Equation de Diffusion 1D

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EXEMPLE 2

Détermination de la distribution de temperature $T(\mathrm{x})$ à travers une barre de section S, de conductivité thermique w3 et de longueur L dont les extrémités sont soumises à des (C.L.) de Dirichlet et une source de chaleur uniforme q.

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x})\right)+q=0$$

Conditions aux limites (C.L):

$$T(0)=T_A=100,$$
$$T(\mathrm{L})=T_B=200,$$

Solution

```
> Restart : Digits := 4 :
> L := 0.02; λ := 0.5; S := 1; q := 1000000; ndx := 5;
```

$L := 0.02$

$\lambda := 0.5$

$S := 1$

$q := 1000000$

$ndx := 5$ **(1.1)**

```
> Δx := L/ndx ;
```

(1.2)

$$\Delta x := 0.004000 \quad \textbf{(1.2)}$$

> $i_{\max} := ndx;$

$$i_{max} := 5 \quad \textbf{(1.3)}$$

Nombre d'équations:

> $N := i_{\max}$

$$N := 5 \quad \textbf{(1.4)}$$

Abscisses des noeuds:

> $x[0] := 0;$
> **for** i **from** 1 **to** N **do**
> $x[i] := \frac{\Delta x}{2} + (i-1)\cdot\Delta x;$
> **end do**;
> $x[N+1] := L;$

$$x_0 := 0$$
$$x_1 := 0.002000$$
$$x_2 := 0.006000$$
$$x_3 := 0.01000$$
$$x_4 := 0.01400$$
$$x_5 := 0.01800$$
$$x_6 := 0.02 \quad \textbf{(1.5)}$$

Conditions aux Limites:

> $T[0] := 100;$
> $T[N+1] := 200;$

$$T_0 := 100$$
$$T_6 := 200 \quad \textbf{(1.6)}$$

Noeuds internes:

> **for** i **from** 2 **to** $N-1$ **do**
> $Sp[i] := 0;$
> $Su[i] := q\cdot S\cdot\Delta x;$
> $a_W[i] := \frac{\lambda\cdot S}{\Delta x};$
> $a_E[i] := a_W[i];$
> $a_P[i] := a_W[i] + a_E[i] - Sp[i];$
> **end do**;

$$Sp_2 := 0$$
$$Su_2 := 4000.$$
$$a_{W_2} := 125.0$$
$$a_{E_2} := 125.0$$
$$a_{P_2} := 250.0$$
$$Sp_3 := 0$$
$$Su_3 := 4000.$$
$$a_{W_3} := 125.0$$
$$a_{E_3} := 125.0$$
$$a_{P_3} := 250.0$$

$$Sp_4 := 0$$
$$Su_4 := 4000.$$
$$a_{W_4} := 125.0$$
$$a_{E_4} := 125.0$$
$$a_{P_4} := 250.0 \quad \textbf{(1.7)}$$

Noeud gauche:

$$> Sp[1] := -\frac{2\cdot\lambda\cdot S}{\Delta x};$$
$$Su[1] := q\cdot S\cdot\Delta x + \frac{2\cdot\lambda\cdot S}{\Delta x}\cdot T[0];$$
$$a_W[1] := 0;$$
$$a_E[1] := \frac{\lambda\cdot S}{\Delta x};$$
$$a_P[1] := a_W[1] + a_E[1] - Sp[1];$$

$$Sp_1 := -250.0$$
$$Su_1 := 29000.$$
$$a_{W_1} := 0$$
$$a_{E_1} := 125.0$$
$$a_{P_1} := 375.0 \quad \textbf{(1.8)}$$

Noeud droit:

$$> Sp[N] := -\frac{2\cdot\lambda\cdot S}{\Delta x};$$
$$Su[N] := q\cdot S\cdot\Delta x + \frac{2\cdot\lambda\cdot S}{\Delta x}\cdot T[N+1];$$
$$a_W[N] := \frac{\lambda\cdot S}{\Delta x};$$
$$a_E[N] := 0;$$
$$a_P[N] := a_W[N] + a_E[N] - Sp[N];$$

