Equation de Diffusion 2D

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EXAMEN

Détermination de la distribution de temperature $T(\mathrm{x}, \mathrm{y})$ à travers une plaque de largeur L, de hauteur H et d'épaisseur e et de conductivité thermique k soumise aux (C.L.) suivantes.

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(k\frac{\mathrm{d}}{\mathrm{dx}}T(\mathrm{x}, \mathrm{y})\right) + \frac{\mathrm{d}}{\mathrm{dy}}\left(k\frac{\mathrm{d}}{\mathrm{dy}}T(\mathrm{x}, \mathrm{y})\right) = 0$$

Conditions aux limites (C.L):

$$T(\mathrm{x}, H) = T_0 = 100,$$
$$q(0, y) = \frac{500\ kW}{m^2},$$
$$q(\mathrm{x}, 0) = 0,$$
$$q(L, y) = 0,$$

Solution

```
> Restart : Digits := 4 :
> L := 0.3; H := 0.4; e := 0.01; λ := 1000; δx := 0.1; δy := 0.1;
```

$$L := 0.3$$
$$H := 0.4$$
$$e := 0.01$$
$$\lambda := 1000$$

$$\delta x := 0.1$$
$$\delta y := 0.1 \qquad \textbf{(1.1)}$$

$> ndx := \frac{L}{\delta x}\ ;\ ndy := \frac{H}{\delta y}; Se := \delta y \cdot e; Ss := \delta x \cdot e;$

$$ndx := 3.000$$
$$ndy := 4.000$$
$$Se := 0.001$$
$$Ss := 0.001 \qquad \textbf{(1.2)}$$

$> i_{max} := \text{round}(ndx);\ j_{max} := \text{round}(ndy);$

$$i_{max} := 3$$
$$j_{max} := 4 \qquad \textbf{(1.3)}$$

Nombre d'équations:

$> Ne := i_{max} \cdot j_{max}$

$$Ne := 12 \qquad \textbf{(1.4)}$$

Abscisses des noeuds:

$> x[0] := 0;$

**for** $i$ **from** 1 **to** $i_{max}$ **do**

$$x[i] := \frac{\delta x}{2} + (i-1) \cdot \delta x\ ;$$

**end do**;

$x[\ i_{max} + 1\ ] := L;$

$$x_0 := 0$$
$$x_1 := 0.05000$$
$$x_2 := 0.1500$$
$$x_3 := 0.2500$$
$$x_4 := 0.3 \qquad \textbf{(1.5)}$$

Ordonnées des noeuds:

$> y[0] := 0;$

**for** $j$ **from** 1 **to** $j_{max}$ **do**

$$y[j] := \frac{\delta y}{2} + (j-1) \cdot \delta y\ ;$$

**end do**;

$y[\ j_{max} + 1\ ] := H;$

$$y_0 := 0$$
$$y_1 := 0.05000$$
$$y_2 := 0.1500$$
$$y_3 := 0.2500$$
$$y_4 := 0.3500$$
$$y_5 := 0.4 \qquad \textbf{(1.6)}$$

Conditions aux Limites:

$> T0 := 100; q := 500000; q0 := 0$

$$T0 := 100$$
$$q := 500000$$
$$q0 := 0 \qquad \textbf{(1.7)}$$

Noeuds internes:

```
> for j from 2 to j_max − 1 do
  for i from 2 to i_max - 1 do
    Sp[i,j] := 0;
    Su[i,j] := 0;
    a_W[i,j] := λ·Se/δx;
    a_E[i,j] := a_W[i,j];
    a_S[i,j] := λ·Ss/δy;
    a_N[i,j] := a_S[i,j];
    a_P[i,j] := a_W[i,j] + a_E[i,j] + a_S[i,j] + a_N[i,j] - Sp[i,j];
  end do;
  end do;
```

Noeuds gauches:

```
> for j from 2 to j_max − 1 do
  Sp[1,j] := 0;
  Su[1 ,j] := q·Se;
  a_W[1,j] := 0;
  a_E[1,j] := λ·Se/δx;
  a_S[1,j] := λ·Ss/δy;
  a_N[1,j] := a_S[1,j];
  a_P[1,j] := a_W[1,j] + a_E[1,j] + a_S[1,j] + a_N[1,j] − Sp[1,j];
  end do;
```

$$
\begin{aligned}
Sp_{1,2} &:= 0\\
Su_{1,2} &:= 500.0\\
a_{W_{1,2}} &:= 0\\
a_{E_{1,2}} &:= 10.00\\
a_{S_{1,2}} &:= 10.00\\
a_{N_{1,2}} &:= 10.00\\
a_{P_{1,2}} &:= 30.00\\
Sp_{1,3} &:= 0\\
Su_{1,3} &:= 500.0\\
a_{W_{1,3}} &:= 0\\
a_{E_{1,3}} &:= 10.00\\
a_{S_{1,3}} &:= 10.00\\
a_{N_{1,3}} &:= 10.00\\
a_{P_{1,3}} &:= 30.00
\end{aligned}
\qquad \textbf{(1.8)}
$$

