Equation de Diffusion 1D

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EXEMPLE 3

Détermination de la distribution de temperature $T(x)$ à travers une barre de section A, de conductivité thermique k et de longueur L soumise à un flux convectif sur sa surface latérale et dont les extrémités sont soumises l'une à une (C.L.) de Dirichlet et l'autre à condition de neumann (bord isolé).

$$\frac{d}{dx}\left(k\cdot S\cdot \frac{d}{dx}T(x)\right) - h\cdot p\cdot\left(T - T_{inf}\right) = 0$$

Conditions aux limites (C.L):

$$T(0) = T_A = 100,$$
$$T_{inf} = 20,$$

Solution

```
> Restart : Digits := 4 :
> L := 1.0; α := 25; ndx := 5;
```

$$L := 1.0$$
$$\alpha := 25$$
$$ndx := 5 \qquad \textbf{(1.1)}$$

```
> Δx := L/ndx ;
```

$$\Delta x := 0.2000 \qquad \textbf{(1.2)}$$

$> i_{max} := ndx;$

$$i_{max} := 5 \quad (1.3)$$

Nombre d'équations:

$> N := i_{max}$

$$N := 5 \quad (1.4)$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to N do
      x[i] := Δx/2 + (i - 1)·Δx;
  end do;
  x[N + 1] := L;
```

$$x_0 := 0$$
$$x_1 := 0.1000$$
$$x_2 := 0.3000$$
$$x_3 := 0.5000$$
$$x_4 := 0.7000$$
$$x_5 := 0.9000$$
$$x_6 := 1.0 \quad (1.5)$$

Conditions aux Limites:

```
> Ta := 100;
  Ti := 20;
```

$$Ta := 100$$
$$Ti := 20 \quad (1.6)$$

Noeuds internes:

```
> for i from 2 to N - 1 do
    Sp[i] := -α·Δx;
    Su[i] := α·Δx·Ti;
    a_W[i] := 1/Δx;
    a_E[i] := a_W[i];
    a_P[i] := a_W[i] + a_E[i] - Sp[i];
  end do;
```

$$Sp_2 := -5.000$$
$$Su_2 := 100.0$$
$$a_{W_2} := 5.000$$
$$a_{E_2} := 5.000$$
$$a_{P_2} := 15.00$$
$$Sp_3 := -5.000$$
$$Su_3 := 100.0$$
$$a_{W_3} := 5.000$$
$$a_{E_3} := 5.000$$
$$a_{P_3} := 15.00$$
$$Sp_4 := -5.000$$
$$Su_4 := 100.0$$

$$a_{W_4} := 5.000$$
$$a_{E_4} := 5.000$$
$$a_{P_4} := 15.00 \quad \textbf{(1.7)}$$

Noeud gauche:

$$> Sp[1] := -\alpha \cdot \Delta x - \frac{2}{\Delta x};$$
$$Su[1] := \alpha \cdot \Delta x \cdot Ti + \frac{2}{\Delta x} \cdot Ta;$$
$$a_W[1] := 0;$$
$$a_E[1] := \frac{1}{\Delta x};$$
$$a_P[1] := a_W[1] + a_E[1] - Sp[1];$$

$$Sp_1 := -15.00$$
$$Su_1 := 1100.$$
$$a_{W_1} := 0$$
$$a_{E_1} := 5.000$$
$$a_{P_1} := 20.00 \quad \textbf{(1.8)}$$

Noeud droit:

$$> Sp[N] := -\alpha \cdot \Delta x;$$
$$Su[N] := \alpha \cdot \Delta x \cdot Ti;$$
$$a_W[N] := \frac{1}{\Delta x};$$
$$a_E[N] := 0;$$
$$a_P[N] := a_W[N] + a_E[N] - Sp[N];$$

$$Sp_5 := -5.000$$
$$Su_5 := 100.0$$
$$a_{W_5} := 5.000$$
$$a_{E_5} := 0$$
$$a_{P_5} := 10.00 \quad \textbf{(1.9)}$$

Equations:

$$> k := 1$$
$$k := 1 \quad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 1 to N do
      Eq[k] := a_P[i]·T[i] = a_W[i]·T[i-1] + a_E[i]·T[i+1] + Su[i];
      k := k + 1;
  end do;
```

$$Eq_1 := 20.00\, T_1 = 1100. + 5.000\, T_2$$
$$k := 2$$
$$Eq_2 := 15.00\, T_2 = 5.000\, T_1 + 5.000\, T_3 + 100.0$$
$$k := 3$$

$$Eq_3 := 15.00\ T_3 = 5.000\ T_2 + 5.000\ T_4 + 100.0$$
$$k := 4$$
$$Eq_4 := 15.00\ T_4 = 5.000\ T_3 + 5.000\ T_5 + 100.0$$
$$k := 5$$
$$Eq_5 := 10.00\ T_5 = 5.000\ T_4 + 100.0$$
$$k := 6 \qquad \textbf{(1.1.2)}$$