>

$$Sp_5 := -250.0$$
$$Su_5 := 54000.$$
$$a_{W_5} := 125.0$$
$$a_{E_5} := 0$$
$$a_{P_5} := 375.0 \quad \textbf{(1.9)}$$

Equations:

$$> k := 1$$

$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 1 to N do
      Eq[k] := a_P[i]·T[i] = a_W[i]·T[i-1] + a_E[i]·T[i+1] + Su[i];
      k := k + 1;
  end do;
```

$$Eq_1 := 375.0\ T_1 = 29000. + 125.0\ T_2$$
$$k := 2$$
$$Eq_2 := 250.0\ T_2 = 125.0\ T_1 + 125.0\ T_3 + 4000.$$
$$k := 3$$
$$Eq_3 := 250.0\ T_3 = 125.0\ T_2 + 125.0\ T_4 + 4000.$$
$$k := 4$$
$$Eq_4 := 250.0\ T_4 = 125.0\ T_3 + 125.0\ T_5 + 4000.$$
$$k := 5$$
$$Eq_5 := 375.0\ T_5 = 125.0\ T_4 + 54000.$$
$$k := 6 \qquad \textbf{(1.1.2)}$$

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$375.0\ T_1 = 29000. + 125.0\ T_2$$
$$250.0\ T_2 = 125.0\ T_1 + 125.0\ T_3 + 4000.$$
$$250.0\ T_3 = 125.0\ T_2 + 125.0\ T_4 + 4000.$$
$$250.0\ T_4 = 125.0\ T_3 + 125.0\ T_5 + 4000.$$
$$375.0\ T_5 = 125.0\ T_4 + 54000. \qquad \textbf{(1.1.3)}$$

```
> Eqs := {seq(Eq[k], k=1..N)};
```

$$Eqs := \{375.0\ T_1 = 29000. + 125.0\ T_2, 250.0\ T_2 = 125.0\ T_1 + 125.0\ T_3 + 4000., 250.0\ T_3 = 125.0\ T_2 + 125.0\ T_4 + 4000., 250.0\ T_4 = 125.0\ T_3 + 125.0\ T_5 + 4000., 375.0\ T_5 = 125.0\ T_4 + 54000.\} \qquad \textbf{(1.1.4)}$$

```
> Tmps := [seq(T[i], i=1..N)];
```

$$Tmps := [T_1, T_2, T_3, T_4, T_5] \qquad \textbf{(1.1.5)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_1 = 150., T_2 = 218., T_3 = 254., T_4 = 258., T_5 = 230.]] \qquad \textbf{(1.1.6)}$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Eqs, Tmps)
```