Noeuds droits:

```
> for j from 2 to j_max − 1 do
```

$Sp[i_{max}, j] := 0;$
$Su[i_{max}, j] := 0;$
$a_W[i_{max}, j] := \frac{\lambda \cdot Se}{\delta x};$
$a_E[i_{max}, j] := 0;$
$a_S[i_{max}, j] := \frac{\lambda \cdot Ss}{\delta y};$
$a_N[i_{max}, j] := a_S[i_{max}, j];$
$a_P[i_{max}, j] := a_W[i_{max}, j] + a_E[i_{max}, j] + a_S[i_{max}, j] + a_N[i_{max}, j] - Sp[i_{max}, j];$
**end do**;

$$\begin{aligned} Sp_{3,2} &:= 0 \\ Su_{3,2} &:= 0 \\ a_{W_{3,2}} &:= 10.00 \\ a_{E_{3,2}} &:= 0 \\ a_{S_{3,2}} &:= 10.00 \\ a_{N_{3,2}} &:= 10.00 \\ a_{P_{3,2}} &:= 30.00 \\ Sp_{3,3} &:= 0 \\ Su_{3,3} &:= 0 \\ a_{W_{3,3}} &:= 10.00 \\ a_{E_{3,3}} &:= 0 \\ a_{S_{3,3}} &:= 10.00 \\ a_{N_{3,3}} &:= 10.00 \\ a_{P_{3,3}} &:= 30.00 \end{aligned} \quad \textbf{(1.9)}$$

Noeuds bas:

> **for** $i$ **from** 2 **to** $i_{max} - 1$ **do**
$Sp[i, 1] := 0;$
$Su[i, 1] := 0;$
$a_W[i, 1] := \frac{\lambda \cdot Se}{\delta x};$
$a_E[i, 1] := a_W[i, 1];$
$a_S[i, 1] := 0;$
$a_N[i, 1] := \frac{\lambda \cdot Ss}{\delta y};$
$a_P[i, 1] := a_W[i, 1] + a_E[i, 1] + a_S[i, 1] + a_N[i, 1] - Sp[i, 1];$
**end do**;

$$\begin{aligned} Sp_{2,1} &:= 0 \\ Su_{2,1} &:= 0 \\ a_{W_{2,1}} &:= 10.00 \\ a_{E_{2,1}} &:= 10.00 \end{aligned}$$

$$a_{S_{2,1}} := 0$$
$$a_{N_{2,1}} := 10.00$$
$$a_{P_{2,1}} := 30.00 \qquad \textbf{(1.10)}$$

Noeuds hauts:

```
> for i from 2 to i_max - 1 do
```

$$Sp[i, j_{max}] := -\frac{2 \cdot \lambda \cdot Se}{\delta x};$$
$$Su[i, j_{max}] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0;$$
$$a_W[i, j_{max}] := \frac{\lambda \cdot Se}{\delta x};$$
$$a_E[i, j_{max}] := a_W[i, j_{max}];$$
$$a_S[i, j_{max}] := \frac{\lambda \cdot Ss}{\delta y};$$
$$a_N[i, j_{max}] := 0;$$
$$a_P[i, j_{max}] := a_W[i, j_{max}] + a_E[i, j_{max}] + a_S[i, j_{max}] + a_N[i, j_{max}] - Sp[i, j_{max}];$$

**end do**;

$$Sp_{2,4} := -20.00$$
$$Su_{2,4} := 2000.$$
$$a_{W_{2,4}} := 10.00$$
$$a_{E_{2,4}} := 10.00$$
$$a_{S_{2,4}} := 10.00$$
$$a_{N_{2,4}} := 0$$
$$a_{P_{2,4}} := 50.00 \qquad \textbf{(1.11)}$$

Noeud (1,1):

$$Sp[1, 1] := 0;$$
$$Su[1, 1] := q \cdot Ss;$$
$$a_W[1, 1] := 0;$$
$$a_E[1, 1] := \frac{\lambda \cdot Se}{\delta x};$$
$$a_S[1, 1] := 0;$$
$$a_N[1, 1] := \frac{\lambda \cdot Ss}{\delta y};$$
$$a_P[1, 1] := a_W[1, 1] + a_E[1, 1] + a_S[1, 1] + a_N[1, 1] - Sp[1, 1];$$
$$Sp_{1,1} := 0$$
$$Su_{1,1} := 500.0$$
$$a_{W_{1,1}} := 0$$
$$a_{E_{1,1}} := 10.00$$
$$a_{S_{1,1}} := 0$$
$$a_{N_{1,1}} := 10.00$$

