Equation de Convection-Diffusion 1D

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EXEMPLE 1

Détermination de la distribution de $x6(x)$ transportée par convection-diffusion à travers un domaine 1D dont les extrémités sont soumises aux (C.L.):
Schéma centré pour le terme convectif.

$$\frac{d}{dx}\left(\rho \cdot u \cdot \phi(x)\right) - \frac{d}{dx}\left(\Gamma \cdot \frac{d}{dx}\phi(x)\right) = 0$$

Conditions aux limites (C.L):

$$\phi(0) = \phi_0 = 1,$$
$$\phi(L) = \phi_L = 0,$$

u = 0.1 m/s , δx=0.2:

```
> Restart : Digits := 4 :
> u := 0.1; L := 1.0; ρ := 1.0; Gam := 0.1; S := 1.0; ndx := 5;
```

$$u := 0.1$$
$$L := 1.0$$
$$\rho := 1.0$$
$$Gam := 0.1$$
$$S := 1.0$$
$$ndx := 5 \qquad \textbf{(1.1)}$$

```
> δx := L/ndx ;
```

$F := \rho \cdot u \cdot S;$

$d := \frac{Gam \cdot S}{\delta x};$

$Pe := \frac{F}{d};$

$$\delta x := 0.2000$$
$$F := 0.100$$
$$d := 0.5000$$
$$Pe := 0.2000 \quad \textbf{(1.2)}$$

$> i_{max} := ndx;$

$$i_{max} := 5 \quad \textbf{(1.3)}$$

Nombre d'équations:

$> Ne := i_{max}$

$$Ne := 5 \quad \textbf{(1.4)}$$

Abscisses des noeuds:

$> x[0] := 0;$

for i **from** 1 **to** i_{max} **do**

$x[i] := \frac{\delta x}{2} + (i-1) \cdot \delta x;$

end do;

$x[i_{max}+1] := L;$

$$x_0 := 0$$
$$x_1 := 0.1000$$
$$x_2 := 0.3000$$
$$x_3 := 0.5000$$
$$x_4 := 0.7000$$
$$x_5 := 0.9000$$
$$x_6 := 1.0 \quad \textbf{(1.5)}$$

Conditions aux Limites:

$> \phi[0] := 1.0;$

$\phi[i_{max}+1] := 0;$

$\phi[L] := \phi[i_{max}+1];$

$$\phi_0 := 1.0$$
$$\phi_6 := 0$$
$$\phi_{1.0} := 0 \quad \textbf{(1.6)}$$

Noeuds internes:

$>$ **for** i **from** 2 **to** $i_{max}-1$ **do**

$Sp[i] := 0;$

$Su[i] := 0;$

$a_W[i] := d + \frac{F}{2};$

$a_E[i] := d - \frac{F}{2};$

$a_P[i] := a_W[i] + a_E[i] - Sp[i];$

end do;

$$Sp_2 := 0$$

$$Su_2 := 0$$
$$a_{W_2} := 0.5500$$
$$a_{E_2} := 0.4500$$
$$a_{P_2} := 1.000$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 0.5500$$
$$a_{E_3} := 0.4500$$
$$a_{P_3} := 1.000$$
$$Sp_4 := 0$$
$$Su_4 := 0$$
$$a_{W_4} := 0.5500$$
$$a_{E_4} := 0.4500$$
$$a_{P_4} := 1.000 \quad \textbf{(1.7)}$$

Noeud gauche:

$> Sp[1] := -(2 \cdot d + F);$

$Su[1] := (2 \cdot d + F) \cdot \phi[0];$

$a_W[1] := 0;$

$a_E[1] := d - \frac{F}{2};$

$a_P[1] := a_W[1] + a_E[1] - Sp[1];$

$$Sp_1 := -1.100$$
$$Su_1 := 1.100$$
$$a_{W_1} := 0$$
$$a_{E_1} := 0.4500$$
$$a_{P_1} := 1.550 \quad \textbf{(1.8)}$$

Noeud droit:

$> Sp[i_{max}] := -(2 \cdot d - F);$

$Su[i_{max}] := (2 \cdot d - F) \cdot \phi[i_{max} + 1];$

$a_W[i_{max}] := d + \frac{F}{2};$

$a_E[i_{max}] := 0;$

$a_P[i_{max}] := a_W[i_{max}] + a_E[i_{max}] - Sp[i_{max}];$

$$Sp_5 := -0.900$$
$$Su_5 := 0.$$
$$a_{W_5} := 0.5500$$
$$a_{E_5} := 0$$
$$a_{P_5} := 1.450 \quad \textbf{(1.9)}$$

Equations:

```
> k := 1
```

$$k := 1 \qquad \textbf{(1.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 1 to Ne do
      Eq[k] := a_P[i]·ϕ[i] = a_W[i]·ϕ[i − 1] + a_E[i]·ϕ[i + 1] + Su[i];
      k := k + 1;
  end do;
```

$$Eq_1 := 1.550\,\phi_1 = 1.100 + 0.4500\,\phi_2$$
$$k := 2$$
$$Eq_2 := 1.000\,\phi_2 = 0.5500\,\phi_1 + 0.4500\,\phi_3$$
$$k := 3$$
$$Eq_3 := 1.000\,\phi_3 = 0.5500\,\phi_2 + 0.4500\,\phi_4$$
$$k := 4$$
$$Eq_4 := 1.000\,\phi_4 = 0.5500\,\phi_3 + 0.4500\,\phi_5$$
$$k := 5$$
$$Eq_5 := 1.450\,\phi_5 = 0.5500\,\phi_4$$
$$k := 6 \qquad \textbf{(1.1.2)}$$

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$1.550\,\phi_1 = 1.100 + 0.4500\,\phi_2$$
$$1.000\,\phi_2 = 0.5500\,\phi_1 + 0.4500\,\phi_3$$
$$1.000\,\phi_3 = 0.5500\,\phi_2 + 0.4500\,\phi_4$$
$$1.000\,\phi_4 = 0.5500\,\phi_3 + 0.4500\,\phi_5$$
$$1.450\,\phi_5 = 0.5500\,\phi_4 \qquad \textbf{(1.1.3)}$$

```
> Eqs := {seq(Eq[k], k = 1 ..Ne)};
```

$$Eqs := \{1.550\,\phi_1 = 1.100 + 0.4500\,\phi_2,\ 1.000\,\phi_2 = 0.5500\,\phi_1 + 0.4500\,\phi_3,\ 1.000\,\phi_3 = 0.5500\,\phi_2 + 0.4500\,\phi_4,\ 1.000\,\phi_4 = 0.5500\,\phi_3 + 0.4500\,\phi_5,\ 1.450\,\phi_5 = 0.5500\,\phi_4\} \qquad \textbf{(1.1.4)}$$

```
> Tmps := [seq(ϕ[i], i = 1 ..Ne)];
```

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5] \qquad \textbf{(1.1.5)}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[\phi_1 = 0.9421,\ \phi_2 = 0.8006,\ \phi_3 = 0.6276,\ \phi_4 = 0.4163,\ \phi_5 = 0.1579]] \qquad \textbf{(1.1.6)}$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Tmps)
```

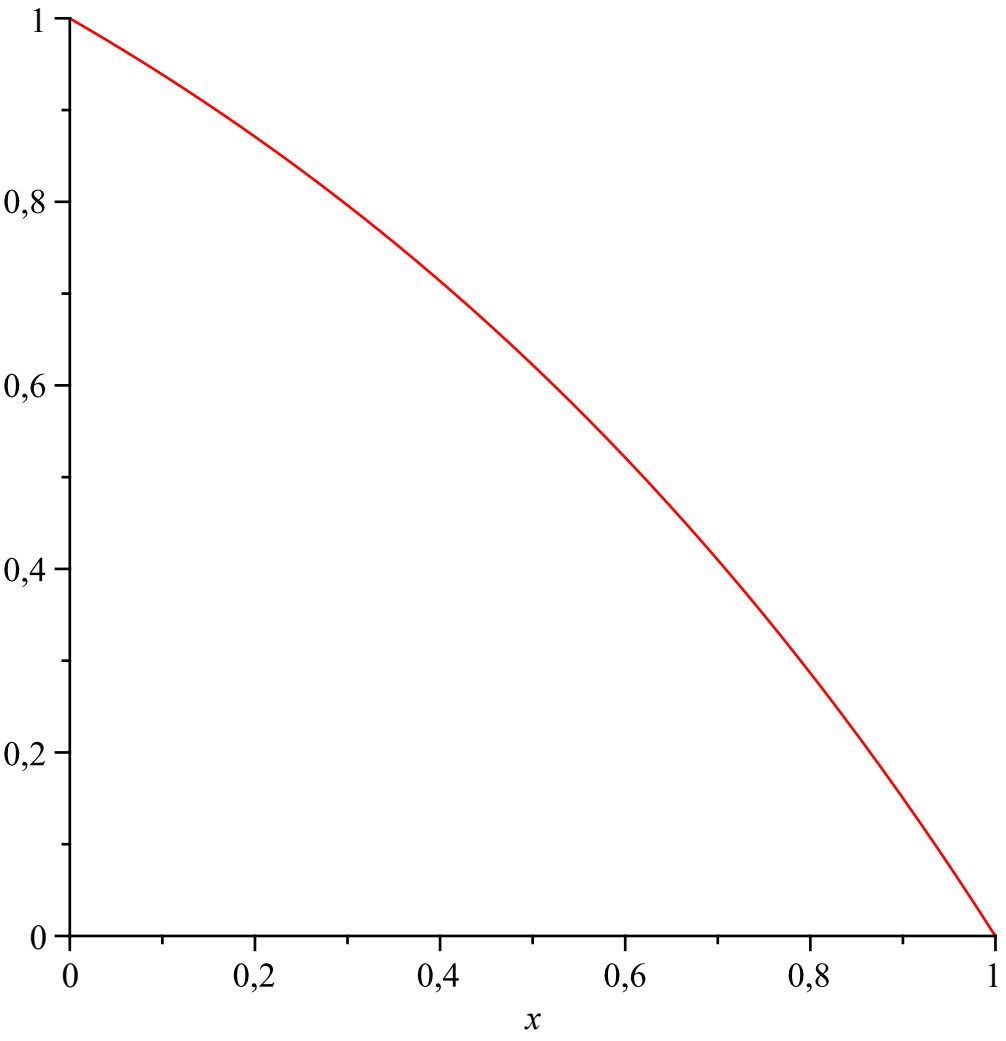
$$A, b := \begin{bmatrix} 1.550 & -0.4500 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5500 & 1.450 \\ -0.5500 & 1.000 & -0.4500 & 0 & 0 \\ 0 & -0.5500 & 1.000 & -0.4500 & 0 \\ 0 & 0 & -0.5500 & 1.000 & -0.4500 \end{bmatrix}, \begin{bmatrix} 1.100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(1.1.7)}$$