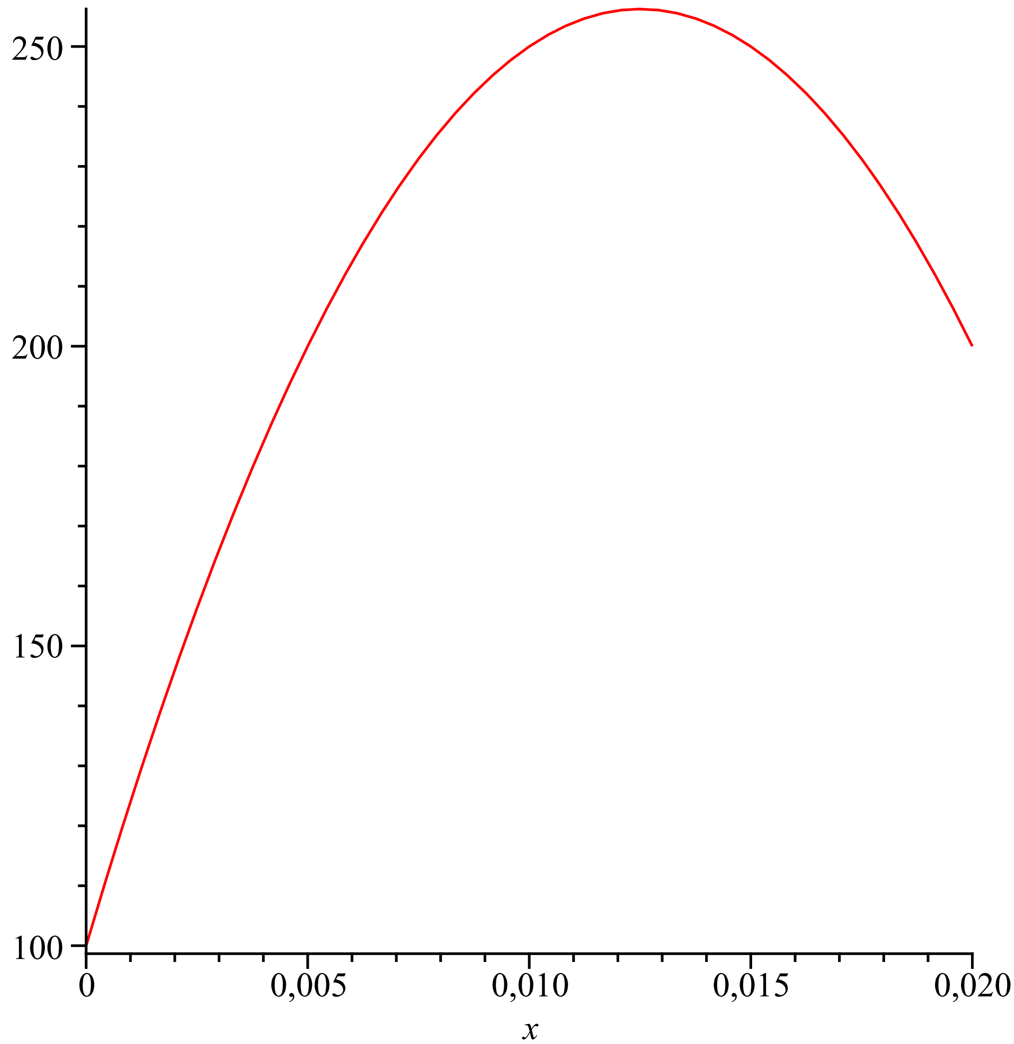
$$A, b := \begin{bmatrix} 375.0 & -125.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -125.0 & 375.0 \\ -125.0 & 250.0 & -125.0 & 0 & 0 \\ 0 & -125.0 & 250.0 & -125.0 & 0 \\ 0 & 0 & -125.0 & 250.0 & -125.0 \end{bmatrix}, \begin{bmatrix} 29000. \\ 54000. \\ 4000. \\ 4000. \\ 4000. \end{bmatrix} \qquad \textbf{(1.1.7)}$$

Solution exacte:

$$> F(x) := \left(\frac{T[N+1] - T[0]}{L} + \frac{q}{2 \cdot \lambda} \cdot (L - x) \right) \cdot x + T[0];$$

$$F := x \rightarrow \left(\frac{T_{N+1} - T_0}{L} + \frac{1}{2} \frac{q\ (L - x)}{\lambda} \right) x + T_0 \qquad \textbf{(1.1.8)}$$

```
> with(plots) :
> plot(F(x), x=0..L);
```



> **for** i **from** 1 **to** N **do**
  $T[i] := rhs(SolT_{1,\, i})$
end do;