Ecriture du système d'équations:

```
> for k from 1 to N do Eq[k] end do;
```

$$20.00\ T_1 = 1100. + 5.000\ T_2$$
$$15.00\ T_2 = 5.000\ T_1 + 5.000\ T_3 + 100.0$$
$$15.00\ T_3 = 5.000\ T_2 + 5.000\ T_4 + 100.0$$
$$15.00\ T_4 = 5.000\ T_3 + 5.000\ T_5 + 100.0$$
$$10.00\ T_5 = 5.000\ T_4 + 100.0 \qquad \textbf{(1.1.3)}$$

```
> Eqs := {seq(Eq[i], i = 1 ..N)} :
> Tmps := [seq(T[i], i = 1 ..N)];
```

$$Tmps := [T_1, T_2, T_3, T_4, T_5] \qquad \textbf{(1.1.4)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[T_1 = 64.23,\ T_2 = 36.91,\ T_3 = 26.50,\ T_4 = 22.60,\ T_5 = 21.30]] \qquad \textbf{(1.1.5)}$$

```
> with(LinearAlgebra) :
> A, b := GenerateMatrix(Eqs, Tmps)
```

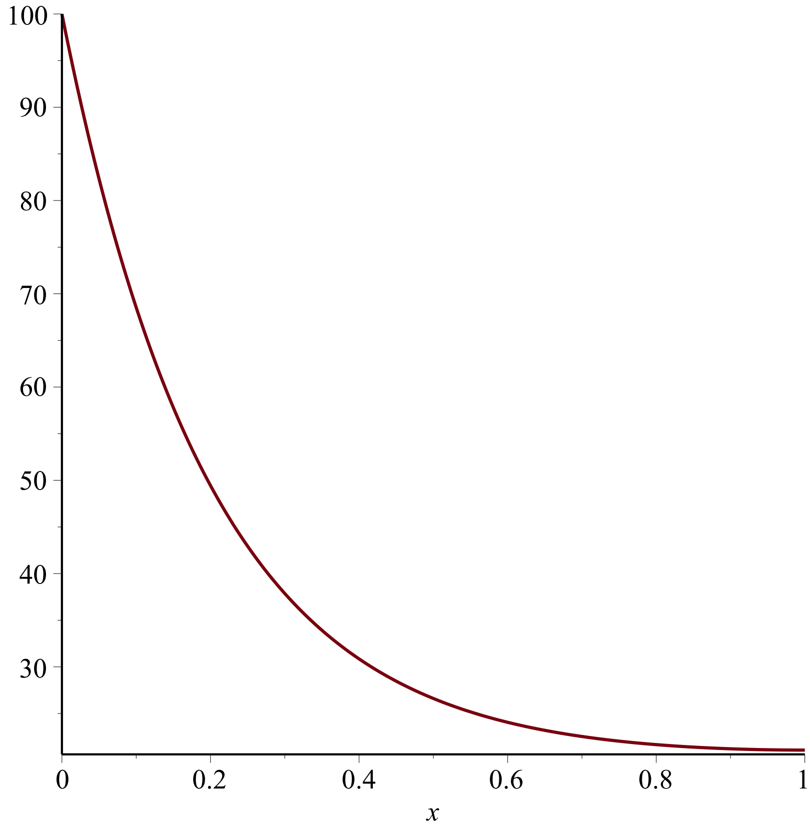
$$A, b := \begin{bmatrix} 20.00 & -5.000 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5.000 & 10.00 \\ -5.000 & 15.00 & -5.000 & 0 & 0 \\ 0 & -5.000 & 15.00 & -5.000 & 0 \\ 0 & 0 & -5.000 & 15.00 & -5.000 \end{bmatrix}, \begin{bmatrix} 1100. \\ 100.0 \\ 100.0 \\ 100.0 \\ 100.0 \end{bmatrix} \qquad \textbf{(1.1.6)}$$