Solution exacte:

```
> F(x) := φ[0] + (φ[L] − φ[0]) · (e^(ρ·u·x/Gam) − 1) / (e^(ρ·u·L/Gam) − 1);
```

$$F := x \to \phi_0 + \frac{\left(\phi_L - \phi_0\right)\left(e^{\frac{\rho u x}{Gam}} - 1\right)}{e^{\frac{\rho u L}{Gam}} - 1} \qquad \textbf{(1.1.8)}$$

```
> with(plots) :
> plot(F(x), x = 0..L);
```




```
> for i from 1 to Ne do
      φ[i] := rhs(SolT[1, i])
  end do;
```

$$\phi_1 := 0.9421$$
$$\phi_2 := 0.8006$$
$$\phi_3 := 0.6276$$
$$\phi_4 := 0.4163$$
$$\phi_5 := 0.1579 \qquad \textbf{(1.1.9)}$$

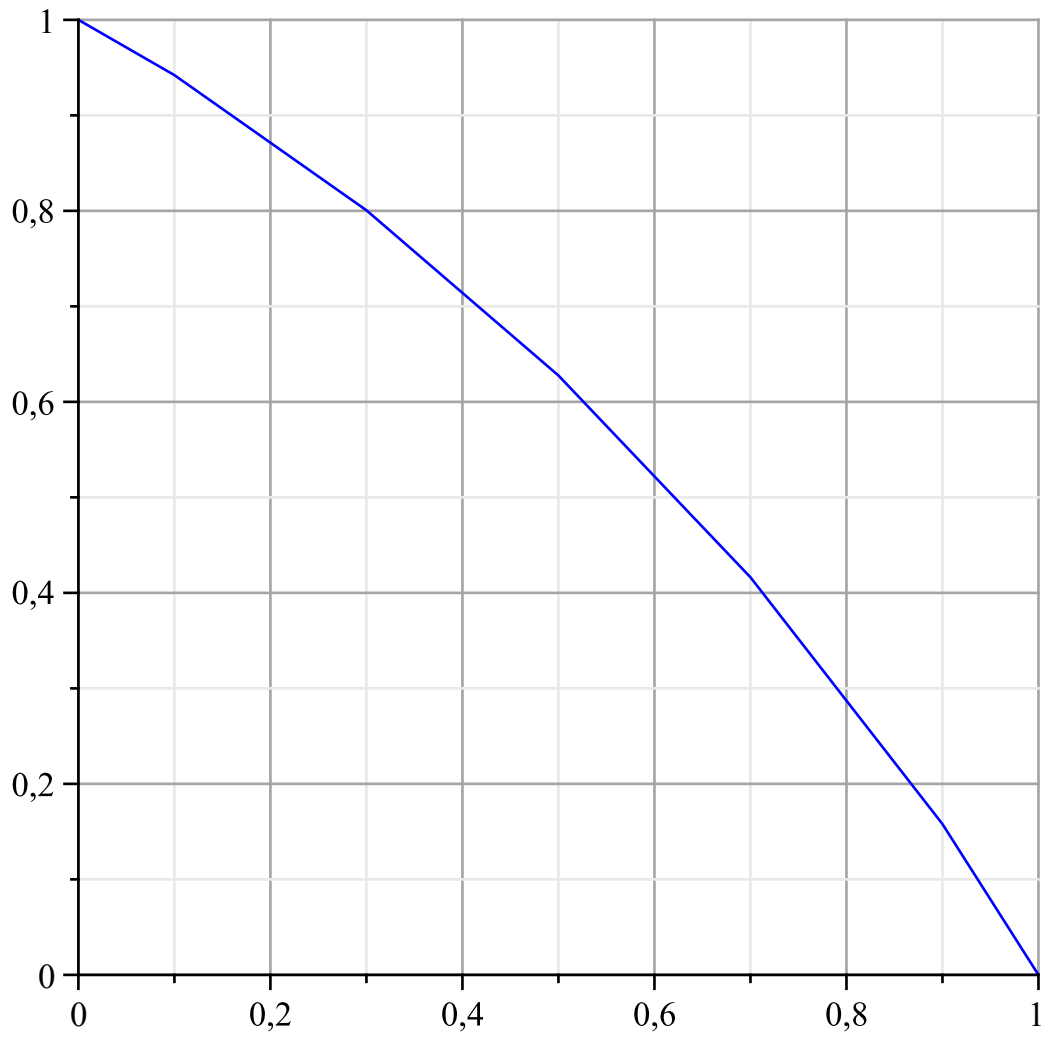
```
> lpN := [ seq( [x[i], φ[i]], i = 0..i_max + 1 ) ]
```

(1.1.10)

$lpN := [[0, 1.0], [0.1000, 0.9421], [0.3000, 0.8006], [0.5000, 0.6276], [0.7000, 0.4163], [0.9000, 0.1579], [1.0, 0]]$ **(1.1.10)**

Courbe Numérique:

$> listplot(lpN, color = blue, gridlines = true)$

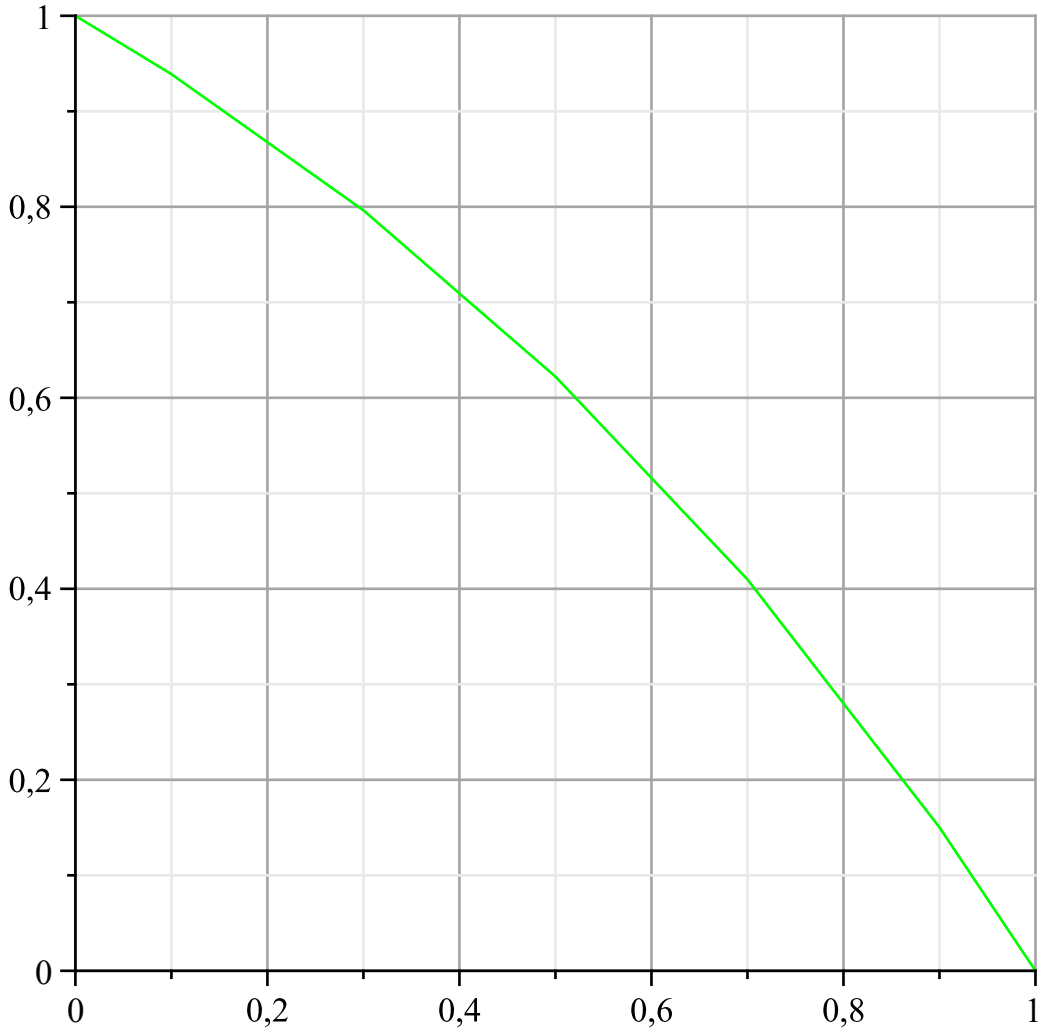


$> lpT := [seq([x[i], F(x[i])], i = 0 .. i_{max} + 1)]$

$lpT := [[0, 1.0], [0.1000, 0.9389], [0.3000, 0.7963], [0.5000, 0.6222], [0.7000, 0.4098], [0.9000, 0.1502], [1.0, 0.]]$ **(1.1.11)**