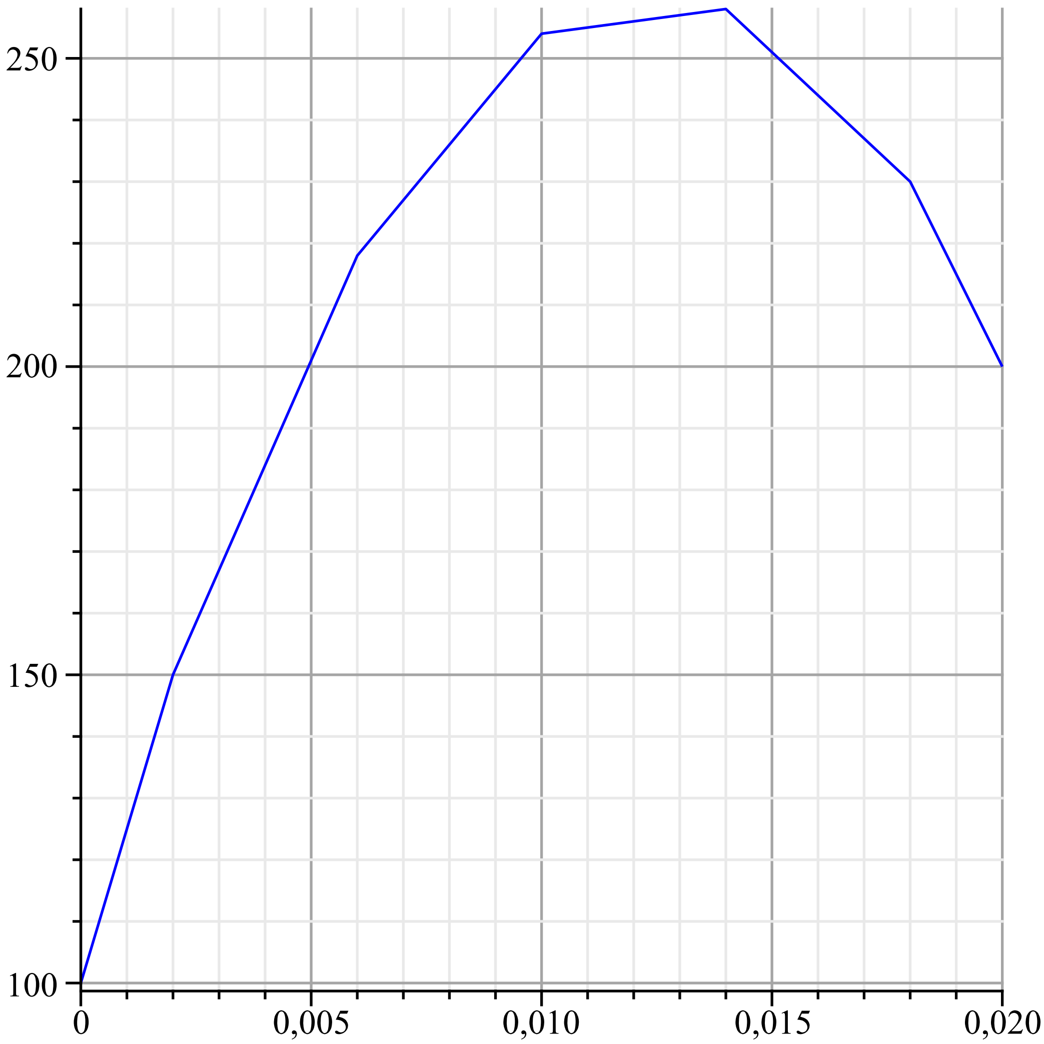
$$T_1 := 150.$$
$$T_2 := 218.$$
$$T_3 := 254.$$
$$T_4 := 258.$$
$$T_5 := 230. \quad \textbf{(1.1.9)}$$

> $lpN := [\ seq([x[i], T[i]], i = 0 ..N + 1)\ ]$

$lpN := [[0, 100], [0.002000, 150.], [0.006000, 218.], [0.01000, 254.], [0.01400, 258.], [0.01800, 230.], [0.02, 200]]$ **(1.1.10)**

Courbe Numérique:

> $listplot(lpN, color = blue, gridlines = true)$

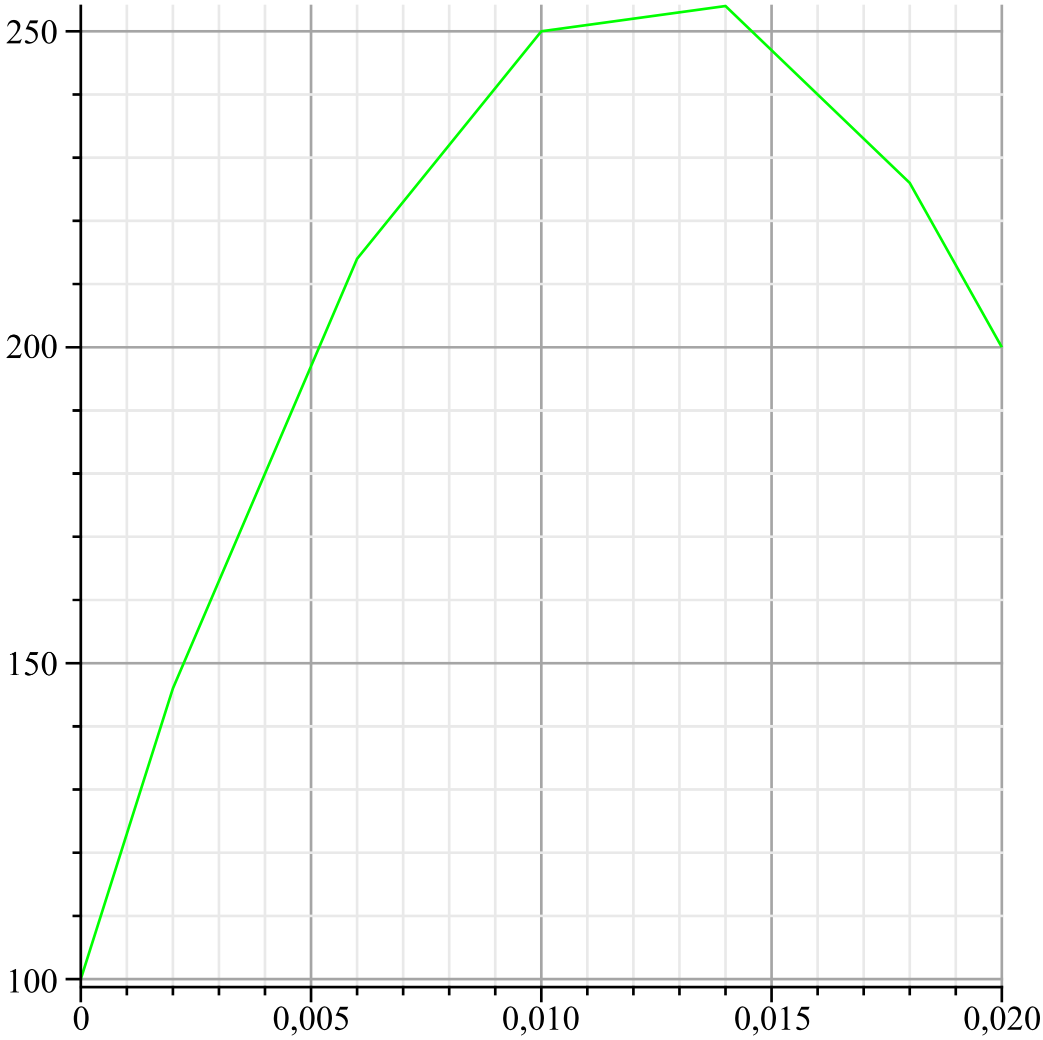


> $lpT := [seq([x[i], F(x[i])], i = 0..N+1)]$

$lpT := [[0, 100.], [0.002000, 146.0], [0.006000, 214.0], [0.01000, 250.0], [0.01400, 254.0], [0.01800, 226.0], [0.02, 200.0]]$ **(1.1.11)**