Solution exacte:

$$> F(x) := (Ta - Ti) \cdot \frac{\cosh\left(\sqrt{\alpha} \cdot (L - x)\right)}{\cosh\left(\sqrt{\alpha} \cdot L\right)} + Ti;$$

$$F := x \rightarrow \frac{(Ta - Ti)\cosh\left(\sqrt{\alpha}\ (L - x)\right)}{\cosh\left(\sqrt{\alpha}\ L\right)} + Ti \qquad \textbf{(1.1.7)}$$

```
> with(plots) :
> plot(F(x), x = 0 ..L);
```




```
> for i from 1 to N do
      T[i] := rhs(SolT[1, i])
  end do;
```

$T_1 := 64.23$

$T_2 := 36.91$

$T_3 := 26.50$

$T_4 := 22.60$

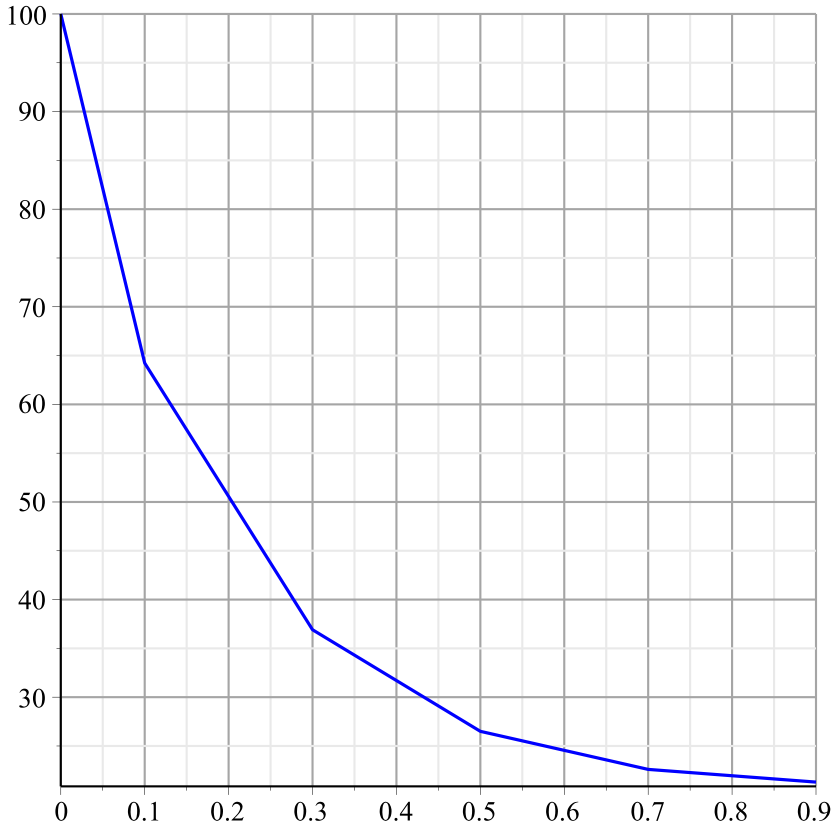
$T_5 := 21.30$ **(1.1.8)**

```
> lpN := [[x[0], Ta], seq([x[i], T[i]], i = 1 ..N)]
```

$lpN := [[0, 100], [0.1000, 64.23], [0.3000, 36.91], [0.5000, 26.50], [0.7000, 22.60], [0.9000, 21.30]]$ **(1.1.9)**

