0.1000 \quad \textbf{(1.2)}$$

> $i_{\max} := ndx;$

$$i_{max} := 5 \quad \textbf{(1.3)}$$

Nombre d'équations:

> $N := i_{\max}$

$$N := 5 \quad \textbf{(1.4)}$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to N do
      x[i] := Δx/2 + (i − 1)·Δx ;
  end do;
  x[N + 1] := L;
```

$$x_0 := 0$$
$$x_1 := 0.05000$$
$$x_2 := 0.1500$$
$$x_3 := 0.2500$$
$$x_4 := 0.3500$$
$$x_5 := 0.4500$$
$$x_6 := 0.5 \quad \textbf{(1.5)}$$

Conditions aux Limites:

```
> T[0] := 100;
  T[N + 1] := 500;
```

$$T_0 := 100$$
$$T_6 := 500 \quad \textbf{(1.6)}$$

Noeuds internes:

```
> for i from 2 to N − 1 do
     Sp[i] := 0;
     Su[i] := 0;
     a_W[i] := λ·S/Δx;
     a_E[i] := a_W[i];
     a_P[i] := a_W[i] + a_E[i] − Sp[i];
  end do;
```

$$Sp_2 := 0$$
$$Su_2 := 0$$
$$a_{W_2} := 100.0$$
$$a_{E_2} := 100.0$$
$$a_{P_2} := 200.0$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 100.0$$
$$a_{E_3} := 100.0$$
$$a_{P_3} := 200.0$$

$$Sp_4 := 0$$
$$Su_4 := 0$$
$$a_{W_4} := 100.0$$
$$a_{E_4} := 100.0$$