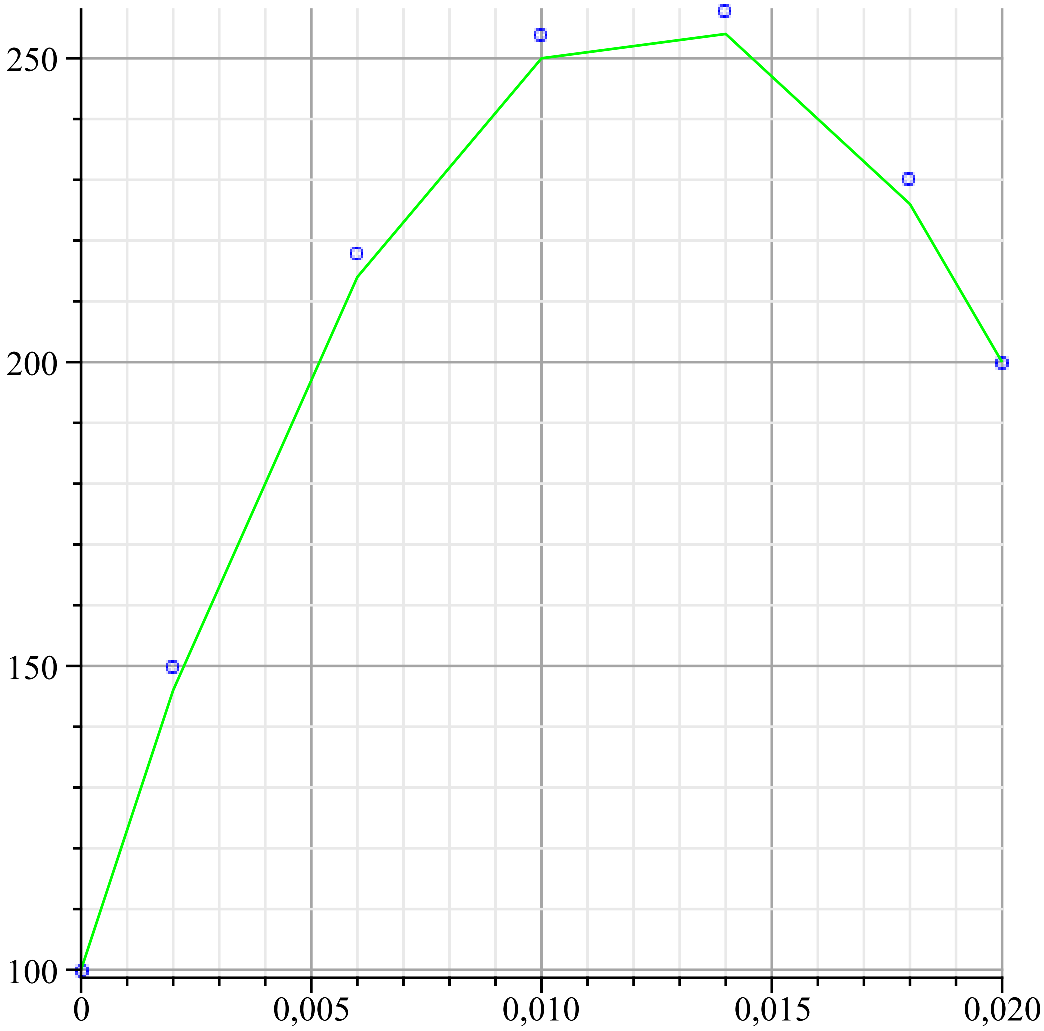
Courbe Théorique avec une liste de points:

> $listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color = green, style = line], color = black, gridlines = true)