Courbe Numérique:

```
> listplot(lpN, color = blue, gridlines = true)
```

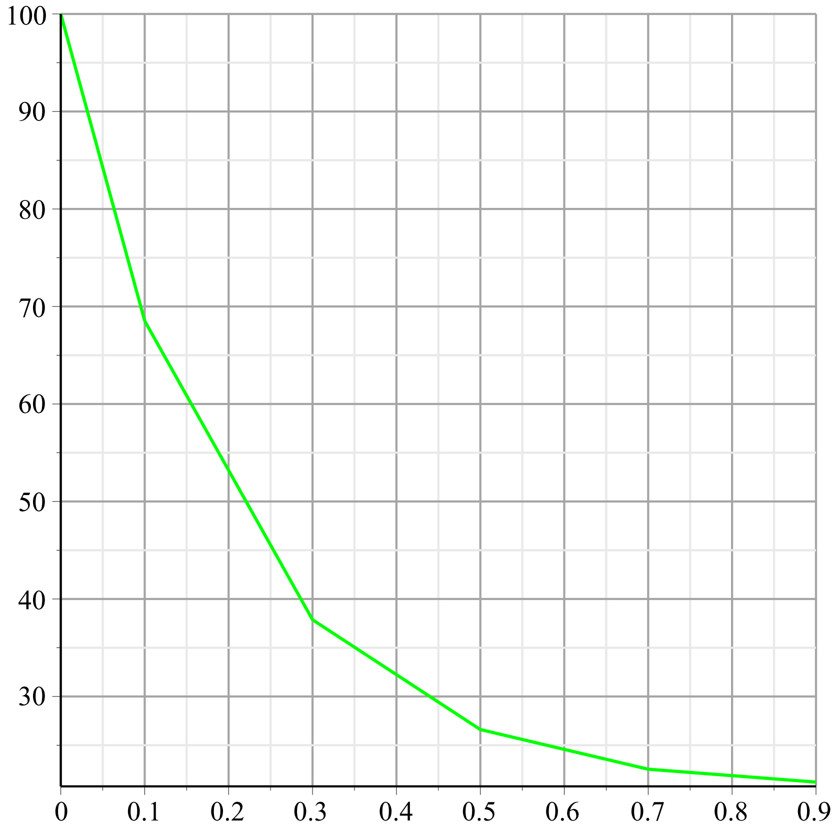


> $lpT := [seq([x[i], F(x[i])], i = 0..N)]$

$lpT := [[0, 100.0], [0.1000, 68.52], [0.3000, 37.87], [0.5000, 26.61], [0.7000, 22.54], [0.9000, 21.22]]$ **(1.1.10)**

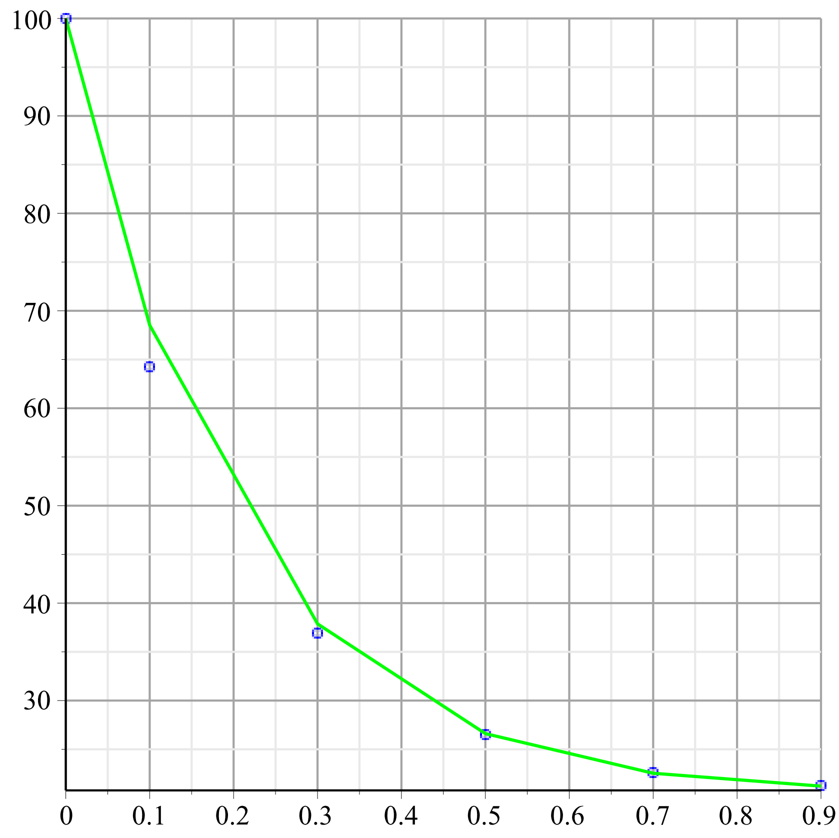
Courbe Théorique avec une liste de points:

> $listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color = green, style = line], color = black, gridlines = true)



Erreur relative:

> **for** i **from** 1 **to** N **do**
$x[i]$:
$T[i]$:
$F(x[i])$:
$\frac{T[i]-F(x[i])}{F(x[i])}\cdot 100$
end do:

Erreure relative

$x[i]$	$T[i]$	$F(x[i])$	Erreure relative (%)
0.1000	64.23	68.52	-6.261
0.3000	36.91	37.87	-2.535
0.5000	26.50	26.61	-0.4134
0.7000	22.60	22.54	0.2662

0.9000	21.30	21.22	0.3770

```
> P1 := plot(F(x), x = 0 .. L, labels = [x, T], legend = Solution exacte) :
  P2 := plot(lpN, color = blue, style = point, legend = Solution numérique) :
  display({P1, P2}, gridlines = true);
```