$$a_{P_4} := 200.0 \qquad (1.7)$$

Noeud gauche:

$> Sp[1] := -\frac{2 \cdot \lambda \cdot S}{\Delta x};$

$Su[1] := \frac{2 \cdot \lambda \cdot S}{\Delta x} \cdot T[0];$

$a_W[1] := 0;$

$a_E[1] := \frac{\lambda \cdot S}{\Delta x};$

$a_P[1] := a_W[1] + a_E[1] - Sp[1];$

$$Sp_1 := -200.0$$
$$Su_1 := 20000.$$
$$a_{W_1} := 0$$
$$a_{E_1} := 100.0$$
$$a_{P_1} := 300.0 \qquad (1.8)$$

Noeud droit:

$> Sp[N] := -\frac{2 \cdot \lambda \cdot S}{\Delta x};$

$Su[N] := \frac{2 \cdot \lambda \cdot S}{\Delta x} \cdot T[N+1];$

$a_W[N] := \frac{\lambda \cdot S}{\Delta x};$

$a_E[N] := 0;$

$a_P[N] := a_W[N] + a_E[N] - Sp[N];$

$$Sp_5 := -200.0$$
$$Su_5 := 1.000\ 10^5$$
$$a_{W_5} := 100.0$$
$$a_{E_5} := 0$$
$$a_{P_5} := 300.0 \qquad (1.9)$$

Equations:

$> k := 1$

$$k := 1 \qquad (1.1.1)$$

Résolution pour les noeuds internes:

> **for** i **from** 1 **to** N **do**

$Eq[k] := a_P[i] \cdot T[i] = a_W[i] \cdot T[i-1] + a_E[i] \cdot T[i+1] + Su[i];$

$k := k + 1;$

end do;