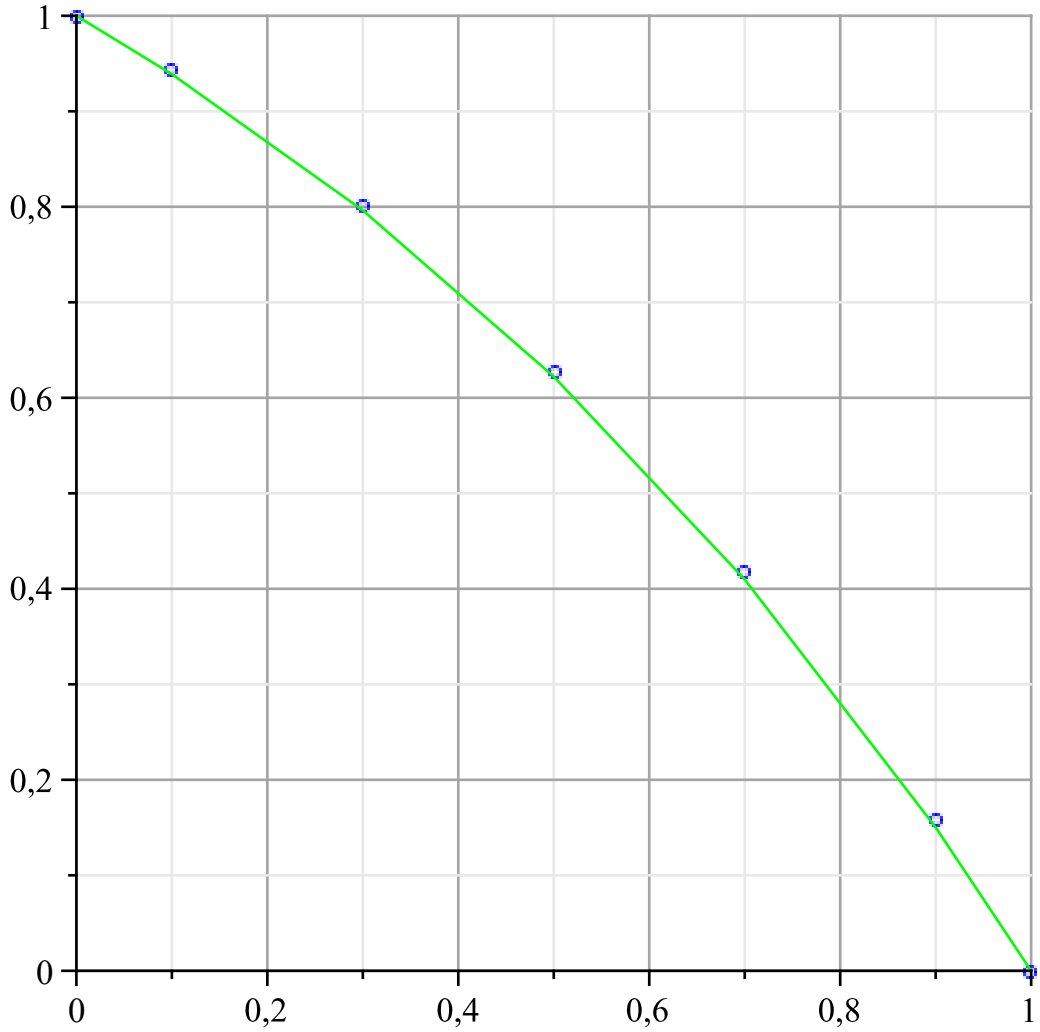
Courbe Théorique avec une liste de points:

$> listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

```
> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color
      = green, style = line], color = black, gridlines = true)
```



Erreur relative:

```
> for i from 1 to Ne do
  x[i];
  ϕ[i];
  F(x[i]);
  (ϕ[i] − F(x[i])) / F(x[i]) · 100
  end do
```

0.1000
0.9421
0.9389
0.3408
0.3000
0.8006
0.7963
0.5400
0.5000
0.6276
0.6222
0.8679
0.7000
0.4163
0.4098
1.586

0.9000
0.1579
0.1502
5.126 **(1.1.12)**

u = 2.5 m/s , δx=0.2:

```
> Restart : Digits := 4 :
> u := 2.5; L := 1.0; ρ := 1.0; Gam := 0.1; S := 1.0; ndx := 5;
```

$$u := 2.5$$
$$L := 1.0$$
$$\rho := 1.0$$
$$Gam := 0.1$$
$$S := 1.0$$
$$ndx := 5 \quad \textbf{(2.1)}$$

```
> δx := L/ndx ;
  F := ρ·u·S;
  d := Gam·S/δx;
  Pe := F/d;
```

$$\delta x := 0.2000$$
$$F := 2.500$$
$$d := 0.5000$$
$$Pe := 5.000 \quad \textbf{(2.2)}$$

```
> i_max := ndx;
```

$$i_{max} := 5 \quad \textbf{(2.3)}$$

Nombre d'équations:

```
> Ne := i_max
```

$$Ne := 5 \quad \textbf{(2.4)}$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to i_max do
      x[i] := δx/2 + (i − 1)·δx ;
  end do;
  x[i_max + 1] := L;
```

$$x_0 := 0$$
$$x_1 := 0.1000$$
$$x_2 := 0.3000$$
$$x_3 := 0.5000$$
$$x_4 := 0.7000$$
$$x_5 := 0.9000$$
$$x_6 := 1.0 \quad \textbf{(2.5)}$$

Conditions aux Limites:

```
> ϕ[0] := 1.0;
```

$\phi[i_{max}+1] := 0;$

$\phi[L] := \phi[i_{max}+1];$

$$\phi_0 := 1.0$$
$$\phi_6 := 0$$
$$\phi_{1.0} := 0 \qquad \textbf{(2.6)}$$

Noeuds internes:

```
> for i from 2 to i_max - 1 do
    Sp[i] := 0;
    Su[i] := 0;
    a_W[i] := d + F/2;
    a_E[i] := d - F/2;
    a_P[i] := a_W[i] + a_E[i] - Sp[i];
  end do;
```

$$Sp_2 := 0$$
$$Su_2 := 0$$
$$a_{W_2} := 1.750$$
$$a_{E_2} := -0.7500$$
$$a_{P_2} := 1.000$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 1.750$$
$$a_{E_3} := -0.7500$$
$$a_{P_3} := 1.000$$
$$Sp_4 := 0$$
$$Su_4 := 0$$
$$a_{W_4} := 1.750$$
$$a_{E_4} := -0.7500$$
$$a_{P_4} := 1.000 \qquad \textbf{(2.7)}$$

Noeud gauche:

```
> Sp[1] := -(2·d + F);
  Su[1] := (2·d + F)·φ[0];
  a_W[1] := 0;
  a_E[1] := d - F/2;
  a_P[1] := a_W[1] + a_E[1] - Sp[1];
```

$$Sp_1 := -3.500$$
$$Su_1 := 3.500$$
$$a_{W_1} := 0$$
$$a_{E_1} := -0.7500$$

(2.8)

$$a_{P_1} := 2.750 \quad \textbf{(2.8)}$$

Noeud droit:

```
> Sp[i_max] := -(2·d-F);
  Su[i_max] := (2·d-F)·ϕ[i_max+1];
  a_W[i_max] := d + F/2;
  a_E[i_max] := 0;
  a_P[i_max] := a_W[i_max] + a_E[i_max] - Sp[i_max];
```

$$Sp_5 := 1.500$$
$$Su_5 := -0.$$
$$a_{W_5} := 1.750$$
$$a_{E_5} := 0$$
$$a_{P_5} := 0.250 \quad \textbf{(2.9)}$$

Equations:

```
> k := 1
```

$$k := 1 \quad \textbf{(2.1.1)}$$

Résolution pour les noeuds internes:

```
> for i from 1 to Ne do
      Eq[k] := a_P[i]·ϕ[i] = a_W[i]·ϕ[i-1] + a_E[i]·ϕ[i+1] + Su[i];
      k := k+1;
  end do;
```

$$Eq_1 := 2.750\,\phi_1 = 3.500 - 0.7500\,\phi_2$$
$$k := 2$$
$$Eq_2 := 1.000\,\phi_2 = 1.750\,\phi_1 - 0.7500\,\phi_3$$
$$k := 3$$
$$Eq_3 := 1.000\,\phi_3 = 1.750\,\phi_2 - 0.7500\,\phi_4$$
$$k := 4$$
$$Eq_4 := 1.000\,\phi_4 = 1.750\,\phi_3 - 0.7500\,\phi_5$$
$$k := 5$$
$$Eq_5 := 0.250\,\phi_5 = 1.750\,\phi_4$$
$$k := 6 \quad \textbf{(2.1.2)}$$

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$2.750\,\phi_1 = 3.500 - 0.7500\,\phi_2$$
$$1.000\,\phi_2 = 1.750\,\phi_1 - 0.7500\,\phi_3$$
$$1.000\,\phi_3 = 1.750\,\phi_2 - 0.7500\,\phi_4$$
$$1.000\,\phi_4 = 1.750\,\phi_3 - 0.7500\,\phi_5$$
$$0.250\,\phi_5 = 1.750\,\phi_4 \quad \textbf{(2.1.3)}$$

```
> Eqs := {seq(Eq[k], k=1..Ne)};
```

$$Eqs := \{2.750\,\phi_1 = 3.500 - 0.7500\,\phi_2,\ 1.000\,\phi_2 = 1.750\,\phi_1 - 0.7500\,\phi_3,\ 1.000\,\phi_3 \quad \textbf{(2.1.4)}$$

$$= 1.750\,\phi_2 - 0.7500\,\phi_4,\ 1.000\,\phi_4 = 1.750\,\phi_3 - 0.7500\,\phi_5,\ 0.250\,\phi_5 = 1.750\,\phi_4\}$$

```
> Tmps := [seq(φ[i], i = 1 ..Ne)];
```

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5] \tag{2.1.5}$$

```
> SolT := solve(Eqs, Tmps);
```

$$SolT := [[\phi_1 = 1.036, \phi_2 = 0.8694, \phi_3 = 1.257, \phi_4 = 0.3521, \phi_5 = 2.464]] \tag{2.1.6}$$

```
> with(LinearAlgebra) :
```

Forme matricielle:

```
> A, b := GenerateMatrix(Eqs, Tmps)
```

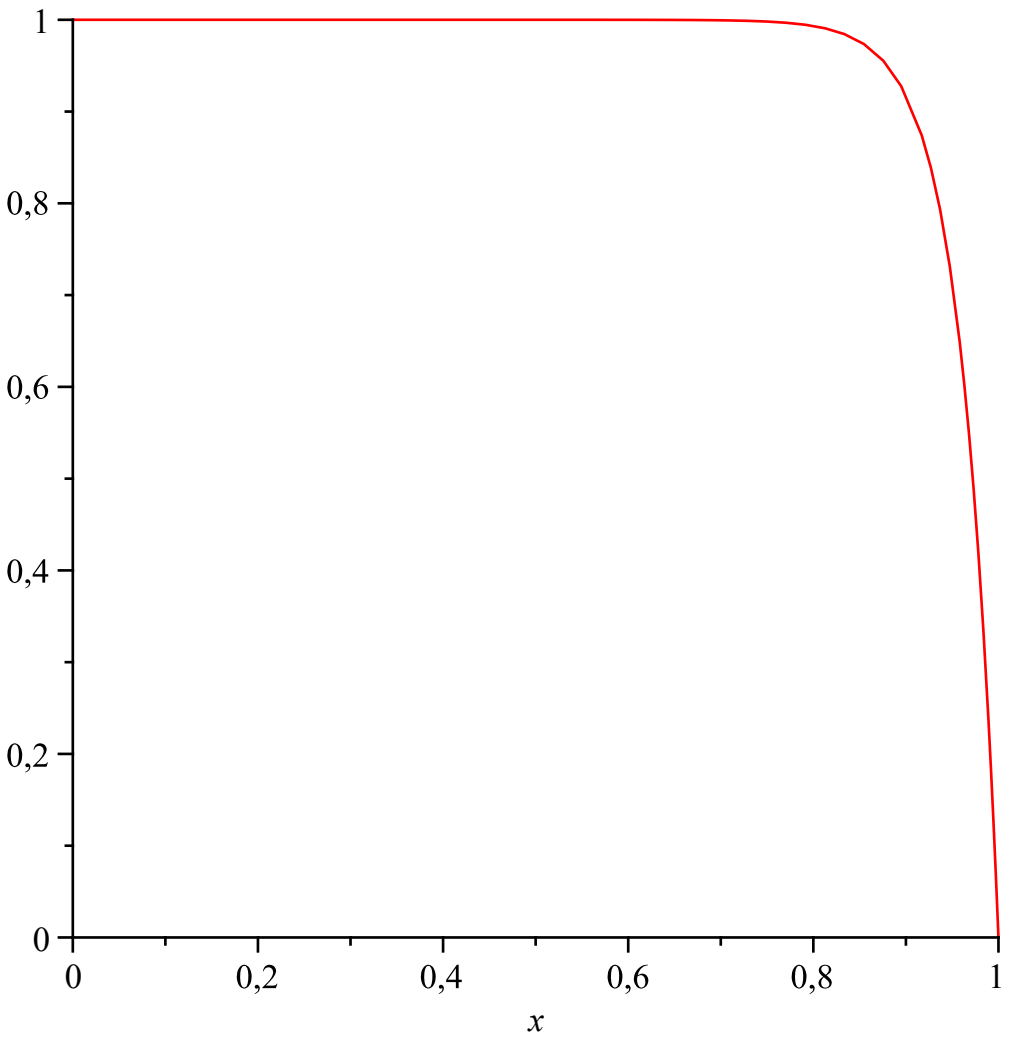
$$A, b := \begin{bmatrix} 2.750 & 0.7500 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.750 & 0.250 \\ -1.750 & 1.000 & 0.7500 & 0 & 0 \\ 0 & -1.750 & 1.000 & 0.7500 & 0 \\ 0 & 0 & -1.750 & 1.000 & 0.7500 \end{bmatrix}, \begin{bmatrix} 3.500 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.1.7}$$

Solution exacte:

```
> F(x) := φ[0] + (φ[L] − φ[0]) · (e^(ρ·u·x/Gam) − 1) / (e^(ρ·u·L/Gam) − 1);
```

$$F := x \to \phi_0 + \frac{(\phi_L - \phi_0)\left(e^{\frac{\rho u x}{Gam}} - 1\right)}{e^{\frac{\rho u L}{Gam}} - 1} \tag{2.1.8}$$

```
> with(plots) :
> plot(F(x), x = 0 ..L);
```




```
> for i from 1 to Ne do
      ϕ[i] := rhs(SolT1, i)
  end do;
```

$$\phi_1 := 1.036$$

$$\phi_2 := 0.8694$$

$$\phi_3 := 1.257$$

$$\phi_4 := 0.3521$$

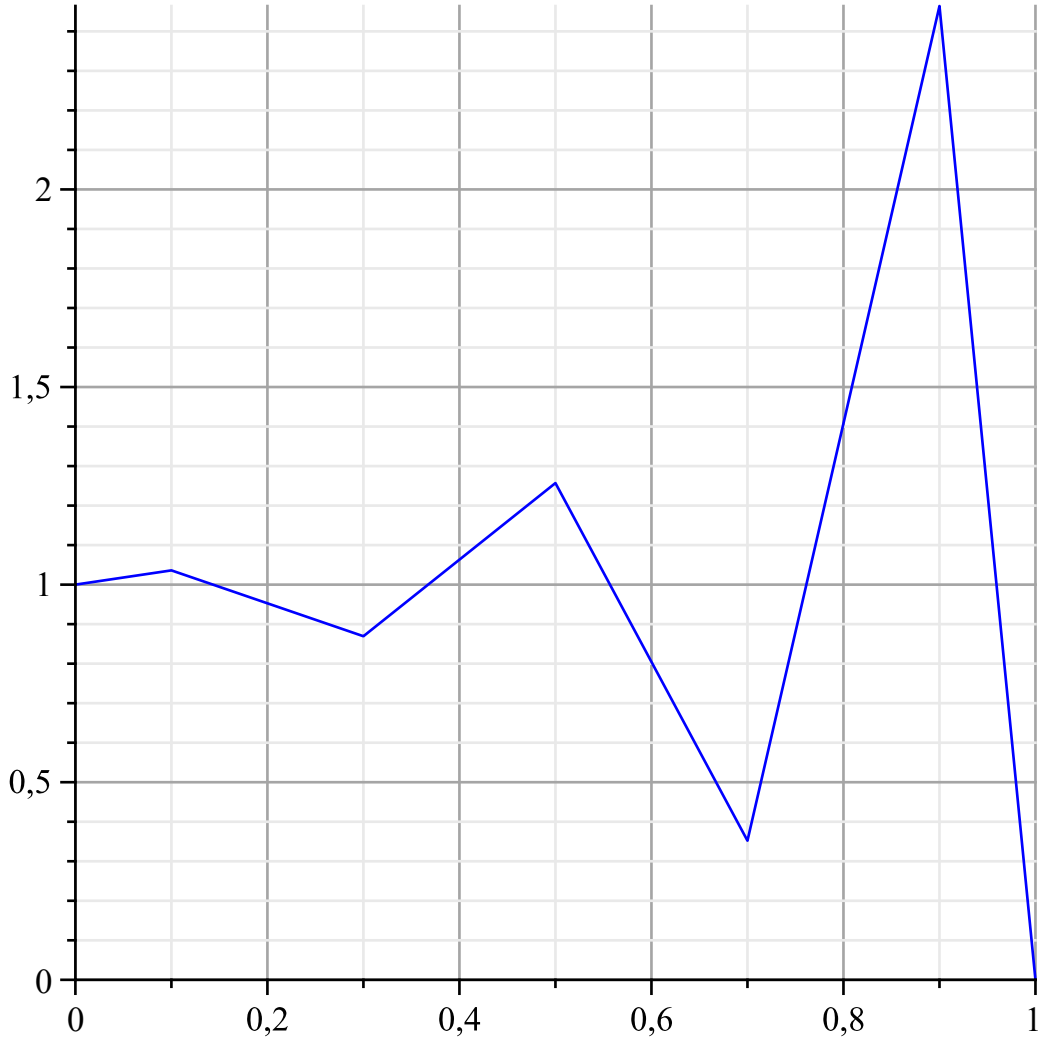
$$\phi_5 := 2.464 \qquad \textbf{(2.1.9)}$$

```
> lpN := [ seq( [x[i], ϕ[i]], i = 0 .. imax + 1 ) ]
```

$$lpN := [[0, 1.0], [0.1000, 1.036], [0.3000, 0.8694], [0.5000, 1.257], [0.7000, 0.3521], [0.9000, 2.464], [1.0, 0]] \qquad \textbf{(2.1.10)}$$

Courbe Numérique:

```
> listplot(lpN, color = blue, gridlines = true)
```

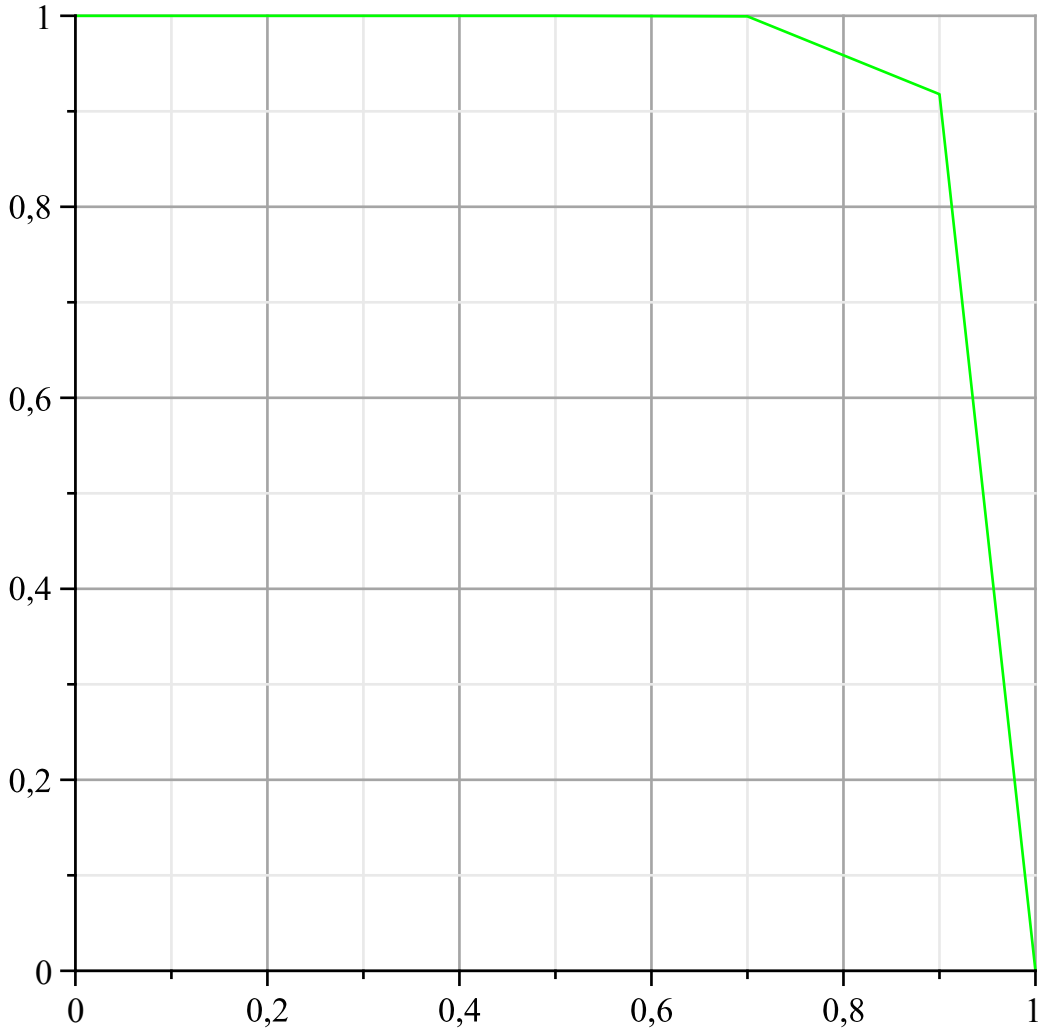


> $lpT := [seq([x[i], F(x[i])], i = 0..i_{max}+1)]$

$lpT := [[0, 1.0], [0.1000, 1.0], [0.3000, 1.0], [0.5000, 1.0], [0.7000, 0.9994], [0.9000, 0.9179], [1.0, 0.]]$ **(2.1.11)**

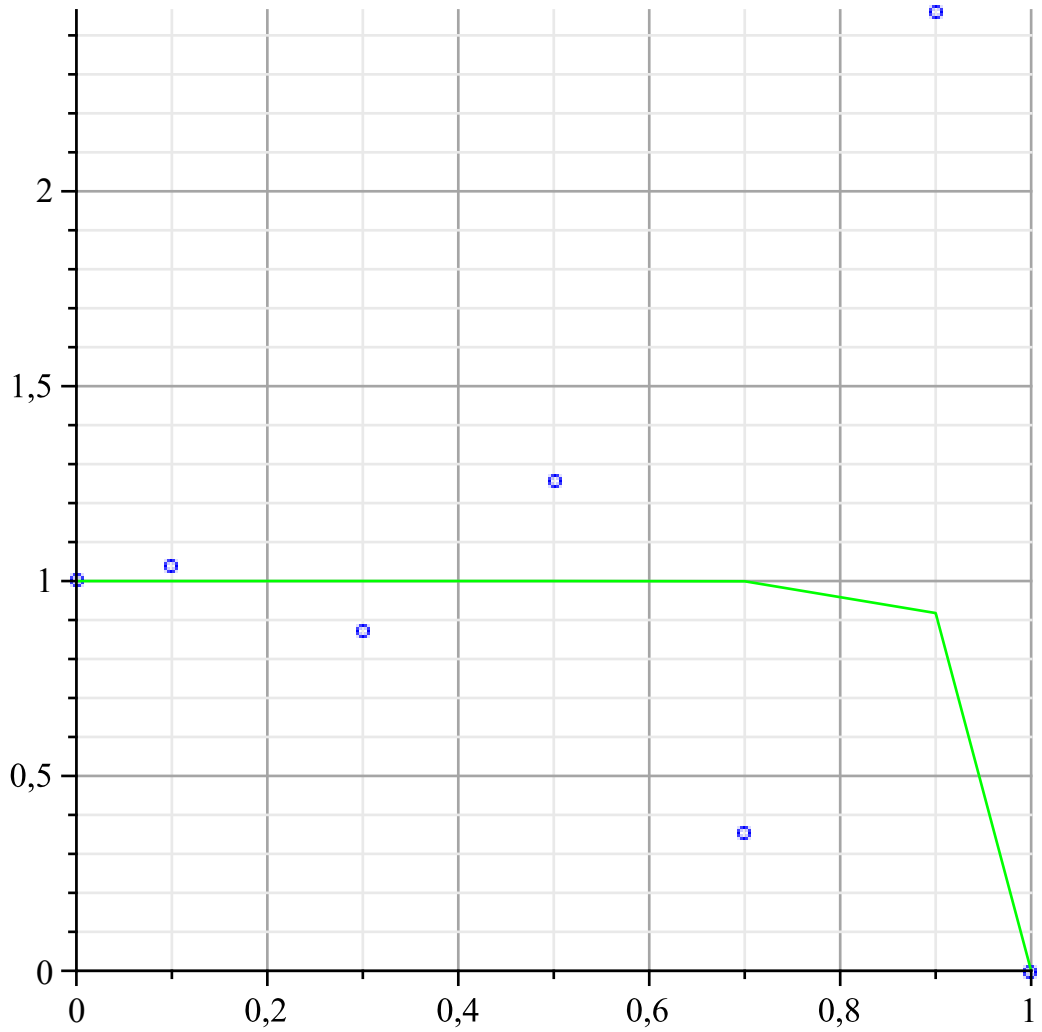
Courbe Théorique avec une liste de points:

> $listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

```
> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color
      = green, style = line], color = black, gridlines = true)
```



Erreur relative:

```
> for i from 1 to Ne do
  x[i];
  ϕ[i];
  F(x[i]);
  (ϕ[i] − F(x[i])) / F(x[i]) · 100
  end do
```

0.1000
1.036
1.0
3.600
0.3000
0.8694
1.0
-13.06
0.5000
1.257
1.0
25.70
0.7000
0.3521
0.9994
-64.77

0.9000
2.464
0.9179
168.4 **(2.1.12)**

>

u = 2.5 m/s , δx=0.05:

```
> Restart : Digits := 4 :
> u := 2.5; L := 1.0; ρ := 1.0; Gam := 0.1; S := 1.0; ndx := 20;
```

$$u := 2.5$$
$$L := 1.0$$
$$\rho := 1.0$$
$$Gam := 0.1$$
$$S := 1.0$$
$$ndx := 20 \quad \textbf{(3.1)}$$

```
> δx := L/ndx ;
  F := ρ·u·S;
  d := Gam·S/δx;
  Pe := F/d;
```

$$\delta x := 0.05000$$
$$F := 2.500$$
$$d := 2.000$$
$$Pe := 1.250 \quad \textbf{(3.2)}$$

```
> i_max := ndx;
```

$$i_{max} := 20 \quad \textbf{(3.3)}$$

Nombre d'équations:

```
> Ne := i_max
```

$$Ne := 20 \quad \textbf{(3.4)}$$

Abscisses des noeuds:

```
> x[0] := 0;
  for i from 1 to i_max do
      x[i] := δx/2 + (i − 1)·δx ;
  end do;
  x[i_max + 1] := L;
```

$$x_0 := 0$$
$$x_1 := 0.02500$$
$$x_2 := 0.07500$$
$$x_3 := 0.1250$$
$$x_4 := 0.1750$$
$$x_5 := 0.2250$$
$$x_6 := 0.2750$$
$$x_7 := 0.3250$$

$$x_8 := 0.3750$$
$$x_9 := 0.4250$$
$$x_{10} := 0.4750$$
$$x_{11} := 0.5250$$
$$x_{12} := 0.5750$$
$$x_{13} := 0.6250$$
$$x_{14} := 0.6750$$
$$x_{15} := 0.7250$$
$$x_{16} := 0.7750$$
$$x_{17} := 0.8250$$
$$x_{18} := 0.8750$$
$$x_{19} := 0.9250$$
$$x_{20} := 0.9750$$
$$x_{21} := 1.0 \qquad \textbf{(3.5)}$$

Conditions aux Limites:

```
> ϕ[0] := 1.0;
  ϕ[i_max + 1] := 0;
  ϕ[L] := ϕ[i_max + 1];
```

$$\phi_0 := 1.0$$
$$\phi_{21} := 0$$
$$\phi_{1.0} := 0 \qquad \textbf{(3.6)}$$

Noeuds internes:

```
> for i from 2 to i_max - 1 do
    Sp[i] := 0;
    Su[i] := 0;
    a_W[i] := d + F/2;
    a_E[i] := d - F/2;
    a_P[i] := a_W[i] + a_E[i] - Sp[i];
  end do;
```

$$Sp_2 := 0$$
$$Su_2 := 0$$
$$a_{W_2} := 3.250$$
$$a_{E_2} := 0.750$$
$$a_{P_2} := 4.000$$
$$Sp_3 := 0$$
$$Su_3 := 0$$
$$a_{W_3} := 3.250$$
$$a_{E_3} := 0.750$$
$$a_{P_3} := 4.000$$
$$Sp_4 := 0$$

$$Su_4 := 0$$
$$a_{W_4} := 3.250$$
$$a_{E_4} := 0.750$$
$$a_{P_4} := 4.000$$
$$Sp_5 := 0$$
$$Su_5 := 0$$
$$a_{W_5} := 3.250$$
$$a_{E_5} := 0.750$$
$$a_{P_5} := 4.000$$
$$Sp_6 := 0$$
$$Su_6 := 0$$
$$a_{W_6} := 3.250$$
$$a_{E_6} := 0.750$$
$$a_{P_6} := 4.000$$
$$Sp_7 := 0$$
$$Su_7 := 0$$
$$a_{W_7} := 3.250$$
$$a_{E_7} := 0.750$$
$$a_{P_7} := 4.000$$
$$Sp_8 := 0$$
$$Su_8 := 0$$
$$a_{W_8} := 3.250$$
$$a_{E_8} := 0.750$$
$$a_{P_8} := 4.000$$
$$Sp_9 := 0$$
$$Su_9 := 0$$
$$a_{W_9} := 3.250$$
$$a_{E_9} := 0.750$$
$$a_{P_9} := 4.000$$
$$Sp_{10} := 0$$
$$Su_{10} := 0$$
$$a_{W_{10}} := 3.250$$
$$a_{E_{10}} := 0.750$$
$$a_{P_{10}} := 4.000$$
$$Sp_{11} := 0$$
$$Su_{11} := 0$$
$$a_{W_{11}} := 3.250$$

$$a_{E_{11}} := 0.750$$
$$a_{P_{11}} := 4.000$$
$$Sp_{12} := 0$$
$$Su_{12} := 0$$
$$a_{W_{12}} := 3.250$$
$$a_{E_{12}} := 0.750$$
$$a_{P_{12}} := 4.000$$
$$Sp_{13} := 0$$
$$Su_{13} := 0$$
$$a_{W_{13}} := 3.250$$
$$a_{E_{13}} := 0.750$$
$$a_{P_{13}} := 4.000$$
$$Sp_{14} := 0$$
$$Su_{14} := 0$$
$$a_{W_{14}} := 3.250$$
$$a_{E_{14}} := 0.750$$
$$a_{P_{14}} := 4.000$$
$$Sp_{15} := 0$$
$$Su_{15} := 0$$
$$a_{W_{15}} := 3.250$$
$$a_{E_{15}} := 0.750$$
$$a_{P_{15}} := 4.000$$
$$Sp_{16} := 0$$
$$Su_{16} := 0$$
$$a_{W_{16}} := 3.250$$
$$a_{E_{16}} := 0.750$$
$$a_{P_{16}} := 4.000$$
$$Sp_{17} := 0$$
$$Su_{17} := 0$$
$$a_{W_{17}} := 3.250$$
$$a_{E_{17}} := 0.750$$
$$a_{P_{17}} := 4.000$$
$$Sp_{18} := 0$$
$$Su_{18} := 0$$
$$a_{W_{18}} := 3.250$$
$$a_{E_{18}} := 0.750$$
$$a_{P_{18}} := 4.000$$

$$Sp_{19} := 0$$
$$Su_{19} := 0$$
$$a_{W_{19}} := 3.250$$
$$a_{E_{19}} := 0.750$$
$$a_{P_{19}} := 4.000 \quad \textbf{(3.7)}$$

Noeud gauche:

$> Sp[1] := -(2 \cdot d + F);$

$Su[1] := (2 \cdot d + F) \cdot \phi[0];$

$a_W[1] := 0;$

$a_E[1] := \mathrm{d} - \frac{F}{2};$

$a_P[1] := a_W[1] + a_E[1] - Sp[1];$

$$Sp_1 := -6.500$$
$$Su_1 := 6.500$$
$$a_{W_1} := 0$$
$$a_{E_1} := 0.750$$
$$a_{P_1} := 7.250 \quad \textbf{(3.8)}$$

Noeud droit:

$> Sp[i_{\max}] := -(2 \cdot \mathrm{d} - \mathrm{F});$

$Su[i_{\max}] := (2 \cdot \mathrm{d} - \mathrm{F}) \cdot \phi[i_{\max} + 1];$

$a_W[i_{\max}] := \mathrm{d} + \frac{F}{2};$

$a_E[i_{\max}] := 0;$

$a_P[i_{\max}] := a_W[i_{\max}] + a_E[i_{\max}] - Sp[i_{\max}];$

$$Sp_{20} := -1.500$$
$$Su_{20} := 0.$$
$$a_{W_{20}} := 3.250$$
$$a_{E_{20}} := 0$$
$$a_{P_{20}} := 4.750 \quad \textbf{(3.9)}$$

Equations:

$> k := 1$

$$k := 1 \quad \textbf{(3.1.1)}$$

Résolution pour les noeuds internes:

$>$ **for** i **from** 1 **to** Ne **do**

$Eq[\mathrm{k}] := a_P[i] \cdot \phi[\mathrm{i}] = a_W[i] \cdot \phi[\mathrm{i} - 1] + a_E[i] \cdot \phi[\mathrm{i} + 1] + Su[i];$

$k := k + 1;$

end do;

$$Eq_1 := 7.250\ \phi_1 = 6.500 + 0.750\ \phi_2$$
$$k := 2$$
$$Eq_2 := 4.000\ \phi_2 = 3.250\ \phi_1 + 0.750\ \phi_3$$
$$k := 3$$

$$Eq_3 := 4.000\ \phi_3 = 3.250\ \phi_2 + 0.750\ \phi_4$$
$$k := 4$$
$$Eq_4 := 4.000\ \phi_4 = 3.250\ \phi_3 + 0.750\ \phi_5$$
$$k := 5$$
$$Eq_5 := 4.000\ \phi_5 = 3.250\ \phi_4 + 0.750\ \phi_6$$
$$k := 6$$
$$Eq_6 := 4.000\ \phi_6 = 3.250\ \phi_5 + 0.750\ \phi_7$$
$$k := 7$$
$$Eq_7 := 4.000\ \phi_7 = 3.250\ \phi_6 + 0.750\ \phi_8$$
$$k := 8$$
$$Eq_8 := 4.000\ \phi_8 = 3.250\ \phi_7 + 0.750\ \phi_9$$
$$k := 9$$
$$Eq_9 := 4.000\ \phi_9 = 3.250\ \phi_8 + 0.750\ \phi_{10}$$
$$k := 10$$
$$Eq_{10} := 4.000\ \phi_{10} = 3.250\ \phi_9 + 0.750\ \phi_{11}$$
$$k := 11$$
$$Eq_{11} := 4.000\ \phi_{11} = 3.250\ \phi_{10} + 0.750\ \phi_{12}$$
$$k := 12$$
$$Eq_{12} := 4.000\ \phi_{12} = 3.250\ \phi_{11} + 0.750\ \phi_{13}$$
$$k := 13$$
$$Eq_{13} := 4.000\ \phi_{13} = 3.250\ \phi_{12} + 0.750\ \phi_{14}$$
$$k := 14$$
$$Eq_{14} := 4.000\ \phi_{14} = 3.250\ \phi_{13} + 0.750\ \phi_{15}$$
$$k := 15$$
$$Eq_{15} := 4.000\ \phi_{15} = 3.250\ \phi_{14} + 0.750\ \phi_{16}$$
$$k := 16$$
$$Eq_{16} := 4.000\ \phi_{16} = 3.250\ \phi_{15} + 0.750\ \phi_{17}$$
$$k := 17$$
$$Eq_{17} := 4.000\ \phi_{17} = 3.250\ \phi_{16} + 0.750\ \phi_{18}$$
$$k := 18$$
$$Eq_{18} := 4.000\ \phi_{18} = 3.250\ \phi_{17} + 0.750\ \phi_{19}$$
$$k := 19$$
$$Eq_{19} := 4.000\ \phi_{19} = 3.250\ \phi_{18} + 0.750\ \phi_{20}$$
$$k := 20$$
$$Eq_{20} := 4.750\ \phi_{20} = 3.250\ \phi_{19}$$
$$k := 21$$

(3.1.2)

Ecriture du système d'équations:

```
> for k from 1 to Ne do Eq[k] end do;
```

$$7.250\ \phi_1 = 6.500 + 0.750\ \phi_2$$
$$4.000\ \phi_2 = 3.250\ \phi_1 + 0.750\ \phi_3$$
$$4.000\ \phi_3 = 3.250\ \phi_2 + 0.750\ \phi_4$$
$$4.000\ \phi_4 = 3.250\ \phi_3 + 0.750\ \phi_5$$
$$4.000\ \phi_5 = 3.250\ \phi_4 + 0.750\ \phi_6$$

$$4.000\,\phi_6 = 3.250\,\phi_5 + 0.750\,\phi_7$$
$$4.000\,\phi_7 = 3.250\,\phi_6 + 0.750\,\phi_8$$
$$4.000\,\phi_8 = 3.250\,\phi_7 + 0.750\,\phi_9$$
$$4.000\,\phi_9 = 3.250\,\phi_8 + 0.750\,\phi_{10}$$
$$4.000\,\phi_{10} = 3.250\,\phi_9 + 0.750\,\phi_{11}$$
$$4.000\,\phi_{11} = 3.250\,\phi_{10} + 0.750\,\phi_{12}$$
$$4.000\,\phi_{12} = 3.250\,\phi_{11} + 0.750\,\phi_{13}$$
$$4.000\,\phi_{13} = 3.250\,\phi_{12} + 0.750\,\phi_{14}$$
$$4.000\,\phi_{14} = 3.250\,\phi_{13} + 0.750\,\phi_{15}$$
$$4.000\,\phi_{15} = 3.250\,\phi_{14} + 0.750\,\phi_{16}$$
$$4.000\,\phi_{16} = 3.250\,\phi_{15} + 0.750\,\phi_{17}$$
$$4.000\,\phi_{17} = 3.250\,\phi_{16} + 0.750\,\phi_{18}$$
$$4.000\,\phi_{18} = 3.250\,\phi_{17} + 0.750\,\phi_{19}$$
$$4.000\,\phi_{19} = 3.250\,\phi_{18} + 0.750\,\phi_{20}$$
$$4.750\,\phi_{20} = 3.250\,\phi_{19} \qquad \textbf{(3.1.3)}$$

> $Eqs := \{seq(Eq[k], k = 1..Ne)\};$

$$Eqs := \{7.250\,\phi_1 = 6.500 + 0.750\,\phi_2, 4.000\,\phi_2 = 3.250\,\phi_1 + 0.750\,\phi_3, 4.000\,\phi_3 = 3.250\,\phi_2 + 0.750\,\phi_4, 4.000\,\phi_4 = 3.250\,\phi_3 + 0.750\,\phi_5, 4.000\,\phi_5 = 3.250\,\phi_4 + 0.750\,\phi_6, 4.000\,\phi_6 = 3.250\,\phi_5 + 0.750\,\phi_7, 4.000\,\phi_7 = 3.250\,\phi_6 + 0.750\,\phi_8, 4.000\,\phi_8 = 3.250\,\phi_7 + 0.750\,\phi_9, 4.000\,\phi_9 = 3.250\,\phi_8 + 0.750\,\phi_{10}, 4.000\,\phi_{10} = 3.250\,\phi_9 + 0.750\,\phi_{11}, 4.000\,\phi_{11} = 3.250\,\phi_{10} + 0.750\,\phi_{12}, 4.000\,\phi_{12} = 3.250\,\phi_{11} + 0.750\,\phi_{13}, 4.000\,\phi_{13} = 3.250\,\phi_{12} + 0.750\,\phi_{14}, 4.000\,\phi_{14} = 3.250\,\phi_{13} + 0.750\,\phi_{15}, 4.000\,\phi_{15} = 3.250\,\phi_{14} + 0.750\,\phi_{16}, 4.000\,\phi_{16} = 3.250\,\phi_{15} + 0.750\,\phi_{17}, 4.000\,\phi_{17} = 3.250\,\phi_{16} + 0.750\,\phi_{18}, 4.000\,\phi_{18} = 3.250\,\phi_{17} + 0.750\,\phi_{19}, 4.000\,\phi_{19} = 3.250\,\phi_{18} + 0.750\,\phi_{20}, 4.750\,\phi_{20} = 3.250\,\phi_{19}\} \qquad \textbf{(3.1.4)}$$

> $Tmps := [seq(\phi[i], i = 1..Ne)];$

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}, \phi_{11}, \phi_{12}, \phi_{13}, \phi_{14}, \phi_{15}, \phi_{16}, \phi_{17}, \phi_{18}, \phi_{19}, \phi_{20}] \qquad \textbf{(3.1.5)}$$

> $SolT := solve(Eqs, Tmps);$

$$SolT := [[\phi_1 = 1.000, \phi_2 = 1.000, \phi_3 = 1.000, \phi_4 = 1.000, \phi_5 = 1.000, \phi_6 = 1.000, \phi_7 = 1.000, \phi_8 = 1.000, \phi_9 = 1.000, \phi_{10} = 1.000, \phi_{11} = 1.000, \phi_{12} = 1.000, \phi_{13} = 1.000, \phi_{14} = 0.9999, \phi_{15} = 0.9998, \phi_{16} = 0.9989, \phi_{17} = 0.9954, \phi_{18} = 0.9800, \phi_{19} = 0.9135, \phi_{20} = 0.6250]] \qquad \textbf{(3.1.6)}$$

> $with(LinearAlgebra):$

Forme matricielle:

> $A, b := GenerateMatrix(Eqs, Tmps)$

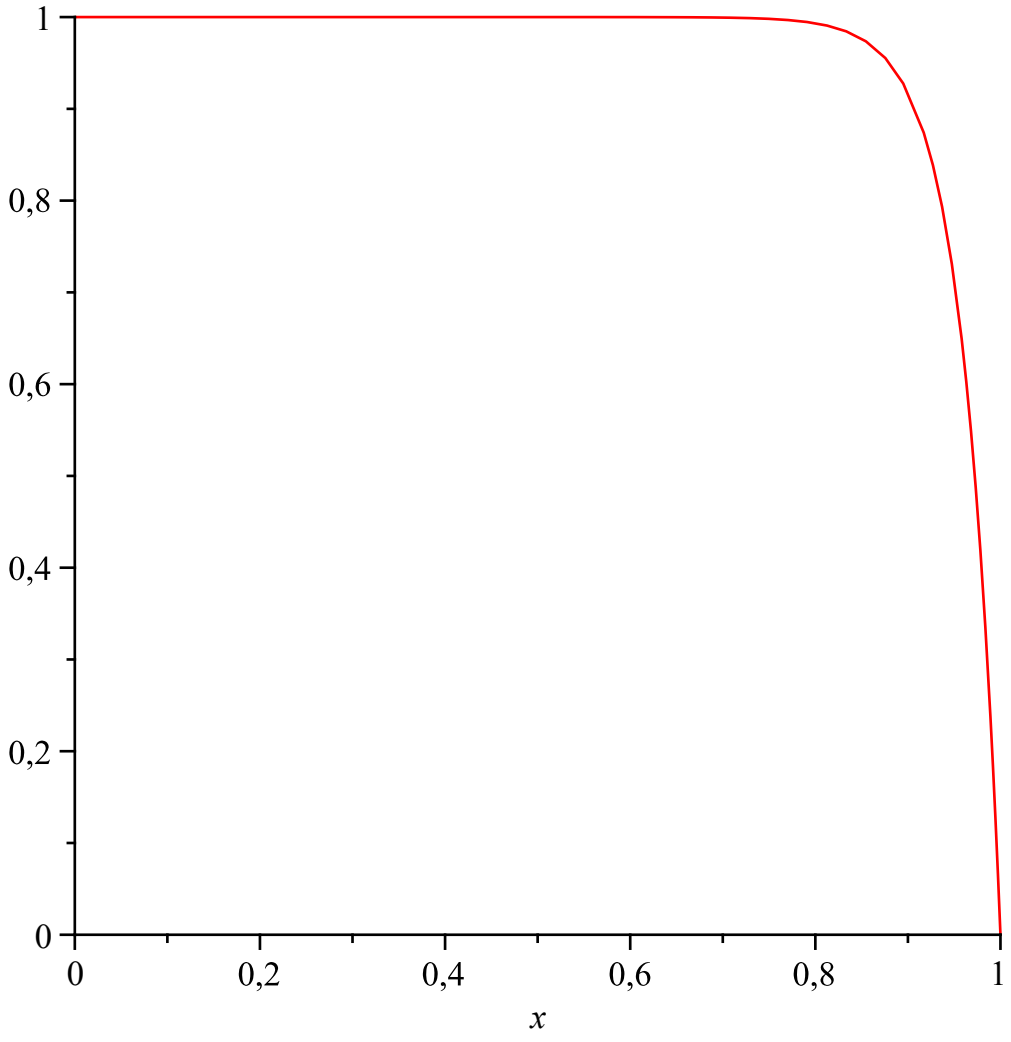
$$A, b := \left[\begin{array}{c} 20\ x\ 20\ Matrix \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right], \left[\begin{array}{c} 1\ ..\ 20\ Vector_{column} \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right] \quad \textbf{(3.1.7)}$$

Solution exacte:

```
> F(x) := phi[0] + (phi[L] - phi[0]) * (exp(rho*u*x/Gam) - 1) / (exp(rho*u*L/Gam) - 1);
```

$$F := x \rightarrow \phi_0 + \frac{\left(\phi_L - \phi_0\right)\left(e^{\frac{\rho u x}{Gam}} - 1\right)}{e^{\frac{\rho u L}{Gam}} - 1} \quad \textbf{(3.1.8)}$$