**(1.12)**

$$a_{P_{1,1}} := 20.00 \qquad \textbf{(1.12)}$$

Noeud (imax,1):

$> Sp[\, i_{max}, 1\,] := 0;$

$Su[\, i_{max}, 1\,] := 0;$

$a_W[\, i_{max}, 1\,] := \frac{\lambda \cdot Se}{\delta x};$

$a_E[\, i_{max}, 1\,] := 0;$

$a_S[\, i_{max}, 1\,] := 0;$

$a_N[\, i_{max}, 1\,] := \frac{\lambda \cdot Ss}{\delta y};$

$a_P[\, i_{max}, 1\,] := a_W[\, i_{max}, 1\,] + a_E[\, i_{max}, 1\,] + a_S[\, i_{max}, 1\,] + a_N[\, i_{max}, 1\,] - Sp[\, i_{max}, 1\,];$

$$Sp_{3,1} := 0$$
$$Su_{3,1} := 0$$
$$a_{W_{3,1}} := 10.00$$
$$a_{E_{3,1}} := 0$$
$$a_{S_{3,1}} := 0$$
$$a_{N_{3,1}} := 10.00$$
$$a_{P_{3,1}} := 20.00 \qquad \textbf{(1.13)}$$

Noeud (1,jmax):

$> Sp[1, j_{max}] := -\frac{2 \cdot \lambda \cdot Se}{\delta x};$

$Su[1\,, j_{max}] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0 + q \cdot Ss;$

$a_W[1, j_{max}] := 0;$

$a_E[1, j_{max}] := \frac{\lambda \cdot Se}{\delta x};$

$a_S[1, j_{max}] := \frac{\lambda \cdot Ss}{\delta y};$

$a_N[1, j_{max}] := 0;$

$a_P[1, j_{max}] := a_W[1, j_{max}] + a_E[1, j_{max}] + a_S[1, j_{max}] + a_N[1, j_{max}] - Sp[1, j_{max}];$

$$Sp_{1,4} := -20.00$$
$$Su_{1,4} := 2500.$$
$$a_{W_{1,4}} := 0$$
$$a_{E_{1,4}} := 10.00$$
$$a_{S_{1,4}} := 10.00$$
$$a_{N_{1,4}} := 0$$
$$a_{P_{1,4}} := 40.00 \qquad \textbf{(1.14)}$$

Noeud (imax,jmax):

$> Sp[i_{max}, j_{max}] := -\frac{2 \cdot \lambda \cdot Se}{\delta x};$

$$Su[i_{max}, j_{max}] := \frac{2 \cdot \lambda \cdot Se}{\delta x} \cdot T0;$$

$$a_W[i_{max}, j_{max}] := \frac{\lambda \cdot Se}{\delta x};$$

$$a_E[i_{max}, j_{max}] := 0;$$

$$a_S[i_{max}, j_{max}] := \frac{\lambda \cdot Ss}{\delta y};$$

$$a_N[i_{max}, j_{max}] := 0;$$

$$a_P[i_{max}, j_{max}] := a_W[i_{max}, j_{max}] + a_E[i_{max}, j_{max}] + a_S[i_{max}, j_{max}] + a_N[i_{max}, j_{max}] - Sp[i_{max}, j_{max}];$$

$$Sp_{3,4} := -20.00$$
$$Su_{3,4} := 2000.$$
$$a_{W_{3,4}} := 10.00$$
$$a_{E_{3,4}} := 0$$
$$a_{S_{3,4}} := 10.00$$
$$a_{N_{3,4}} := 0$$
$$a_{P_{3,4}} := 40.00 \qquad \textbf{(1.15)}$$