$$Eq_1 := 300.0\ T_1 = 20000. + 100.0\ T_2$$
$$k := 2$$
$$Eq_2 := 200.0\ T_2 = 100.0\ T_1 + 100.0\ T_3$$
$$k := 3$$
$$Eq_3 := 200.0\ T_3 = 100.0\ T_2 + 100.0\ T_4$$
$$k := 4$$
$$Eq_4 := 200.0\ T_4 = 100.0\ T_3 + 100.0\ T_5$$
$$k := 5$$
$$Eq_5 := 300.0\ T_5 = 100.0\ T_4 + 1.000\ 10^5$$
$$k := 6 \tag{1.1.2}$$

Ecriture du système d'équations:

> **for** *k* **from** 1 **to** *N* **do** *Eq*[k] **end do**;

$$300.0\ T_1 = 20000. + 100.0\ T_2$$
$$200.0\ T_2 = 100.0\ T_1 + 100.0\ T_3$$
$$200.0\ T_3 = 100.0\ T_2 + 100.0\ T_4$$
$$200.0\ T_4 = 100.0\ T_3 + 100.0\ T_5$$
$$300.0\ T_5 = 100.0\ T_4 + 1.000\ 10^5 \tag{1.1.3}$$

> *Eqs* := {*seq*(*Eq*[*k*], *k* = 1 ..*N*) };

$$Eqs := \{300.0\ T_1 = 20000. + 100.0\ T_2, 200.0\ T_2 = 100.0\ T_1 + 100.0\ T_3, 200.0\ T_3 = 100.0\ T_2 + 100.0\ T_4, 200.0\ T_4 = 100.0\ T_3 + 100.0\ T_5, 300.0\ T_5 = 100.0\ T_4 + 1.000\ 10^5\} \tag{1.1.4}$$

> *Tmps* := [*seq*(*T*[*i*], *i* = 1 ..*N*)];

$$Tmps := [T_1, T_2, T_3, T_4, T_5] \tag{1.1.5}$$