```
> with(plots):
> plot(F(x), x=0..L);
```




```
> for i from 1 to Ne do
      phi[i] := rhs(SolT[1, i])
  end do;
```

$$\phi_1 := 1.000$$

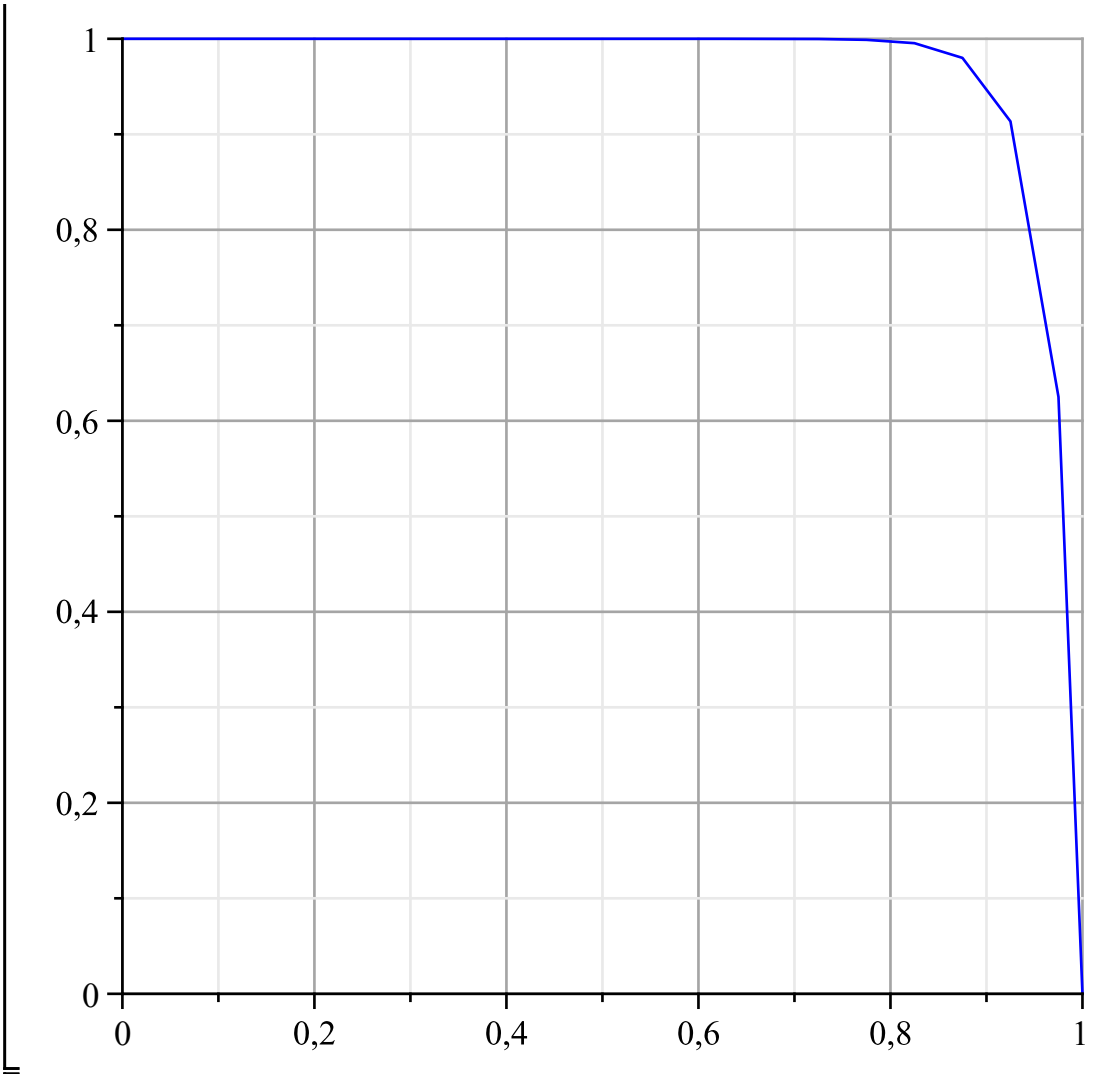
$$\phi_2 := 1.000$$
$$\phi_3 := 1.000$$
$$\phi_4 := 1.000$$
$$\phi_5 := 1.000$$
$$\phi_6 := 1.000$$
$$\phi_7 := 1.000$$
$$\phi_8 := 1.000$$
$$\phi_9 := 1.000$$
$$\phi_{10} := 1.000$$
$$\phi_{11} := 1.000$$
$$\phi_{12} := 1.000$$
$$\phi_{13} := 1.000$$
$$\phi_{14} := 0.9999$$
$$\phi_{15} := 0.9998$$
$$\phi_{16} := 0.9989$$
$$\phi_{17} := 0.9954$$
$$\phi_{18} := 0.9800$$
$$\phi_{19} := 0.9135$$
$$\phi_{20} := 0.6250 \qquad \textbf{(3.1.9)}$$

> $lpN := \left[seq\left(\left[x[i], \phi[i] \right], i = 0 .. i_{max} + 1 \right) \right]$

$lpN := [[0, 1.0], [0.02500, 1.000], [0.07500, 1.000], [0.1250, 1.000], [0.1750, 1.000], [0.2250, 1.000], [0.2750, 1.000], [0.3250, 1.000], [0.3750, 1.000], [0.4250, 1.000], [0.4750, 1.000], [0.5250, 1.000], [0.5750, 1.000], [0.6250, 1.000], [0.6750, 0.9999], [0.7250, 0.9998], [0.7750, 0.9989], [0.8250, 0.9954], [0.8750, 0.9800], [0.9250, 0.9135], [0.9750, 0.6250], [1.0, 0]]$ **(3.1.10)**

Courbe Numérique:

> $listplot(lpN, color = blue, gridlines = true)$

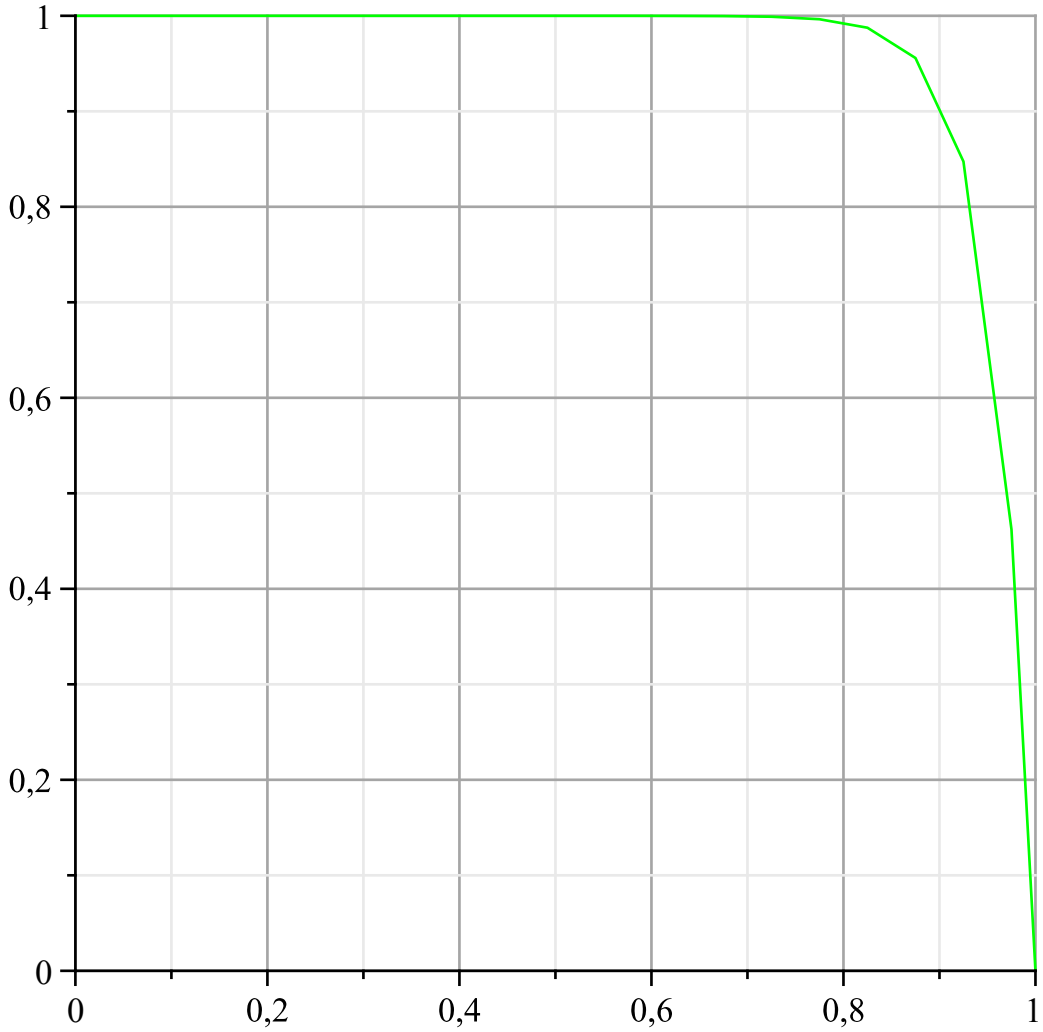


> $lpT := [seq([x[i], F(x[i])], i = 0..i_{max}+1)]$

lpT := [[0, 1.0], [0.02500, 1.0], [0.07500, 1.0], [0.1250, 1.0], [0.1750, 1.0], [0.2250, 1.0], [0.2750, 1.0], [0.3250, 1.0], [0.3750, 1.0], [0.4250, 1.0], [0.4750, 1.0], [0.5250, 1.0], [0.5750, 1.000], [0.6250, 0.9999], [0.6750, 0.9997], [0.7250, 0.9990], [0.7750, 0.9964], [0.8250, 0.9875], [0.8750, 0.9558], [0.9250, 0.8474], [0.9750, 0.4621], [1.0, 0.]] **(3.1.11)**

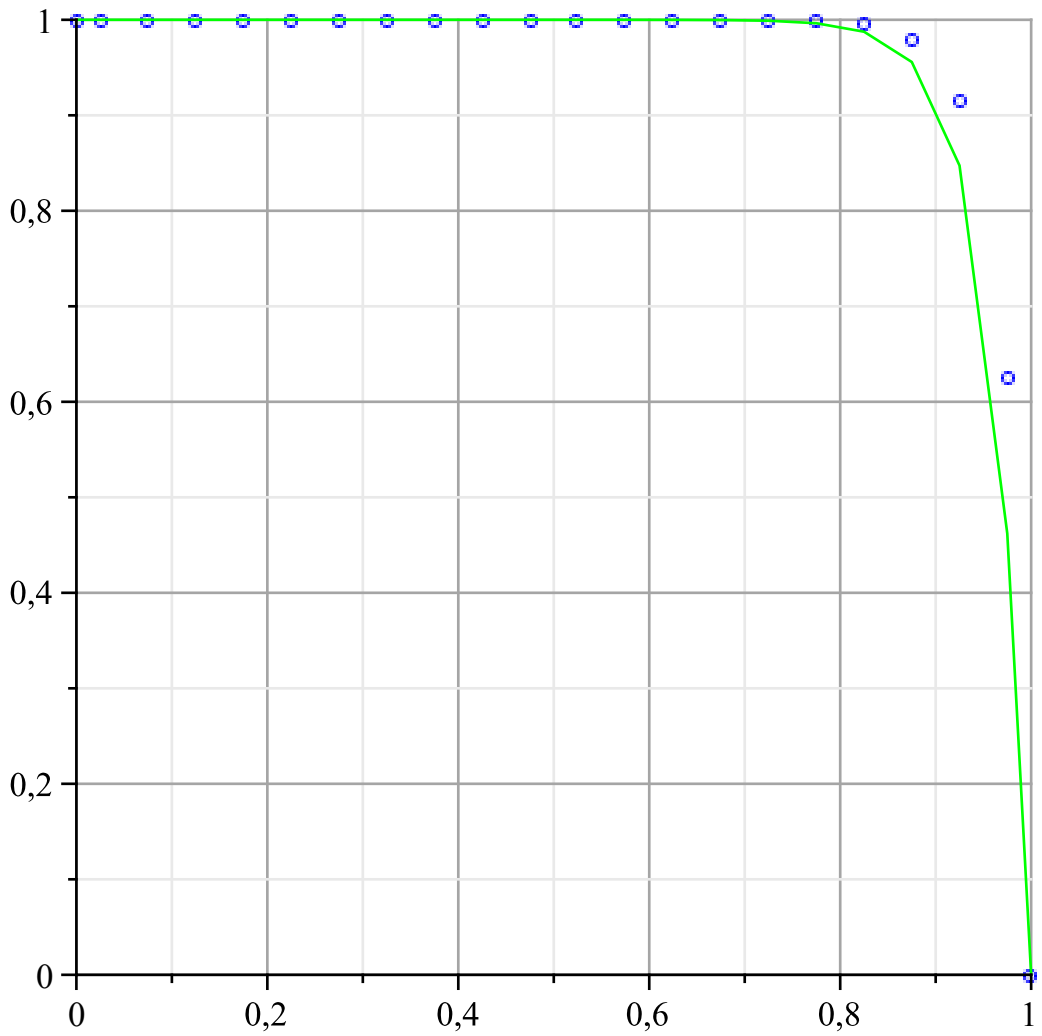
Courbe Théorique avec une liste de points:

> $listplot(lpT, color = green, gridlines = true)$



Tracé des deux courbes ensembles:

```
> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color
      = green, style = line], color = black, gridlines = true)
```

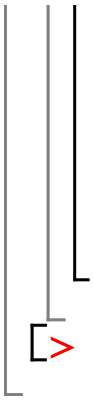


Erreur relative:

```
> for i from 1 to Ne do
  x[i];
  ϕ[i];
  F(x[i]);
  (ϕ[i] − F(x[i])) / F(x[i]) · 100
  end do
```

0.02500
1.000
1.0
0.
0.07500
1.000
1.0
0.
0.1250
1.000
1.0
0.
0.1750
1.000
1.0
0.

0.2250
1.000
1.0
0.
0.2750
1.000
1.0
0.
0.3250
1.000
1.0
0.
0.3750
1.000
1.0
0.
0.4250
1.000
1.0
0.
0.4750
1.000
1.0
0.
0.5250
1.000
1.0
0.
0.5750
1.000
1.000
0.
0.6250
1.000
0.9999
0.01000
0.6750
0.9999
0.9997
0.02001
0.7250
0.9998
0.9990
0.08008
0.7750
0.9989
0.9964
0.2509
0.8250
0.9954
0.9875
0.8000
0.8750
0.9800
0.9558
2.532



0.9250
0.9135
0.8474
7.800
0.9750
0.6250
0.4621
35.25

(3.1.12)