**Equations**:

> $k := 1$

$$k := 1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for j from 1 to j_max do
  for i from 1 to i_max do
      Eq[k] := a_P[i,j]·T[i,j] = a_W[i,j]·T[i-1,j] + a_E[i,j]·T[i+1,j] + a_S[i,j]·T[i,j-1] + a_N[i,j]·T[i,j+1] + Su[i,j];
      k := k + 1;
  end do;
  end do;
```

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$20.00\,T_{1,1} = 500.0 + 10.00\,T_{2,1} + 10.00\,T_{1,2}$$
$$30.00\,T_{2,1} = 10.00\,T_{1,1} + 10.00\,T_{3,1} + 10.00\,T_{2,2}$$
$$20.00\,T_{3,1} = 10.00\,T_{2,1} + 10.00\,T_{3,2}$$
$$30.00\,T_{1,2} = 500.0 + 10.00\,T_{2,2} + 10.00\,T_{1,1} + 10.00\,T_{1,3}$$
$$40.00\,T_{2,2} = 10.00\,T_{1,2} + 10.00\,T_{3,2} + 10.00\,T_{2,1} + 10.00\,T_{2,3}$$
$$30.00\,T_{3,2} = 10.00\,T_{2,2} + 10.00\,T_{3,1} + 10.00\,T_{3,3}$$
$$30.00\,T_{1,3} = 500.0 + 10.00\,T_{2,3} + 10.00\,T_{1,2} + 10.00\,T_{1,4}$$
$$40.00\,T_{2,3} = 10.00\,T_{1,3} + 10.00\,T_{3,3} + 10.00\,T_{2,2} + 10.00\,T_{2,4}$$
$$30.00\,T_{3,3} = 10.00\,T_{2,3} + 10.00\,T_{3,2} + 10.00\,T_{3,4}$$
$$40.00\,T_{1,4} = 2500. + 10.00\,T_{2,4} + 10.00\,T_{1,3}$$
$$50.00\,T_{2,4} = 10.00\,T_{1,4} + 10.00\,T_{3,4} + 10.00\,T_{2,3} + 2000.$$
$$40.00\,T_{3,4} = 10.00\,T_{2,4} + 2000. + 10.00\,T_{3,3} \qquad \textbf{(1.1.2)}$$

```
> Eqs := {seq(Eq[k], k = 1 ..Ne)};
```

$$Eqs := \{20.00\,T_{1,1} = 500.0 + 10.00\,T_{2,1} + 10.00\,T_{1,2},\ 30.00\,T_{1,2} = 500.0 + 10.00\,T_{2,2} + 10.00\,T_{1,1} + 10.00\,T_{1,3},\ 30.00\,T_{1,3} = 500.0 + 10.00\,T_{2,3} + 10.00\,T_{1,2} + 10.00\,T_{1,4},\ 40.00\,T_{1,4} = 2500. + 10.00\,T_{2,4} + 10.00\,T_{1,3},\ 30.00\,T_{2,1} = 10.00\,T_{1,1} + 10.00\,T_{3,1} + 10.00\,T_{2,2},\ 40.00\,T_{2,2} = 10.00\,T_{1,2} + 10.00\,T_{3,2} + 10.00\,T_{2,1} + 10.00\,T_{2,3},\ 40.00\,T_{2,3} = 10.00\,T_{1,3} + 10.00\,T_{3,3} + 10.00\,T_{2,2} + 10.00\,T_{2,4},\ 50.00\,T_{2,4} = 10.00\,T_{1,4} + 10.00\,T_{3,4} + 10.00\,T_{2,3} + 2000.,\ 20.00\,T_{3,1} = 10.00\,T_{2,1} + 10.00\,T_{3,2},\ 30.00\,T_{3,2} = 10.00\,T_{2,2} + 10.00\,T_{3,1} + 10.00\,T_{3,3},\ 30.00\,T_{3,3} = 10.00\,T_{2,3} + 10.00\,T_{3,2} + 10.00\,T_{3,4},\ 40.00\,T_{3,4} = 10.00\,T_{2,4} + 2000. + 10.00\,T_{3,3}\} \quad (1.1.3)$$

```
> SolT := solve(Eqs);
```

$$SolT := \{T_{1,1} = 260.0,\ T_{1,2} = 242.3,\ T_{1,3} = 205.6,\ T_{1,4} = 146.3,\ T_{2,1} = 227.8,\ T_{2,2} = 211.2,\ T_{2,3} = 178.2,\ T_{2,4} = 129.7,\ T_{3,1} = 212.2,\ T_{3,2} = 196.5,\ T_{3,3} = 166.2,\ T_{3,4} = 124.0\} \quad (1.1.4)$$

```
> Tmps := [seq(lhs(SolT_i), i = 1 ..Ne )];
```

$$Tmps := [T_{1,1}, T_{1,2}, T_{1,3}, T_{1,4}, T_{2,1}, T_{2,2}, T_{2,3}, T_{2,4}, T_{3,1}, T_{3,2}, T_{3,3}, T_{3,4}] \quad (1.1.5)$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Tmps)
```

$$A, b := \begin{bmatrix} 12 \text{ x } 12 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}, \begin{bmatrix} 1 \text{ .. } 12 \text{ Vector}_{column} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (1.1.6)$$

```
> b[8]
```

$$2000. \quad (1.1.7)$$