> *SolT* := *solve*(*Eqs*, *Tmps*);

$$SolT := [[T_1 = 140., T_2 = 220., T_3 = 300., T_4 = 380., T_5 = 460.]] \tag{1.1.6}$$

> *with*(*LinearAlgebra*) :

Forme matricielle:

> *A*, *b* := *GenerateMatrix*(*Eqs*, *Tmps*)

$$A, b := \begin{bmatrix} 300.0 & -100.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -100.0 & 300.0 \\ -100.0 & 200.0 & -100.0 & 0 & 0 \\ 0 & -100.0 & 200.0 & -100.0 & 0 \\ 0 & 0 & -100.0 & 200.0 & -100.0 \end{bmatrix}, \begin{bmatrix} 20000. \\ 1.000\ 10^5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.1.7}$$

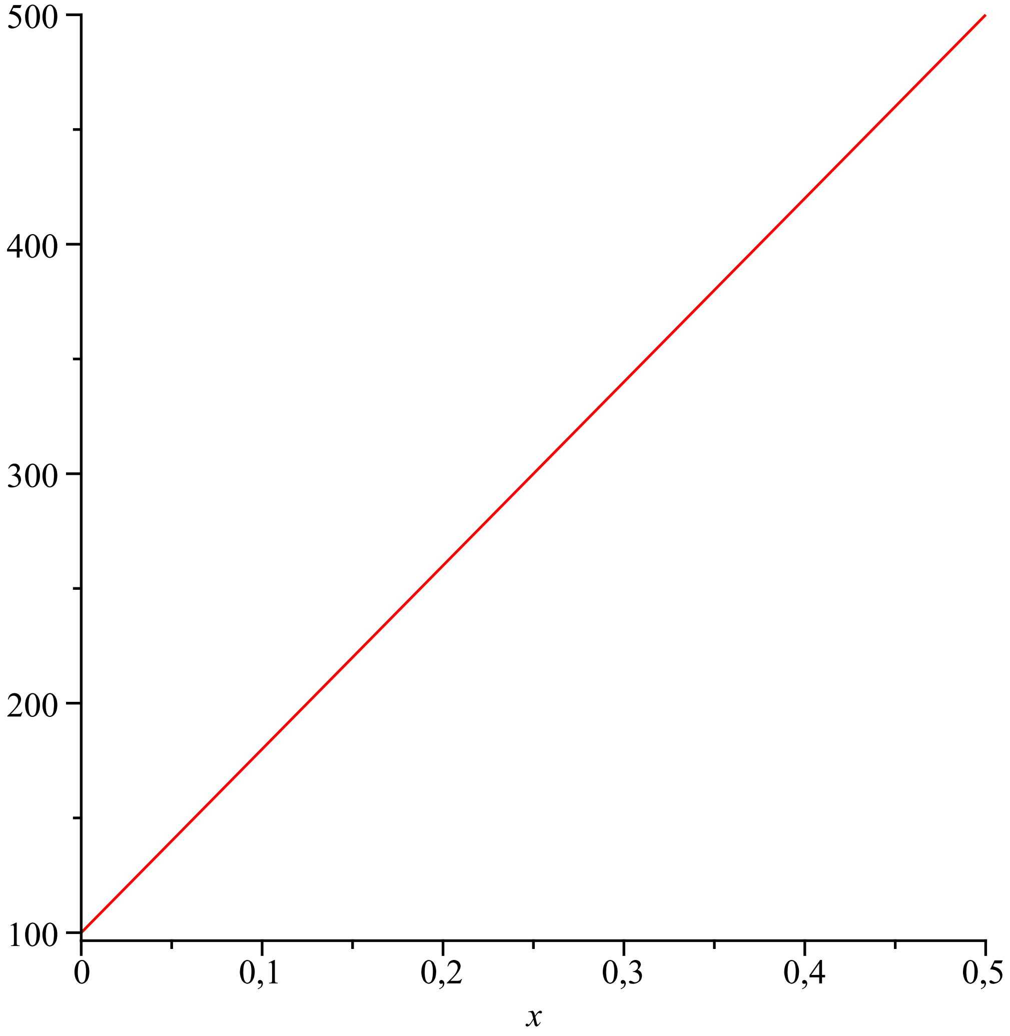
Solution exacte:

> *F*(*x*) := 800·*x* + 100;

$$F := x \rightarrow 800\ x + 100 \tag{1.1.8}$$

> *with*(*plots*) :

> *plot*(*F*(*x*), *x* = 0 ..*L*);



> **for** i **from** 1 **to** N **do**
> $T[i] := rhs(SolT_{1,\,i})$
> **end do**;

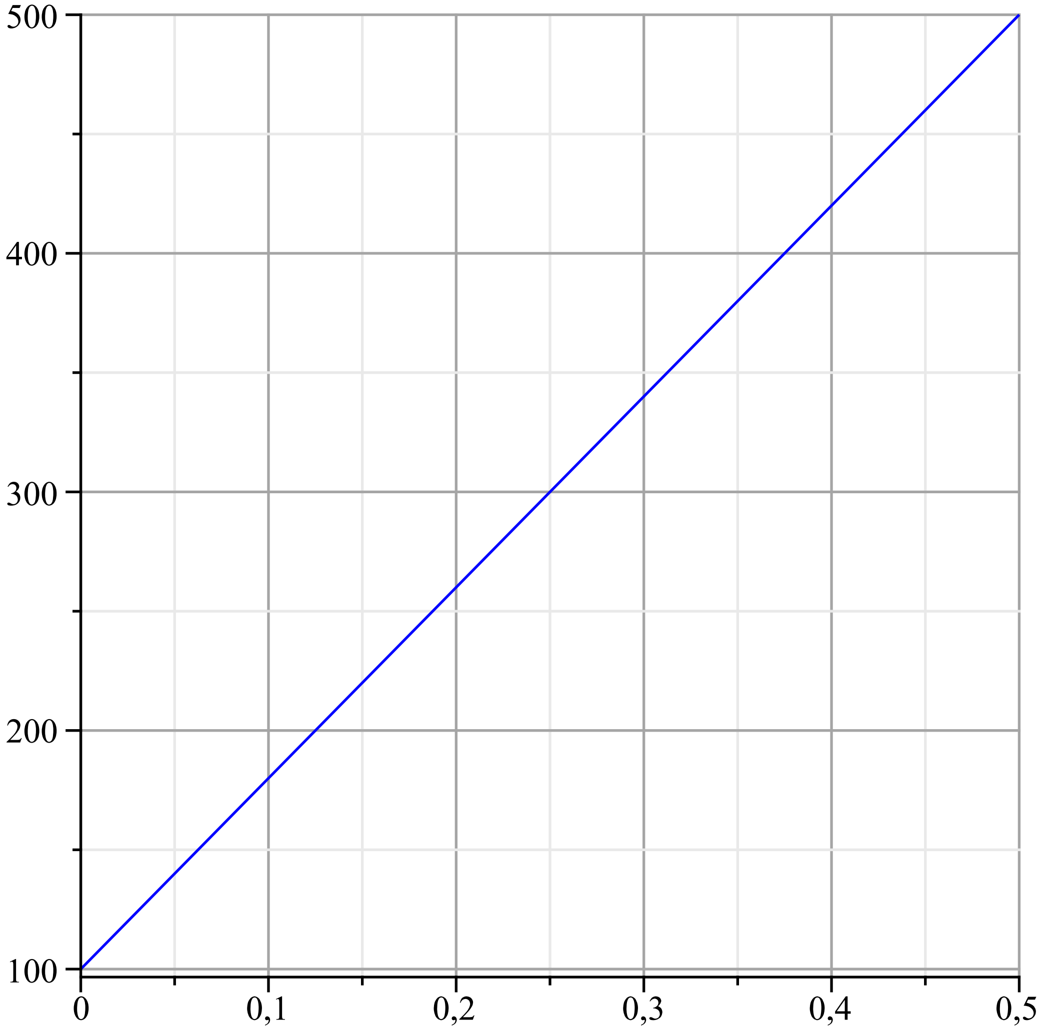
$$T_1 := 140.$$
$$T_2 := 220.$$
$$T_3 := 300.$$
$$T_4 := 380.$$
$$T_5 := 460. \quad \textbf{(1.1.9)}$$