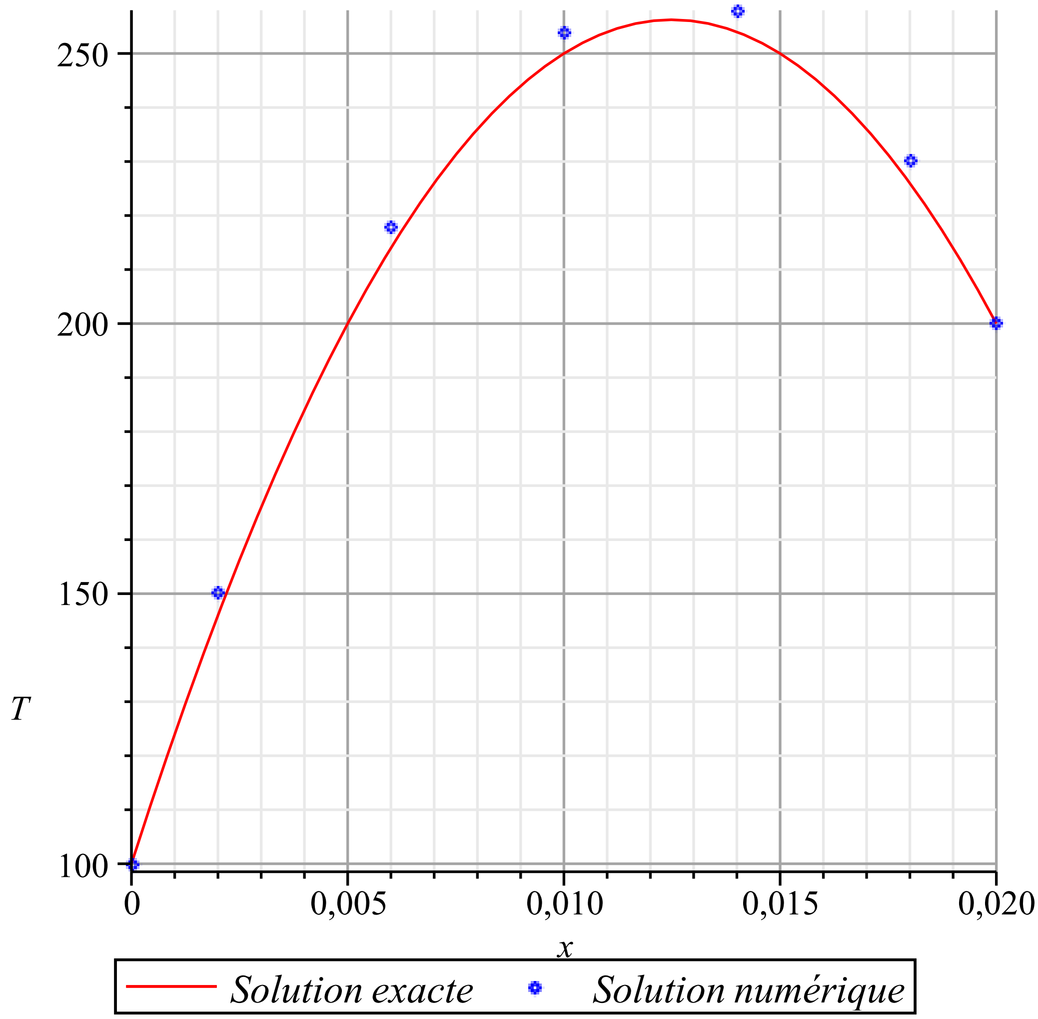
> Erreur relative:

> **for** i **from** 1 **to** N **do**
$x[i]$;
$T[i]$;
$F(x[i])$;
$\frac{T[\mathrm{i}]-F(x[\mathrm{i}])}{F(x[\mathrm{i}])}\cdot 100$
end do

0.002000
150.
146.0
2.740
0.006000
218.
214.0
1.869
0.01000
254.
250.0
1.600
0.01400
258.
254.0
1.575
0.01800

$$230.$$
$$226.0$$
$$1.770 \qquad (1.1.12)$$

```
> P1 := plot(F(x), x = 0 .. L, labels = [x, T], legend = Solution exacte) :
  P2 := plot(lpN, color = blue, style = point, legend = Solution numérique) :
  display({P1, P2}, gridlines = true);
```



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