> $lpN := [\,seq([x[i], T[i]], i = 0..N+1)\,]$

$lpN := [[0, 100], [0.05000, 140.], [0.1500, 220.], [0.2500, 300.], [0.3500, 380.], [0.4500, 460.], [0.5, 500]]$ **(1.1.10)**

Courbe Numérique:

> $listplot(lpN, color = blue, gridlines = true)$

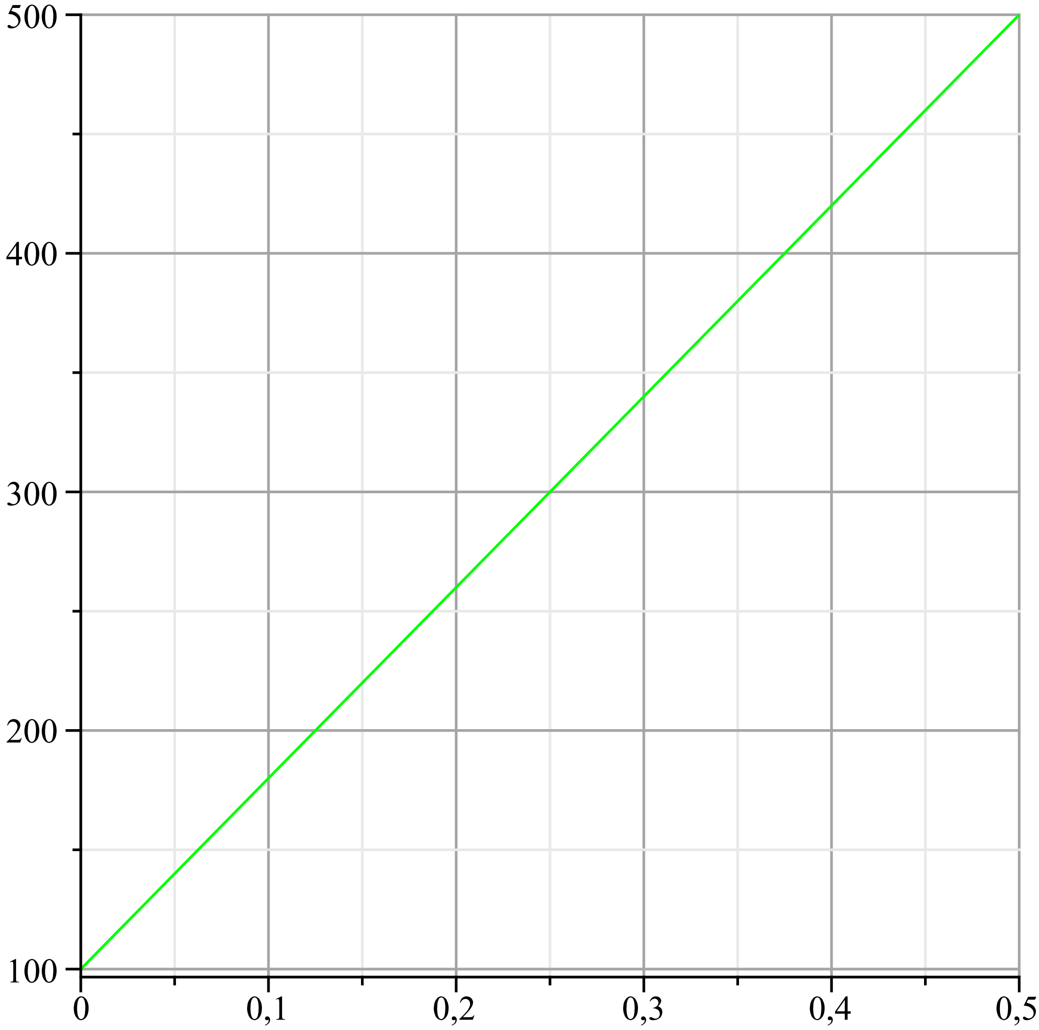


> $lpT := [seq([x[i], F(x[i])], i=0..N+1)]$

$lpT := [[0, 100], [0.05000, 140.0], [0.1500, 220.0], [0.2500, 300.0], [0.3500, 380.0], [0.4500, 460.0], [0.5, 500.0]]$ **(1.1.11)**

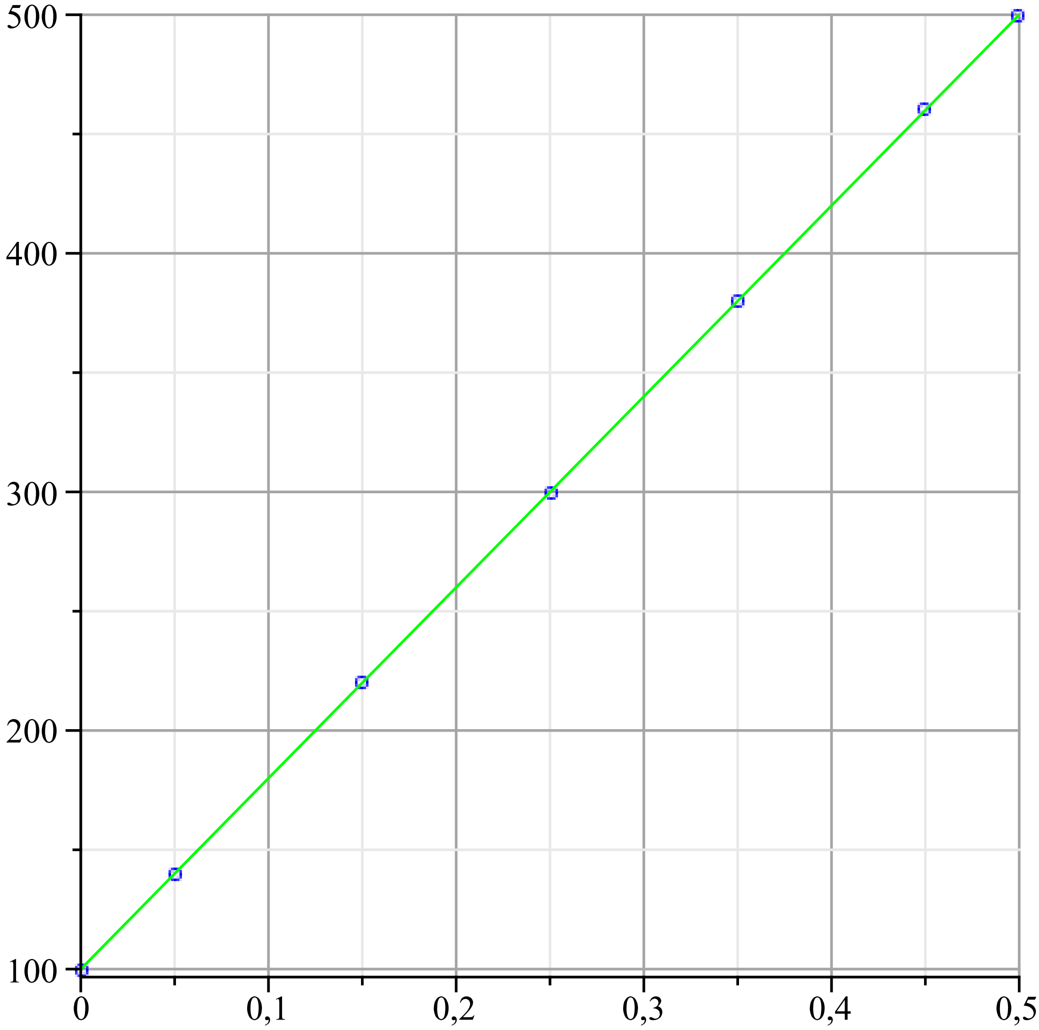
Courbe Théorique avec une liste de points:

> $listplot(lpT, color=green, gridlines=true)$



Tracé des deux courbes ensembles:

> *multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color = green, style = line], color = black, gridlines = true)*



Erreur relative:

```
> for i from 1 to N do
    x[i];
    T[i];
    F(x[i]);
    (T[i] - F(x[i]))/F(x[i]) ·100
  end do
```

0.05000
140.
140.0
0.
0.1500
220.
220.0
0.
0.2500
300.
300.0
0.
0.3500
380.
380.0
0.
0.4500

460.
460.0
0. **(1.1.12)**

>